Heat cost of parametric generation of microwave squeezed states

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In parametric systems, squeezed states of radiation can be generated via extra work done by external sources. This eventually increases the entropy of the system despite the fact that squeezing is reversible. We investigate the entropy increase due to squeezing and show that it is quadratic in the squeezing rate and may become important in the repeated operation of tunable oscillators (quantum buses) used to connect qubits in various proposed schemes for quantum computing.

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I. INTRODUCTION

Parametric devices using the nonlinear inductance and natural protection from decoherence of Josephson junctions are considered as prospective elements of quantum circuits [1]. In particular, these have been proposed and implemented as quantum buses, providing tunable coupling between solid state qubits (see, e.g., Refs. [2–6]). On the other hand, parametric Josephson devices can produce squeezed states of microwave radiation [7–11]. Such experiments were performed in the 1980s [12,13] and more recently [14].

In this paper we investigate the generation of squeezed states during the operation of a quantum bus and estimate their additional contribution to the entropy of the system and associated heat production. We will see that this contribution is quadratic in the squeezing rate and should be taken into account if the system operates near its limit of efficiency determined by the Landauer erasure principle (see, e.g., Refs. [15,16]).

A quantum bus can be considered as a harmonic oscillator with tunable frequency $\omega(t)$; tuning it in and out of resonance with qubits allows the manipulation of their states (e.g., by performing two-qubit gates). The changes of $\omega(t)$ should be fast compared to the characteristic frequencies of qubits and the bus (i.e., $\dot{\omega}/\omega \gg \omega$); the decoherence rate in the circuit, however, must be much smaller than ω. But these are the very conditions that lead to the generation of squeezed states. Indeed, a coherent state of a harmonic oscillator is squeezed by a sudden change of the oscillator frequency [17]. Recall that a coherent state is a state with equal and minimal uncertainties: $\langle \Delta P^2\rangle = \langle \Delta Q^2\rangle = 1/2$, where $P = (a - a^\dagger)/i\sqrt{2}$ and $Q = (a + a^\dagger)/\sqrt{2}$ are dimensionless momentum and position expressed through bosonic creation and annihilation operators.

A. Wigner functions of oscillator states

It is convenient to describe oscillator states by their Wigner functions (see, e.g., Ref. [18])

$$W(\alpha,\alpha^*) = \frac{1}{2\pi^2}\int d\lambda\, d\lambda^*\, e^{(-\lambda\alpha^*+\lambda^*\alpha)}\,\mathrm{tr}[e^{(\lambda a^\dagger - \lambda^* a)}\rho], \quad (1)$$

which have the advantage of reducing to classical distribution functions in the classical limit. Here ρ is the density matrix of the system. For a coherent state, the Wigner function is Gaussian,

$$W_{\alpha_0}(\alpha,\alpha^*) = \frac{2}{\pi}\exp[-2|\alpha-\alpha_0|^2].$$

The amplitude and phase of the complex parameter α_0 (which in the Schrödinger representation behaves as $\alpha_0(t) = \alpha_0(0)\exp[-i\omega t]$) describe the classical limit of the oscillator state. A squeezed state will have instead the Wigner function

$$W(\alpha,\alpha^*) = \frac{2}{\pi}\exp\Big\{-2s\,[(x-x_0)\cos\theta + (y-y_0)\sin\theta]^2 - \frac{2}{s}[(y-y_0)\cos\theta - (x-x_0)\sin\theta]^2\Big\}. \quad (2)$$

Here $x = \mathrm{Re}\,\alpha$, $y = \mathrm{Im}\,\alpha$, s is the squeezing parameter, and θ determines the direction of the squeezing axis. Similarly, the thermal state

$$W_{\mathrm{th}}(\alpha,\alpha^*) = \frac{2/\pi}{1+2\bar{n}}\exp[-2|\alpha|^2/(1+2\bar{n})],$$

characterized by the average photon number $\bar{n} = \{\exp[\omega/T] - 1\}^{-1}$, can be squeezed to [19]

$$W(\alpha,\alpha^*) = \frac{2/\pi}{1+2\bar{n}}\exp\left\{-\frac{2}{1+2\bar{n}}\left[s(x\cos\theta + y\sin\theta)^2 + \frac{1}{s}(y\cos\theta - x\sin\theta)^2\right]\right\}. \quad (3)$$

