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Modulated electromechanics: large enhancements of nonlinearities

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Abstract
It is well-known that the nonlinear coupling between a mechanical oscillator and a superconducting resonator or optical cavity can be used to generate a Kerr-nonlinearity for the cavity mode. We show that the strength of this Kerr-nonlinearity, as well as the effect of the photon-pressure force can be enormously increased by modulating the strength of the nonlinear coupling. We describe an electromechanical circuit where this enhancement could be readily realized.

Keywords: electromechanics, modulation enhancement, optical nonlinearity

1. Introduction

Micro- and nano-fabricated electromechanical systems have been realized in the laboratory [1–4] using superconducting microwave resonant cavities coupled to tiny, low-frequency mechanical oscillators. The coupling between a single LC-resonator (the cavity) and a mechanical oscillator is nonlinear because the frequency of the LC-resonator depends on the position of the oscillator. This nonlinear coupling is the same as that in opto-mechanical systems, in which one of the mirrors of an optical cavity is allowed to oscillate [2, 5].

Experiments to-date in superconducting and optical systems have exploited the mechanical interaction in its linear regime to great effect, realizing, e.g., high-precision measurements [6, 7], cooling [8, 9], state-transfer [10, 11], amplification [12] and slow light [13]. A further set of quantum effects could be realized in the nonlinear regime [14–22], of which we consider here the Kerr effect and the single-photon pressure force. The main obstacle to observing these effects is that they occur much more slowly than the coupling rate for the nonlinear interaction itself and typical dissipation rates of the electrical or optical cavity. If we denote the coupling rate by g (defined precisely below), then the rate of the induced Kerr nonlinearity is (g^2/Ω), where Ω is the mechanical frequency, and the displacement of the mechanical resonator induced by the photons is proportional to (g/Ω) [14, 15]. Since in present systems $g \ll \kappa \ll \Omega$, where κ is the photon loss rate, these nonlinear effects are negligible.

The Kerr nonlinearity is interesting because it can be used to prepare nonclassical states—in particular superpositions of coherent states—of the cavity mode, and these can also be transferred to the mechanical mode [11]. The photon-pressure force can be used to observe the mechanical effects of light, but it also creates entanglement between the Fock states of the LC-resonator and coherent states of the mechanics. This could potentially be used to perform quantum non-demolition measurements of the photons via the mechanics, and to probe foundational questions in quantum theory [23, 24].

Here, we show that for electro-mechanical systems the Kerr nonlinearity and the displacement induced by the photon-pressure force can be enhanced by a factor of $\Omega/g \sim 10^2 - 10^4$ by modulating g at a frequency close to the mechanical frequency. As described below, this can be achieved by modulating the cavity frequency. (We note that this technique could also be applied to opto-mechanical systems if g could be modulated with sufficient amplitude. While at present this is precluded in opto-mechanics by the relatively large ratio of cavity length to wave-length, it might be possible in recently demonstrated cavities nano-fabricated in silicon [25].)

Given the state-of-the art in experimental technology, with the enhancement provided by the modulation, we expect that it will be feasible to observe signatures of the nonlinear interaction. However, we find that to generate and observe photon-phonon entanglement, an increase is still required in the nonlinear coupling rate g and/or in the lifetime $1/\kappa$ of the superconducting resonator. Specifically, the ratio g/κ needs to be increased by a factor of about 10^3.

We can understand why modulating the electro-mechanical coupling enhances the nonlinear effects by considering the force exerted by the light on the mechanics. When there are n photons in the cavity this force is $F \propto (n + 1/2)$, where the 1/2 comes from the zero-point energy that contributes to the Casimir force [26]. For fixed n the force F is constant and so is far off-resonance with the mechanics. If we modulate g then we imprint an oscillation onto F so that it can drive the mechanics at or near its resonant frequency, greatly enhancing the effect of the photons (and the zero-point fluctuations) on the mechanics. This enhanced effect then acts back on the LC-resonator generating an enhanced Kerr nonlinearity. Note that the zero-point fluctuations do not drive the mechanics, but merely transfer the drive applied to g onto the mechanics.

