Virtual photons in ultra-strongly coupled systems

or

Quantum Nonlinear Optics without Photons

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in collaboration with:

S. Savasta, R. Stassi, A.F. Kockum, L. Garziano, O.D. Stefano,
E. Macri, and A. Miranowicz
RIKEN and University of Messina.

Pedagogical review:

A.F. Kockum, A. Miranowicz, S.D. Liberato, S. Savasta, F. Nori

Ultrastrong coupling between light and matter

Nature Reviews Physics **1**, pp. 19–40 (2019).

Related works are available in our web site

1. L. Garziano, R. Stassi, V. Macrì, A.F. Kockum, S. Savasta, F. Nori
   Multiphoton quantum Rabi oscillations in ultrastrong cavity QED
   Phys. Rev. A **92**, 063830 (2015). [PDF][Link][arXiv]
2. L. Garziano, V. Macrì, R. Stassi, O.D. Stefano, F. Nori, S. Savasta
   One Photon Can Simultaneously Excite Two or More Atoms
   Phys. Rev. Lett. **117**, 043601 (2016). [PDF][Link][arXiv][Suppl. Info.]
   Featured in Physics, Editors' Suggestion
3. R. Stassi, S. Savasta, L. Garziano, B. Spagnolo, F. Nori
   Output field-quadrature measurements and squeezing in ultrastrong cavity-QED
   New Journal of Physics **18**, 123005 (2016). [PDF][Link][arXiv]
4. O.D. Stefano, R. Stassi, L. Garziano, A.F. Kockum, S. Savasta, F. Nori
   Feynman-diagrams approach to the quantum Rabi model for ultrastrong cavity QED: stimulated emission and reabsorption of virtual particles dressing a physical excitation
   New Journal of Physics **19**, 053010 (2017). [PDF][Link][arXiv]
5. A.F. Kockum, A. Miranowicz, V. Macrì, S. Savasta, F. Nori
   Deterministic quantum nonlinear optics with single atoms and virtual photons
   Phys. Rev. A **95**, 063849 (2017). [PDF][Link][arXiv]
6. A.F. Kockum, V. Macrì, L. Garziano, S. Savasta, F. Nori
   Frequency conversion in ultrastrong cavity QED
   Scientific Reports **7**, 5313 (2017). [PDF][Link][arXiv][Suppl. Info.]
7. Z. Chen, Y. Wang, T. Li, L. Tian, Y. Qiu, K. Inomata, F. Yoshihara, S. Han, F. Nori, J.S. Tsai, J.Q. You
   Single-photon-driven high-order sideband transitions in an ultrastrongly coupled circuit-quantum-electrodynamics system
   Phys. Rev. A **96**, 012325 (2017). [PDF][Link][arXiv]
8. R. Stassi, V. Macrì, A.F. Kockum, O.D. Stefano, A. Miranowicz, S. Savasta, F. Nori
   Quantum Nonlinear Optics without Photons
   Phys. Rev. A **96**, 023818 (2017). [PDF][Link][arXiv]
9. R. Stassi, F. Nori
   Long-lasting quantum memories: Extending the coherence time of superconducting artificial atoms in the ultrastrong-coupling regime
   Phys. Rev. A **97**, 033823 (2018). [PDF][Link][arXiv]
10. A. Settineri, V. Macrí, A. Ridolfo, O.D. Stefano, A.F. Kockum, F. Nori, S. Savasta
    Dissipation and thermal noise in hybrid quantum systems in the ultrastrong-coupling regime
    Phys. Rev. A **98**, 053834 (2018). [PDF][Link][arXiv]
11. O.D. Stefano, A.F. Kockum, A. Ridolfo, S. Savasta, F. Nori
    Photodetection probability in quantum systems with arbitrarily strong light-matter interaction
    Scientific Reports **8**, 17825 (2018). [PDF][Link 1][Link 2][arXiv]
12. V. Macrì, F. Nori, A.F. Kockum
    Simple preparation of Bell and Greenberger-Horne-Zeilinger states using ultrastrong-coupling circuit QED
    Phys. Rev. A **98**, 062327 (2018). [PDF][Link][arXiv]
13. A.F. Kockum, A. Miranowicz, S.D. Liberato, S. Savasta, F. Nori
    Ultrastrong coupling between light and matter
    Nature Reviews Physics **1**, pp. 19–40 (2019). [PDF][Link][arXiv]
14. O.D. Stefano, A. Settineri, V. Macrì, A. Ridolfo, R. Stassi, A.F. Kockum, S. Savasta, F. Nori
    Interaction of Mechanical Oscillators Mediated by the Exchange of Virtual Photon Pairs
    Phys. Rev. Lett. **122**, 030402 (2019). [PDF][Link][arXiv][Suppl. Info.]