II. SQUEEZING OF OSCILLATOR STATES BY SUDDEN FREQUENCY CHANGE

After being introduced in Ref. [17], squeezing the oscillator states by a sudden change of oscillator frequency was considered in a number of papers [8,11,20–24]. In particular, it was shown [11,21,24] that, in the absence of decoherence, repeated abrupt small changes of the oscillator frequency can produce arbitrarily large squeezing. Here we will mainly follow the approach of [11].

Consider an arbitrary time dependence of the system frequency $\omega(t)$. Let us also denote the creation and annihilation operators belonging to the Fock state of an oscillator with the frequency $\omega(t=0)$ by a_0, $a_0^\dagger$, and those corresponding to $\omega(t)$, by a_ω, $a_\omega^\dagger$. The Hamiltonian keeps its standard form,

$$H(t) = \hbar\omega(t)\left(a_\omega^\dagger a_\omega + \tfrac{1}{2}\right), \tag{4}$$

and the commutation relations between a_ω, $a_\omega^\dagger$ hold, if the old and new operators are related via a Bogoliubov transformation

$$a_\omega = \frac{[\omega(t)+\omega(0)]a_0 - [\omega(t)-\omega(0)]\,a_0^\dagger}{2\sqrt{\omega(t)\omega(0)}}, \tag{5}$$

which can be explicitly written as [17]

$$a_0 = V^\dagger(t) a_\omega V(t); \quad a_0^\dagger = V^\dagger(t) a_\omega^\dagger V(t), \tag{6}$$

where

$$\begin{aligned} V(t) &= \exp\left\{-\frac{1}{4}\left[\ln\frac{\omega(0)}{\omega(t)}\right]\left[a_\omega^2 - (a_\omega^\dagger)^2\right]\right\}; \\ V^\dagger(t) &= \exp\left\{\frac{1}{4}\left[\ln\frac{\omega(0)}{\omega(t)}\right]\left[a_\omega^2 - (a_\omega^\dagger)^2\right]\right\}. \end{aligned} \tag{7}$$

In order to work in the initial Fock space at $t = 0$, we apply the transformation $V^{-1}(t) = V^\dagger(t)$, which gives

$$\begin{aligned} H(t) \to \tilde{H}(t) &= V(t)H(t)V^\dagger(t) - i\hbar V(t)\frac{\partial}{\partial t}V^\dagger(t) \\ &= \hbar\omega(t)\left(a_0^\dagger a_0 + \frac{1}{2}\right) + i\hbar\frac{\dot{\omega}(t)}{\omega(t)}\left[a_0^2 - (a_0^\dagger)^2\right]. \end{aligned} \tag{8}$$

Hereafter we correct an error made in Ref. [11], where the transformation (8) was effectively applied twice in the same direction, which quantitatively (but not qualitatively) affected the results. For the Wigner function, with α, α^* always referring to the coherent states in the same Fock space (at $t = 0$), we find the master equation (Ref. [11], *mutatis mutandis*)

$$\begin{aligned} \frac{\partial}{\partial t}W(\alpha,\alpha^*,t) &= 2\omega(t)\mathrm{Im}\left(\alpha^*\frac{\partial}{\partial\alpha^*}\right)W(\alpha,\alpha^*,t) \\ &\quad + \frac{\partial\ln\omega(t)}{\partial t}\mathrm{Re}\left(\alpha\frac{\partial}{\partial\alpha^*}\right)W(\alpha,\alpha^*,t), \end{aligned} \tag{9}$$

or

$$\begin{aligned} \frac{\partial}{\partial t}W(x,y,t) &= \omega(t)\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)W(x,y,t) \\ &\quad + \frac{1}{2}\frac{\partial\ln\omega(t)}{\partial t}\left(x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}\right)W(x,y,t). \end{aligned} \tag{10}$$