Recently a number of schemes have been presented to enhance the opto-mechanical interaction by using more than one optical (or electrical) mode [27–29]. These show that two optical modes can be used to enhance the nonlinearities. The analysis in [30] suggests another way to understand the method we present here: the modulation is equivalent to adding a third oscillator at the modulation frequency. The present scheme can therefore be viewed as adding a third classical mode to the configuration, rather than the third quantum mode used in [27–29]. Conversely the method we present here reveals that the schemes in [27–29] can also be understood in terms of resonance; they effectively create a new mode that is on or near resonance with the mechanics.

In the next section we review the electromechanical/optomechanical coupling Hamiltonian, and explain how it generates an effective Kerr nonlinearity and displaces the mechanical resonator via the photon-pressure force. In section 3 we show how the modulation of g enhances the Kerr nonlinearity and the photon-pressure induced displacement. In section 4 we discuss the requirements on the resonator and cavity damping rates to observe the photon-induced displacement of the mechanics. In section 5 we consider two methods for modulating g in electro-mechanical systems. Using the second of these methods we present a readily realizable circuit where the affects of photon-pressure force are enhanced by more than three orders of magnitude. Section 6 concludes with a brief summary.

2. The electromechanical/optomechanical Hamiltonian

The electromechanical coupling is given by the Hamiltonian H, where [31]

$$H/\hbar = \omega a^\dagger a + \Omega b^\dagger b + g\left(a^\dagger a + 1/2\right)\left(b + b^\dagger\right), \tag{1}$$

in which a and b are the annihilation operators for the LC-resonator and mechanical modes, respectively, ω and Ω are their respective frequencies, and g is the elecro-mechanical coupling rate. The final term, $(g/2)(b + b^\dagger)$, is the contribution from the zero-point fluctuations of the LC-resonator. It is often discarded but is required in our analysis here. The unitary evolution operator generated by this Hamiltonian, $U(t) = e^{-iHt/\hbar}$ can be written, up to a phase factor, in the form [14, 15]:

$$\begin{aligned} U(t) = \exp\left[-i(\omega t - \mu)a^\dagger a\right] \exp\left[i\mu\left(a^\dagger a\right)^2\right] \\ \times \exp\left[-i\left(a^\dagger a + 1/2\right)\left(\lambda_x x - \lambda_p p\right)\right] \exp\left(-i\Omega b^\dagger b t\right), \end{aligned} \tag{2}$$

with $\lambda_x = (g/\Omega)\sin(\Omega t)$, $\lambda_p = (g/\Omega)[1 - \cos(\Omega t)]$, and

$$\mu = \left(g^2/\Omega\right)[t - \sin(\Omega t)/\Omega], \tag{3}$$

and we have defined the dimensionless mechanical position and momentum operators by $x \equiv b + b^\dagger$ and $p \equiv -i(b - b^\dagger)$. Two key effects can be read off from U. The first is that the electrical mode displaces the mechanical mode by the (dimensionless) phase-space distance $\Delta s \equiv \sqrt{\Delta x^2 + \Delta p^2} = (g/\Omega)(4n + 2)$, at time $t = \pi/\Omega$, where n is the number of photons in the electrical mode. The second effect is that at times $\tau = 2\pi m/\Omega$, for integer m, the electrical mode undergoes the evolution

$$U(\tau) = \exp\left[-i(\omega - \chi)a^\dagger a\tau + i\chi\left(a^\dagger a\right)^2\tau\right], \tag{4}$$

where the size of the effective Kerr-nonlinearity is $\chi = g^2/\Omega$. Both the displacement of the mechanical mode, and the Kerr-evolution contain the small factor g/Ω. If we examine the opto-mechanical H above, and move into the interaction picture, then the interaction is

$H_{\rm I} = \hbar g(a^{\dagger}a + 1/2)(be^{-i\Omega t} + b^{\dagger}e^{i\Omega t})$. Both terms oscillate at the mechanical frequency Ω, and are therefore off-resonant for $g \ll \Omega$. As a result the rotating-wave approximation for $g \ll \Omega$ eliminates the interaction.