A brief History of Optics

- Very many photons: Classical Optics
- Few photons: Quantum Optics
- One photon: Quantum Optics
- No photons: Zen Quantum Optics

Quantum Nonlinear Optics without Photons

- We have studied nonlinear optical processes with qubits, where only ***virtual photons*** are involved. (PRA 2016)
- Our results show that *N* spatially-separated and non-degenerate qubits can coherently exchange energy in analogy with light modes in nonlinear optics.
- These processes can produce multi-particle entanglement simply starting from one or more qubits in their excited state and letting the system evolve spontaneously
- We have also studied nonlinear optical processes where ***both*** virtual and real photons are involved.
- Two separate atoms can be jointly excited by a **single** photon and *vice versa*. This joint absorption and emission processes can also occur with three or more atoms (and with atoms in separate cavities).

Quantum Nonlinear Optics without Photons

- Spontaneous time evolution is also able to transfer the entanglement from two qubits to a different one.

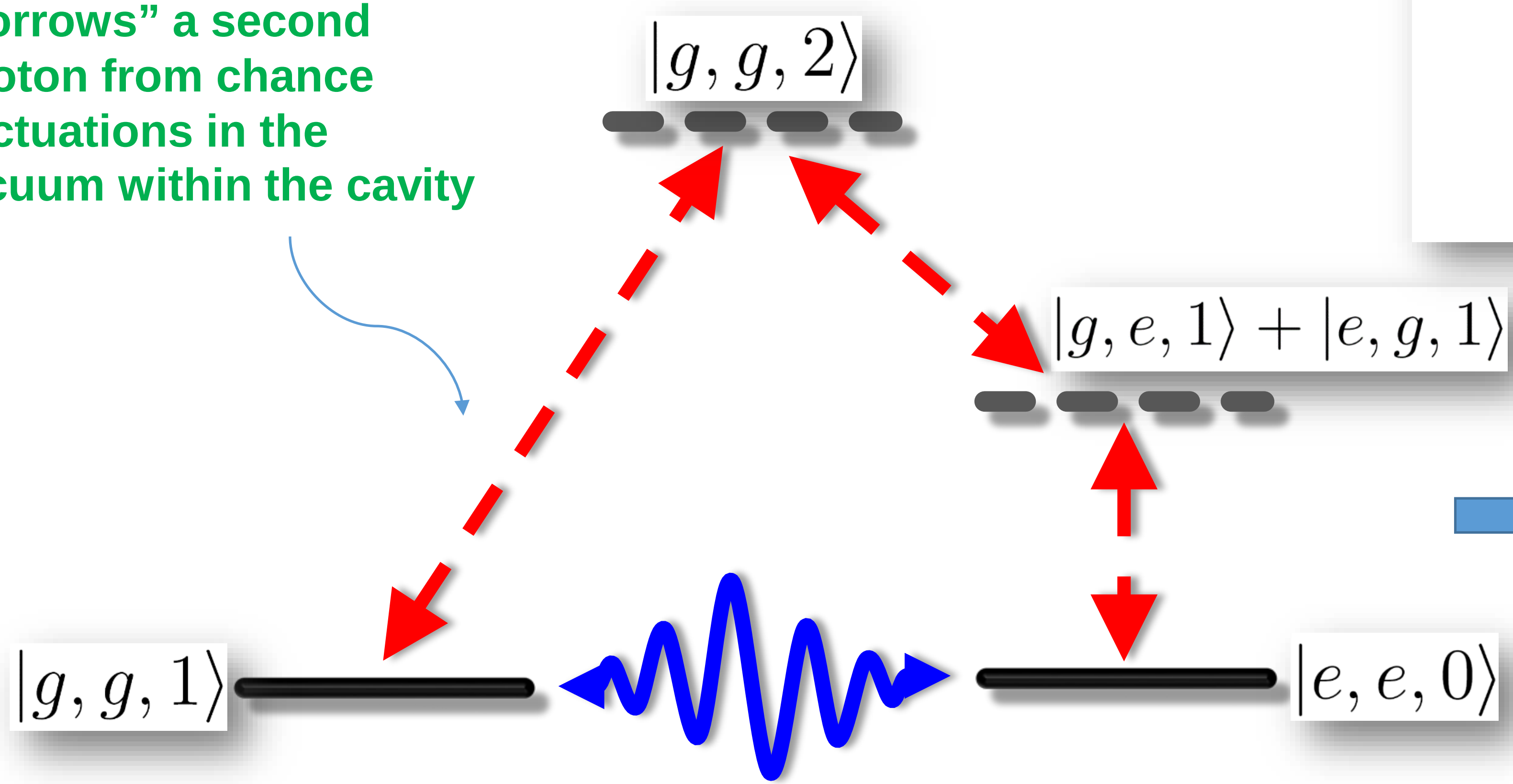
- Maximally-entangled multi-particle states can be obtained by free evolution.