We did not include the diffusive terms, which describe decoherence (including relaxation), thus assuming that the system maintains coherence over many cycles of frequency switching and of its free evolution with frequency $\omega(t)$. Equation (10) is a first-order linear equation and can be solved by the method of characteristics. The characteristic equations are

$$\frac{dx}{dt} = \frac{\dot{\omega}}{2\omega}x - \omega y; \quad \frac{dy}{dt} = \omega x - \frac{\dot{\omega}}{2\omega}y. \tag{11}$$

In the limit of fast frequency change, when the $\dot{\omega}$ terms dominate, these equations lead to squeezing of the quantum state:

$$\frac{d\ln x}{dt} \approx \frac{d\ln\sqrt{\omega}}{dt}; \quad \frac{d\ln y}{dt} \approx -\frac{d\ln\sqrt{\omega}}{dt}, \tag{12}$$

so that $W(x,y,\Delta t) \approx W(\sqrt{s}x, \frac{1}{\sqrt{s}}y, 0)$, with squeezing parameter $s = \omega(\Delta t)/\omega(0)$, and Δt the duration of the fast frequency change [cf. Eqs. (2) and (3)]. In the quasistatic limit, Eq. (11) simply describes the rotation of the Wigner function as a whole: slow changes of parameters do not produce any squeezing, as expected.

III. EFFECT OF REPEATED FREQUENCY CHANGES

As we mentioned above, in the absence of decoherence, periodic changes of the oscillator frequency can produce an arbitrarily high degree of squeezing, even if the difference between the two limiting values, ω_0 and ω_1, is arbitrarily small. It is only necessary that at least one of the transitions ($\omega_0 \to \omega_1$ or $\omega_0 \to \omega_1$) is fast on the scale of ω, and these transitions are tuned to the phase of the oscillator. This may also take place during the operation of a quantum bus, which would normally switch fast on- and off-resonance with the qubits coupled to it. As we shall see, there are entropy costs associated with the production of squeezed states. Therefore this side effect should be avoided in the operation of quantum buses.

Let us return to the characteristics equation (11). Without loss of generality, we can consider two regimes of oscillator frequency switches $\omega_0 \leftrightarrow \omega_1$: the ratchet regime (when one transition, e.g., $\omega_0 \to \omega_1$, is fast, and the opposite one is slow [11,24]), and the seesaw regime (when both transitions are fast [21]). In either case, if two consecutive fast switches occur at the moments t_{n-1} and t_n, the point on a characteristic will evolve according to

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix}_{t_n+0} &= \begin{pmatrix} \sqrt{s_n}\cos\theta_{n,n-1} & \sqrt{s_n}\sin\theta_{n,n-1} \\ -\frac{1}{\sqrt{s_n}}\sin\theta_{n,n-1} & \frac{1}{\sqrt{s_n}}\cos\theta_{n,n-1} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}_{t_{n-1}+0} \\ &\equiv \Lambda_n \begin{pmatrix} x \\ y \end{pmatrix}_{t_{n-1}+0}. \end{aligned} \tag{13}$$

Here $\theta_{n,n-1}$ is the phase angle accumulated during the period of slow evolution, and s_n is the squeezing achieved at the nth step. In the ratchet regime: $s_n = s = \omega_1/\omega_0$ (or vice versa), while in the seesaw case: $s_{2n} = 1/s_{2n+1} = s$. Obviously, $\det\Lambda_n = 1$. After two consecutive switches

$$(\Lambda_n\Lambda_{n-1})^{\mathrm{r}} = \begin{pmatrix} (s-1)\cos\theta_{n,n-1}\cos\theta_{n-1,n-2} + \cos\theta_{n,n-2} & (s-1)\cos\theta_{n,n-1}\sin\theta_{n-1,n-2} + \sin\theta_{n,n-2} \\ -\left(\frac{1}{s}-1\right)\cos\theta_{n,n-1}\sin\theta_{n-1,n-2} - \sin\theta_{n,n-2} & \left(\frac{1}{s}-1\right)\cos\theta_{n,n-1}\cos\theta_{n-1,n-2} + \cos\theta_{n,n-2} \end{pmatrix} \tag{14}$$