3. Modulating the electromechanical coupling

If we modulate the interaction rate g so that

$$g \to \tilde{g}(t) = g\cos(\nu t), \tag{5}$$

with $\nu = \Omega - \delta$, then we can bring the interaction near to resonance, with the remaining detuning equal to δ. If we then move into the interaction picture with respect to the Hamiltonian $H(\nu) = \hbar\nu b^{\dagger}b$, and make the rotating-wave approximation ($g \ll \nu$), the effective Hamiltonian for the joint system is that given by H but with Ω replaced by δ and g replaced by $g_{\rm eff} = g/2$. By choosing $\delta = 2g_{\rm eff}$, the rate of the Kerr nonlinearity becomes

$$\chi = \frac{g^2}{4\delta} = \frac{g}{4} \gg \frac{g^2}{\Omega}, \tag{6}$$

and the mechanical displacement is similarly

$$\Delta s = \frac{1}{2}(4n + 2) \gg \frac{g}{\Omega}(4n + 2). \tag{7}$$

The above magnification of the nonlinear effects is potentially very large, but given the dependence of these rates on δ we might wonder if they could be increased even more by reducing δ further. The choice $\delta = 2g_{\rm eff}$ is not optimal for the photon-pressure force (see below) but it is optimal for the Kerr term, which we now show. First, consider choosing some number r greater than 1/2, and setting $\delta = g_{\rm eff}$. The Kerr term in the evolution operator U now has $\chi = rg_{\rm eff}$, but there is a catch. We must wait for the two oscillators to decouple and this takes the minimum time $\tau = r2\pi/g_{\rm eff}$, which increases with r. As an example, if we want to use the Kerr term to prepare the superposition

$$|\mathrm{cat}(\alpha)\rangle \equiv (|\alpha\rangle - i|-\alpha\rangle)/\sqrt{2}, \tag{8}$$

where $|\pm\alpha\rangle$ denotes a coherent state with complex amplitude $\pm\alpha$, then we need $\chi\tau = \pi/2$. If we choose r to minimize the time taken, then the minimum is at $r = 1/2$, the effective value of χ is $g_{\rm eff}/2 = g/4$, and the time taken to prepare the cat state is $\tau = \pi/g_{\rm eff}$. For preparing $|\mathrm{cat}(\alpha)\rangle$ in the shortest time, the best Kerr rate is therefore half the maximum available.

The above analysis does not exclude the possibility that a shorter time might be obtained by allowing $g_{\rm eff}$ to be an arbitrary function of time. To answer this question we perform a numerical search for time-dependent control strategies (that is, ways to change $g_{\rm eff}$ and δ with time) to generate the unitary $V = \exp[i\pi(a^{\dagger}a)^2/2]$ in the minimum time. If we can prepare $|\mathrm{cat}(\alpha)\rangle$ in time τ, then the realizable Kerr rate is $\chi = \pi/(2\tau)$. To do this we divide the time interval $[0, \tau]$ up into N segments, and allow $g_{\rm eff}$ and δ to take a different value on each segment. We then perform a gradient search to find an optimal set of values for $g_{\rm eff}$ and δ, given a maximum value for $g_{\rm eff}$. Since the system consists of two oscillators the state space is potentially large. Fortunately, the problem allows a simplification: the Hamiltonian commutes with $a^{\dagger}a$, and so preserves the populations of the number states. This means that we lose no accuracy in truncating the LC-resonator in the number basis. The mechanical oscillator on the other hand requires a much larger state-space, because the evolution generates coherent states for this oscillator. To perform the numerical optimization we use just three number states for the LC-resonator, thirty for the mechanics, and set $g_{\rm eff} = \pi$. We choose an arbitrary initial state for the LC-resonator, $|\psi(0)\rangle$, and use the gradient search to maximize the fidelity between $|\psi(\tau)\rangle$ and the desired final state $V|\psi(0)\rangle$, for a range of values of τ. Here, the fidelity between two density matrices ρ and σ is $F = \mathrm{Tr}\,[\sqrt{\sigma^{1/2}\rho\sigma^{1/2}}]$. When the two states are pure this reduces to the absolute value of their inner product [32]. We also run the optimization with two values of N ($N = 10$ and $N = 15$) to ensure that N does not limit the fidelity. We find that for $\tau \geqslant 1$ we can always obtain a fidelity equal to unity, with essentially arbitrary accuracy. As soon as we set $\tau < 1$ this is no longer possible. If we define our figure of merit as $\varepsilon = 1 - F$, where F is the fidelity, then for $\tau = 0.99, 0.95, 0.9, 0.8$ we obtain $\varepsilon = 1.1 \times 10^{-4}, 2.7 \times 10^{-3}, 1 \times 10^{-2}, 3.6 \times 10^{-2}$. This clear change in behavior around $\tau = 1$ gives us considerable confidence that $\chi = g_{\rm eff}/2$ is the maximum effective Kerr rate for the purposes of preparing $|\mathrm{cat}(\alpha)\rangle$.