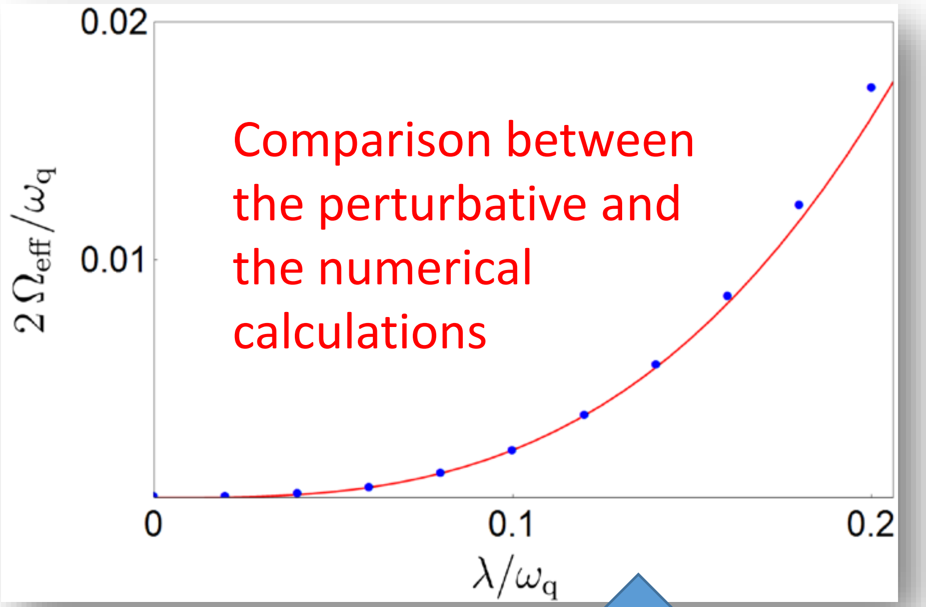
- These effects arise from terms that can change the number of excitations in the system, enabling higher-order processes via virtual photons. We describe a unified picture of this type of processes and their relation to nonlinear optics.

One photon can simultaneously excite two atoms (PRL 2016)

Diagram providing the largest contribution to the coupling strength between the 1-photon & 2-qubits

the system briefly "borrows" a second photon from chance fluctuations in the vacuum within the cavity





$$\frac{\Omega_{eff}}{\omega_{\rm q}} = \frac{8}{3}\sin\theta\cos^2\theta\left(\frac{\lambda}{\omega_{\rm q}}\right)^3$$

Outline

- Introduction: Cavity-QED

  from the weak- to the strong-, ultrastrong-, deep- … -coupling regimes

- Virtual photons in ultra-strongly coupled systems:

  A single photon can simultaneously excite two or more atoms

- Quantum nonlinear optics without photons:

  1) Description of the system
  2) Three-qubit mixing
  3) Four-qubit mixing

- Conclusions

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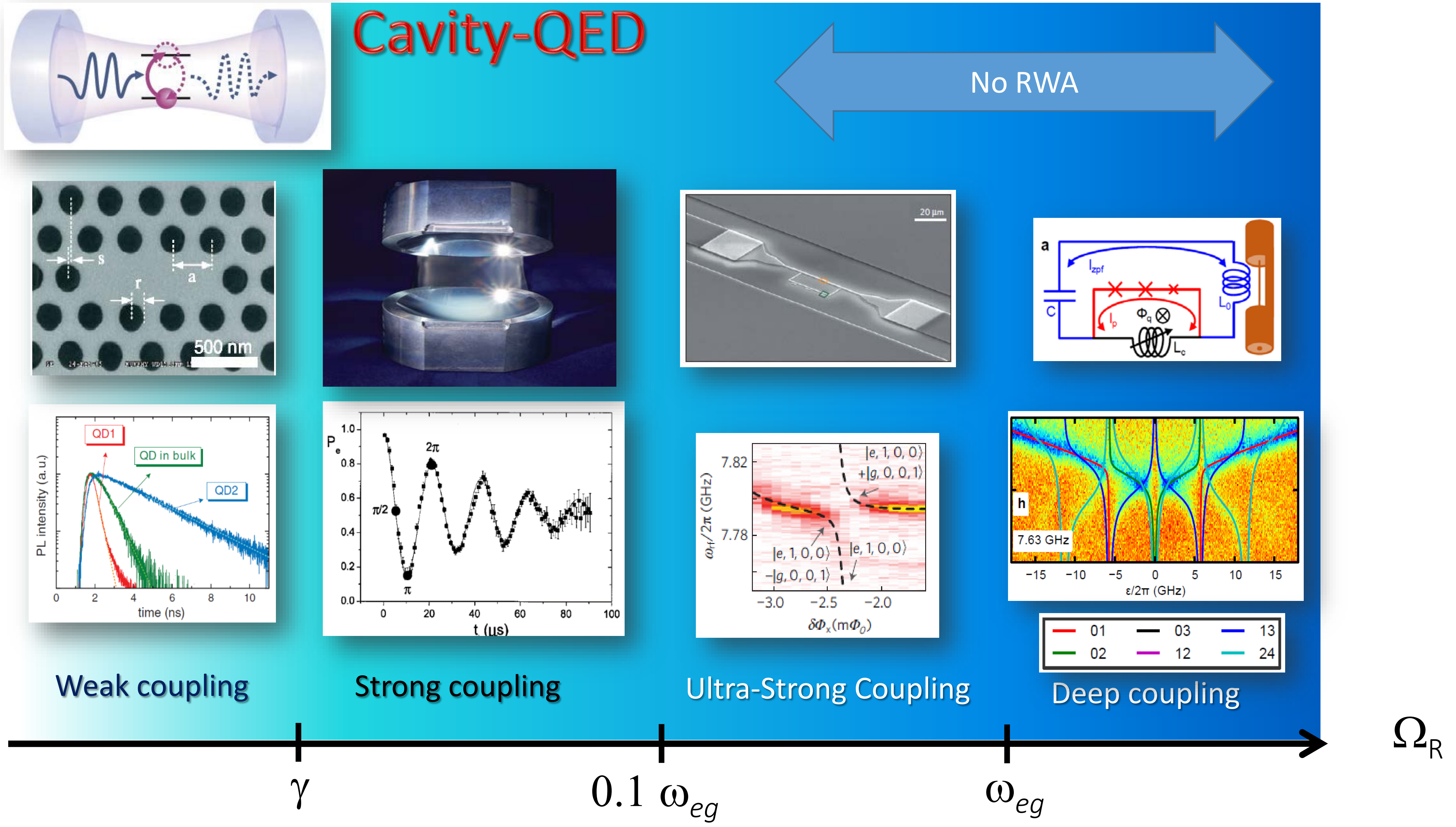
Cavity-QED

Cavity QED investigates the interaction of confined electromagnetic field modes with atoms, where the quantum nature of light affects the system dynamics.