and

$$(\Lambda_n\Lambda_{n-1})^{\mathrm{s}} = \begin{pmatrix} -(s-1)\sin\theta_{n,n-1}\sin\theta_{n-1,n-2} + \cos\theta_{n,n-2} & (s-1)\sin\theta_{n,n-1}\cos\theta_{n-1,n-2} + \sin\theta_{n,n-2} \\ -\left(\frac{1}{s}-1\right)\sin\theta_{n,n-1}\cos\theta_{n-1,n-2} - \sin\theta_{n,n-2} & -\left(\frac{1}{s}-1\right)\sin\theta_{n,n-1}\sin\theta_{n-1,n-2} + \cos\theta_{n,n-2} \end{pmatrix}. \tag{15}$$

Here $\theta_{n,n-2} = \theta_{n,n-1} + \theta_{n-1,n-2}$. In the case of periodic switchings, $\theta_{n+1,n} = \Theta$, this reduces to

$$(\Lambda_n \Lambda_{n-1})^{\mathrm{r}} = \begin{pmatrix} (s-1)\cos^2\Theta + \cos 2\Theta & \frac{s+1}{2}\sin 2\Theta \\ -\frac{1/s+1}{2}\sin 2\Theta & \left(\frac{1}{s}-1\right)\cos^2\Theta + \cos 2\Theta \end{pmatrix} \tag{16}$$

and

$$(\Lambda_n \Lambda_{n-1})^{\mathrm{s}} = \begin{pmatrix} -(s-1)\sin^2\Theta + \cos 2\Theta & \frac{s+1}{2}\sin 2\Theta \\ -\frac{1/s+1}{2}\sin 2\Theta & -\left(\frac{1}{s}-1\right)\sin^2\Theta + \cos 2\Theta \end{pmatrix}. \tag{17}$$

The resonance conditions, $\Theta^{\mathrm{r}} = \pi q$ and $\Theta^{\mathrm{s}} = \pi(q + 1/2)$, when both (16) and (17) diagonalize, ensure that after $2N$ switchings the state will be squeezed exponentially to an arbitrary degree,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \pm s & 0 \\ 0 & \pm\frac{1}{s} \end{pmatrix}^N \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \pm \begin{pmatrix} s^N x_0 \\ s^{-N} y_0 \end{pmatrix}, \tag{18}$$

no matter how small $(s-1)$. For finite values of $(s-1)$, an exact resonance is not necessary. For example, in the ratchet case a runaway squeezing will happen if [24]

$$s > s_c = \frac{1 + |\sin\Theta|}{|\cos\Theta|}. \tag{19}$$

Deviations from periodic switching in the operation of a quantum bus, fluctuations of circuit parameters, and eventually relaxation and decoherence in the system will limit the actual degree of squeezing. Nevertheless, the very operation of a quantum bus will tend to produce squeezed states.

IV. THERMODYNAMIC COSTS OF SQUEEZING

Squeezing is in itself a reversible process. Nevertheless, in the presence of decoherence it will lead to an increase of the system's entropy, which can be associated with internal friction [25]. To be specific, consider the so-called energy entropy [25,26]

$$S_E[\rho] \equiv -\sum_n p_n \ln p_n \geqslant S[\rho] \equiv -\mathrm{tr}\,[\rho \ln \rho]\,, \tag{20}$$