The situation regarding the photon-pressure force is a little different. In this case, the interaction causes the mechanics to be driven by a force proportional to the number of photons, n, in the LC-resonator. To make the most of this force we should arrange it to drive the mechanical oscillator at its resonance, and we do this by choosing $\delta = 0$. The resulting evolution of the (dimensionless) mechanical momentum operator, in the interaction picture, is

$$p(t) = p(0) - g_{\rm eff}\left(a^{\dagger}a + 1/2\right)t. \tag{9}$$

The upper limit to the phase-space displacement is now only that imposed by the damping rate of the mechanics, γ. The steady-state displacement is

$$\Delta s_{\rm ss} = \left|p(t)_{\rm ss} - p(0)\right| = (g_{\rm eff}/\gamma)(n + 1/2), \tag{10}$$

where n is the number of photons in the resonator. Equation (10) assumes that the resonator has reached its steady-state for a given photon number. We note that since the damping of the LC-resonator is faster than the mechanics, obtaining the steady-state for a fixed n would require repeated re-initialization of the LC-resonator in the number state n. Note that the photon force generates a coherent state of the mechanical oscillator. If the oscillator starts in the vacuum state, then this coherent state is $|\beta(t)\rangle$, with $\beta(t) = -i(g_{\rm eff}/2)(2n + 1)t$. The average number of phonons in the coherent state is then $|\beta|^2 = [g_{\rm eff}(2n + 1)t/2]^2$, and the steady-state value is $|\beta|^2_{\rm ss} = [g_{\rm eff}(2n + 1)/(4\gamma)]^2$.

Note that to obtain the largest nonlinear enhancement, for a given value of the interaction rate g, the amplitude of the modulation would need to be equal to g. If this modulation amplitude is instead ηg, so that $g(t) = g[1 + \eta\cos(\nu t)]$, then the resulting Kerr rate and the maximum mechanical displacement are obtained merely by replacing $g_{\rm eff} = g/2$ with $g_{\rm eff} = \eta g/2$. For the Kerr rate we still pick $\delta = g_{\rm eff}$ to achieve the maximum value $\chi = g_{\rm eff} = \eta g/2$, while the photon-pressure force becomes $\mathrm{d}p/\mathrm{d}t = -(\eta g/2)(n + 1/2)$.