where ρ is the density matrix, $\{p_n\}_{n=0}^{\infty}$ are its diagonal terms in the energy basis, and S is the standard (fine-grained) von Neumann entropy. The two entropies coincide if and only if ρ commutes with the Hamiltonian. Obviously, unlike S, S_E can increase in a closed system. This property of S_E makes it analogous to the coarse-grained entropy introduced in the classical case by Gibbs [27], as opposed to the fine-grained classical entropy, which also does not change in a closed system due to Liouville's theorem. Some coarse-graining procedure is therefore essential for the description of closed systems, be they classical or quantum. While there exist different ways of introducing coarse-grained quantum entropy (see, e.g., Ref. [28]), the energy entropy is perhaps the simplest and most logical choice, given the special role played by the Hamiltonian and its eigenstates. The von Neumann entropy naturally approaches S_E as the decoherence processes wipe out the off-diagonal terms of the density matrix in the energy representation. The additional advantage of using S_E in our case is that it can be conveniently expressed through the Wigner function [29], since

$$\begin{aligned} p_n &= 2(-1)^n \int d\alpha\, d\alpha^*\, e^{-2|\alpha|^2}\, L_n(4|\alpha|^2)\, W(\alpha,\alpha^*) \\ &= 2(-1)^n \iint dx\, dy\, e^{-2(x^2+y^2)}\, L_n[4(x^2+y^2)]\, W(x,y). \end{aligned} \tag{21}$$

Here $L_n(x)$ is the Laguerre polynomial. We can associate with the additional entropy, $\delta S(s) = S_E(s) - S_E(0)$, of a squeezed state a specific amount of heat,

$$\delta Q(s) = T \quad \delta S_E(s), \tag{22}$$

which will be eventually released into the system.

As an example, consider a squeezed thermal state (3), for which an exact solution can be actually found [29] in terms of the hypergeometric function [30]

$$\begin{aligned} p_n^{\mathrm{th}} &= \kappa \sum_{q=0}^{n} C_n^q (-1)^q \left(\frac{2}{1+\kappa/s}\right)^{n+1-q} \\ &\quad \times {}_2\mathrm{F}_1\left(\frac{1}{2}, n+1-q; 1; -\frac{\kappa(s-1/s)}{1+\kappa/s}\right), \end{aligned} \tag{23}$$

where $\kappa = \tanh(\omega/2T)$. A more convenient approximation, valid for small squeezing, $\kappa(s-1) \ll 1$, is given by

$$\begin{aligned} p_n &\approx \frac{p_n^{\mathrm{eq}}}{C_1(\varepsilon)} I_0\left(\kappa\varepsilon\left(\frac{n}{1-\kappa} + \frac{n+1}{1+\kappa}\right)\right) \\ &\approx \frac{p_n^{\mathrm{eq}}}{C_2(\varepsilon)} I_0\left(\frac{\omega\varepsilon}{T}(n+1/2)\right). \end{aligned} \tag{24}$$

Here $p_n^{\mathrm{eq}} = (1 - \exp[-\omega/T])\exp[-n\omega/T]$ is the equilibrium population of the nth energy level of the oscillator, $\varepsilon \equiv |s-1|$ characterizes squeezing, $I_0(z)$ is the modified Bessel function, and the last expression in Eq. (24) is valid for $\omega/T \ll 1$. The normalization constants $C_{1,2} = 1 + O(\varepsilon^2)$. The occupation of states with numbers $n < N_0$ will initially decrease, and of those with numbers $n > N_0$ increase with squeezing. Expanding the Bessel function, $I_0(z) = 1 + z^2/4 + \cdots$, we see that $d(p_n - p_n^{\mathrm{eq}})/d\varepsilon = 0$, if

$$n = N_0 \equiv \frac{2T}{\omega} - \frac{1}{2}. \tag{25}$$

This is in good agreement with the results based on the exact formula in Eq. (23) (Fig. 1).