4. Effects of the resonator and cavity damping

In the above analysis we have assumed that the evolution of the resonator and cavity mode is unitary on the timescale of the coupling rate g. While this will be a good approximation so long as the resonator and cavity damping rates, denoted respectively by γ and κ, are much smaller than g, current experiments are not yet in this regime. It is therefore worth elucidating the limiting effects of these damping rates when $\gamma, \kappa \lesssim g$. Damping of the resonator and cavity mode has two primary effects, one being to reduce the number of photons in these oscillators at the respective rates $n\gamma$ and $n\kappa$ (pure damping), where n is the number of phonons/photons, and to destroy the coherences between the Fock states $|n\rangle$ also at this rate (decoherence), with the caveat that these coherences are preserved if the state is a coherent state $|\alpha\rangle$. The impact of these effects differs greatly depending on whether we wish to observe the displacement of the resonator due to the photons, or whether we wish to create entanglement between the two systems. The former allows us to observe directly the effect of the nonlinear coupling and the mechanical effect of light at the single photon level. The relation in equation (10) shows us that to generate a significant displacement we need $\gamma \sim g$. There is, however, no particular requirement on κ. While κ reduces the number n in equation (10) this can be continually replaced by preparing the cavity mode in a number state on a timescale faster than $1/\kappa$. The decohering effects of the damping rates are not important in this case because the state of the resonator is coherent, and that of the cavity mode is a Fock state.

If we wish to create the state $|\mathrm{cat}(\alpha)\rangle$ in the cavity mode, or we wish to generate entanglement between the two systems, then coherences between number states are essential, and the decoherence due to the damping has a direct effect. To engineer entanglement we can prepare the cavity mode in a superposition of two Fock states, and the interaction will create an entangled (and correlated) state by displacing the mechanical oscillator by a different amount for each Fock state. Thus in destroying the coherences in the Fock basis the decoherence destroys the entanglement. Preparing either cat states or entangled states therefore requires $\gamma, \kappa \ll g$.

5. A practical circuit implementation

We now turn to the question of how g might be modulated in real electromechanical circuits. Consider a superconducting LC-resonator capacitively coupled to a mechanical resonator, as shown in figure 1. The mechanical oscillator forms one flexible plate of a capacitor, and thus changes the capacitance as it moves. The Hamiltonian for the circuit is given by equation (1), where the frequency of the LC-resonator is $\omega = 1/\sqrt{L_r C_r}$ with L_r the inductance and C_r the capacitance, and the nonlinear coupling rate is $g = \omega/(2d)\sqrt{\hbar/(2\, m\Omega)}$, with d the distance between the capacitor plates, and m the mass of the mechanical oscillator. It is therefore possible to modulate g by modulating ω or d. In fact the latter is just another way of modulating ω, since d determines C_r. As long as the modulation frequency is small compared to the frequency ω of the LC-resonator, then the adiabatic approximation preserves the state of the system with respect to the eigenvectors of the changing mode operators. The result is that the mode operator a is preserved, and it is merely ω and g that change with time.

The coupling strength can be potentially modulated by varying the distance d between the capacitor plates, or between those of a second capacitor in parallel with the first. This second capacitor can be designed to have a smaller plate separation, which can then be strongly modulated with a bulk-acoustic-wave resonator. This method was suggested recently by Kielpinski *et al* [33] as a way to provide a linear coupling between the motion of a trapped ion and an LC-resonator. Using a 1 GHz LC-resonator, they obtain a modulation amplitude corresponding to $\eta = 0.3$. If we were to use a 10 MHz mechanical resonator with electro-

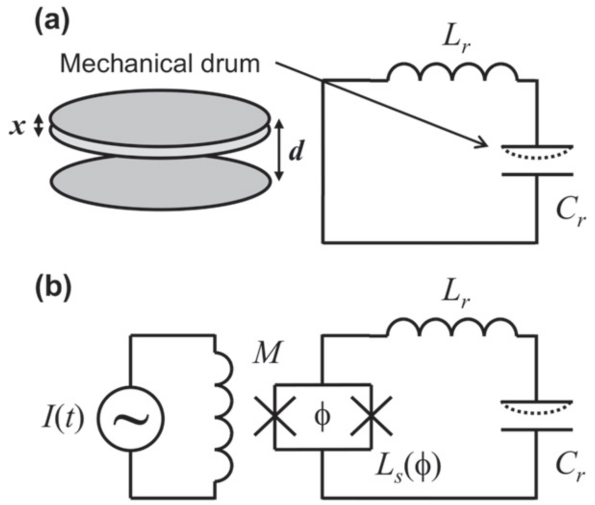

Figure 1. (a) A simple LC-circuit capacitively coupled to a mechanical resonator. The flexible mechanical oscillator with deflection x forms one plate of a capacitor with a static plate separation d. (b) An electromechanical circuit in which the nonlinear coupling rate between the superconducting LC-resonator and the mechanical oscillator can be modulated. Similar to (a), but the total inductance $L_t(\phi) = L_s(\phi) + L_r$ of the LC-resonator is determined by ϕ, the magnetic flux applied to the dc-SQUID loop. This flux is created by the current source on the left and inductively coupled to the dc-SQUID through M.

mechanical coupling rate $g = 2\pi \times 100$ Hz [11], and modulate ω near 10 MHz, we could achieve a maximum Kerr nonlinearity with rate $\chi = \eta g/4 = 47\ \mathrm{s}^{-1}$, an increase in the Kerr rate by a factor of 7500. Note that the connection between the present scheme, and the dual-resonator schemes in [27–30], can be seen by considering one plate of the second capacitor as a quantum oscillator instead of a fixed classical drive. While the use of a bulk-acoustic-wave resonator to strongly modulate a capacitance appears feasible, such a system has not yet be attempted experimentally.

We now consider an all-electrical method of modulating g, easily realizable with standard circuit techniques. This involves adding to the LC-resonator a pair of weakly nonlinear Josephson junctions (JJ's), a dc-SQUID [34], whose inductance can be modulated with an external magnetic field. Of course, JJ's can already be used to create strong Kerr nonlinearities in superconducting resonators [35, 36], so the purpose of our circuit is to enhance the photon-pressure force. If in the future stronger couplings g exist, it may be possible for the enhanced opto-mechanical Kerr effect to compete with that produced by the JJ's in our circuit, the latter having been minimized by design. The circuit we propose is shown in figure 1(b). Our resonator design maximizes frequency tunability, while minimizing the anharmonicity generated by the addition of the JJ's. The dc-SQUID is placed in series with the coil inductor L_r so that the combined inductance, $L_t(\phi) = L_J/\cos(\pi\phi) + L_r$, can be modulated with a magnetic flux $\phi = \Phi/\Phi_0$, where $\Phi_0 = h/(2e)$ is the flux quantum, $L_J = \Phi_0/[4\pi I_0]$ is the Josephson inductance, and I_0 is the critical current of each junction. If we choose $\xi = L_r/L_J \gg 1$, then $L_t(\phi)$ effectively provides a tunable linear inductance for the LC-resonator with relative anharmonicities $(\omega_{12} - \omega_{01})/\omega_{01} < -0.05\ \%$ and Kerr nonlinearities smaller than $-1 \times 10^{-4} \times \omega$, where $\omega_{ij} \approx \omega$ is the transition frequency from energy level i to j. The frequency of the LC-resonator is

$$\omega = 1\Big/\sqrt{L_{\rm t}(\phi)C_{\rm r}} = \omega_{\max} G(\phi) \tag{11}$$

with

$$G(\phi) = \sqrt{(1 + \xi)\cos(\pi\phi)/(1 + \xi\cos(\pi\phi))}, \tag{12}$$

$$\omega_{\max} = \sqrt{\xi\Big/\big[L_{\rm r} C_{\rm r}(1 + \xi)\big]}. \tag{13}$$