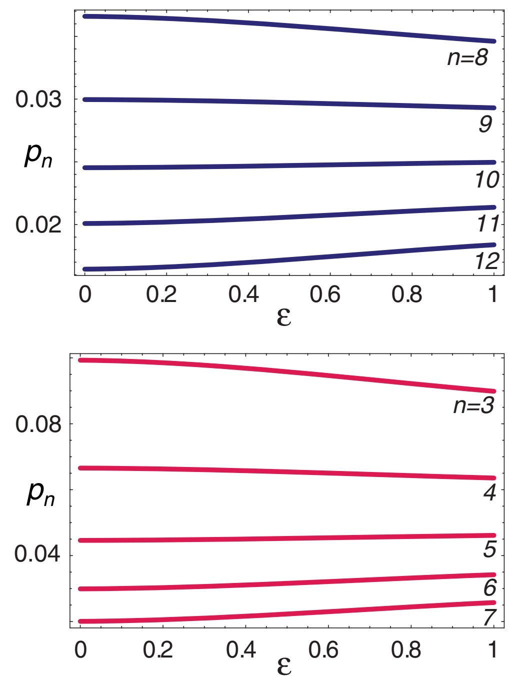

FIG. 1. (Color online) Occupation numbers of the energy levels of a harmonic oscillator in a squeezed thermal state [Eq. (23)] as a function of squeezing $\varepsilon = |s-1|$. (a) $\omega/T = 0.2$, estimated boundary number $N_0 = 2T/\omega - 1/2 = 9.5$; levels (top to bottom) 8, 9, 10, 11, 12. (b) $\omega/T = 0.4$, $N_0 = 4.5$; levels (top to bottom) 3, 4, 5, 6, 7.

The corresponding change in entropy is given by

$$\begin{aligned}\delta S_E(\varepsilon) &= -\sum_n \delta p_n \ln p_n^{\mathrm{eq}} = -\frac{\varepsilon^2}{16} \frac{(\omega/T)^2}{(1-\exp[-\omega/T])^2} \\ &\quad \times \left\{ \ln(1-e^{-\omega/T})(1+6e^{-\omega/T}+e^{-2\omega/T}) \right. \\ &\quad \left. - \frac{\omega}{T} \frac{9e^{-\omega/T}+14e^{-2\omega/T}+e^{-3\omega/T}}{1-e^{-\omega/T}} \right\} \\ &\approx -\frac{\varepsilon^2}{2}\left\{ \left(1-\frac{\omega}{T}\right)\ln\frac{\omega}{T} - 3 + 5\frac{\omega}{T} \right\}\left(\frac{\omega}{T} \ll 1\right). \end{aligned} \tag{26}$$

With the same accuracy, the equilibrium entropy of an unsqueezed thermal state becomes

$$\begin{aligned} S &= -\ln[2\sinh(\omega/2T)] + (\omega/2T)\coth(\omega/2T) \\ &\approx -\ln(\omega/T) + 1. \end{aligned}$$

Therefore in the leading term in $T/\omega \gg 1$ the squeezing contribution to the entropy is

$$\delta S_E(\varepsilon) \sim \frac{\varepsilon^2}{2} S, \tag{27}$$

and the amount of additional heat released into the system due to squeezing will be

$$\delta Q \sim T\,\frac{\varepsilon^2}{2} S = \frac{(s-1)^2}{2} TS \tag{28}$$

per cycle. Detuning between the bus and qubits can be typically 10–30% [6], making feasible the prefactors in Eqs. (27) and (28) as high as 0.25 (i.e., the contribution of squeezing to the system entropy becomes comparable to the equilibrium entropy of the device). Of course, this contribution becomes important only when the system operates near the Landauer erasure limit, with $Q_L = T\ln 2$ [15,16]. If a need ever arises to limit its effect, the quadratic dependence of δQ on the squeezing parameter indicates that even a slight decrease of the difference between the operating frequencies of the bus will drastically reduce it. One also should avoid a periodic operation of the bus, to exclude the resonant increase of the squeezing rate.

V. CONCLUSION

We have shown that the operation of a quantum bus will lead to squeezing of its quantum state and calculated the corresponding additional heat, which will be injected into the system. This contribution is quadratic in the squeezing rate (proportional to the ratio of the working frequencies of the bus) and can become important only in systems operating either near the Landauer erasure limit or under the conditions when there is a resonant increase of squeezing by repeated bus operation.

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