The coupling between the LC-resonator and the mechanical oscillator stems from the fact that the capacitance is inversely proportional to the distance between its plates. If x is the position of the mechanical oscillator, C is the value of $C_{\rm r}$ when $x = 0$, and the amplitude of the oscillation is small compared to the distance between the plates, then $C_{\rm r} \approx C(1 - x/d)$. Including this in the expression for the frequency of the LC-resonator we obtain $\omega = \omega_{\max} G(\phi)[1 + x/(2d)]$. The full Hamiltonian for the two oscillators is given by substituting this expression for ω into the Hamiltonian for the non-interacting oscillators, $H_0 = \hbar\omega a^\dagger a + \hbar\Omega b^\dagger b$. The resulting Hamiltonian is that given in equation (1) with $\omega = \omega_{\max} G(\phi)$, $g = g_{\max} G(\phi)$, where $g_{\max} = \omega_{\max} x_{\rm zp}/(2d)$, $x_{\rm zp} = \sqrt{\hbar/(2m\Omega)}$ is the 'zero-point motion' of the mechanics, and we have used the fact that the mechanical position operator is $x_{\rm zp}(b + b^\dagger)$. If we vary ϕ with time, and ensure that the rate of change of ϕ is small compared to ω, then the adiabatic approximation preserves the state of the system with respect to the eigenvectors of the changing mode operators. The result is that the mode operator a is preserved, and it is merely ω and g that change with time. By varying ϕ we can choose $g(t)$ to be any function of time, within the constraint $0 < g < g_{\max}$.

To obtain the modulation $g(t) = g[1 + \eta\cos(\nu t)]$ we choose $G(\phi) = [1 + \eta\cos(\nu t)]/(1 + \eta)$, in which case $g = g_{\max}/(1 + \eta)$ and $g_{\rm eff} = \eta g_{\max}/(2 + 2\eta)$. The rate at which the optical force increases the momentum of the mirror is then $\mathrm{d}p/\mathrm{d}t = -g_{\rm eff}(n + 1/2)$, with n the number of photons. So at what rate could the momentum be changed with present technology? The largest value of g that has been achieved to-date is $g_{\rm o} = 2\pi \times 230\ \text{Hz} = 1445\ \text{s}^{-1}$ [37] for a 7.5 GHz LC-resonator. If we use a similar flexible capacitor in our new circuit and choose $2I_{\rm o} = 1.5\,\mu$ A, $L_{\rm r} = 2$ nH, giving $\xi = 9$, and $C_{\rm r} \approx 50$ fF, then $\omega_{\max}/(2\pi) \approx 15$ GHz and we expect that modulations with $\eta = 0.1$ should be feasible. In principal, a factor of 2 increase in $x_{\rm zp}$ could be achieved by reducing the mechanical mass by a factor of four (using a thinner plate) and reducing the tension by the same factor to preserve the mechanical frequency. This would give $g_{\max} \approx (15\ \text{GHz}/7.5\ \text{GHz})(8\ \text{fm}/4\ \text{fm})\, g_{\rm o} = 2\pi \times 920\ \text{Hz} = 5780\ \text{s}^{-1}$. Using a single qubit to load photons into the LC-resonator, the preparation of a number-state with $n = 10$ or even $n = 50$ is entirely feasible [38]. With $n = 10$ and the above value for $g_{\max}$, the time taken to displace the mechanical oscillator by an average of $|\beta|^2 = 25$ phonons is $t = \sqrt{\beta}/(g_{\rm eff}[n + 1/2]) \approx 1.8$ ms. Presently, $1/\kappa \approx 1\ \mu$s, three orders of magnitude smaller than t, so future reductions in κ and increases in g will still be required to achieve $t \approx 1/\kappa$ and thus observe photon-phonon entanglement. Given 10 MHz mechanics with a quality factor of $Q = 10^5$, the average steady-state mechanical displacement for $n = 10$ photons is $\approx$5 phonons. Without the modulation the maximum displacement that can be generated for the same resonator with $n = 10$ is $(g_{\max}/\Omega)(4n + 2) = 10^{-3}$ phonons, nearly 4 orders-of-magnitude smaller.

6. Summary

We have shown that modulating the electromechanical coupling rate increases the Kerr nonlinearity and the effect of the photon-pressure force by orders of magnitude, and we have shown how this modulation can be realized. Because this enhancement is limited by the intrinsic g, it allows a unique measure of g based on the nonlinear effects, not the linearized behavior commonly observed. Potential uses include the generation of high-amplitude nonclassical states, observing the mechanical effects of quantum states of light, and realizing non-demolition measurements of photons. It is interesting to note that, in principal, a similar enhancement could also be achieved in opto-mechanical systems.

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