A high degree of control of quantum systems can be reached in the strong-coupling regime of cavity-QED, where the atom-field coupling rate is dominant with respect to the loss and decoherence rates. This paves the way for many interesting physical applications.

Cavity QED can be exploited for the realization of quantum gates and quantum networks for quantum computational tasks

Many of the proposed concepts, pioneered with flying atoms, have been adapted and further developed using superconducting artificial atoms in the electromagnetic field of microwave resonators. This has produced the rapidly growing field of circuit QED, which is useful both for exploring light-mater interactions in a controllable manner, and for future quantum technologies.


Cavity-QED
No RWA
500 nm
s
a
r
20 µm
a
I_{zpf}
C
L_0
I_p
Φ_q
L_c
QD1
QD in bulk
QD2
PL intensity (a.u.)
time (ns)
P_e
π/2
2π
π
t (µs)
7.82
7.78
$\omega_{rf}/2\pi$ (GHz)
|e, 1, 0, 0⟩ +|g, 0, 0, 1⟩
|e, 1, 0, 0⟩ −|g, 0, 0, 1⟩
|e, 1, 0, 0⟩
$\delta\Phi_x(m\Phi_0)$
h
7.63 GHz
ε/2π (GHz)
01
02
03
12
13
24
Weak coupling
Strong coupling
Ultra-Strong Coupling
Deep coupling
γ
$0.1\ \omega_{eg}$
ω_{eg}
Ω_R

About 20 Ultra-Strong Coupling Experiments

Quantum wells

G. Günter *et al.*, Nature **458**, 178 (2009)
A. A. Anappara *et al.*, Phys. Rev. B **79**, 201303 (2009)
Y. Todorov *et al.*, Phys. Rev. Lett. **105**, 196402 (2010)
M. Geiser *et al.*, Phys. Rev. Lett. **108**, 106402 (2012)

Molecules

T. Schwartz *et al.*, Phys. Rev. Lett. **106**, 196405 (2011)
S. Kéna-Cohen *et al.*, Adv. Opt. Mat. **1**, 827 (2013)
Gambino et al. ACS Photonics 1, 1042 (2014)

2DEG

G. Scalari *et al.*, Science **335**, 1323 (2012)
C. Maissen *et al.*, Phys. Rev. B **90**, 205309 (2014)

Organic LEDs

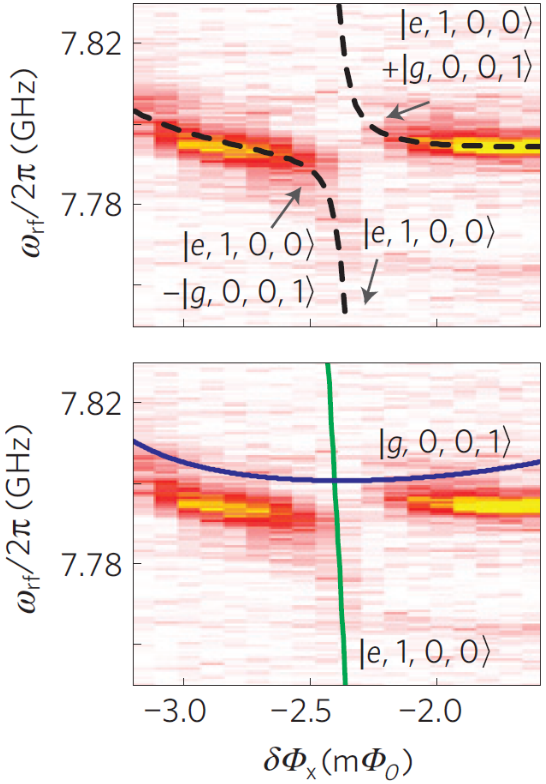
Gubbin *et al.* 104, 233302 (2014)
Mazzeo *et al.* 104, 233303 (2014)

Magnons

M. Goryachev *et al.*, Phys. Rev. Appl. **2**, 054002 (2014)

Circuit QED

P. Forn-Diaz *et al.*, Phys. Rev. Lett. **105**, 237001 (2010)
→ T. Niemczyk *et al.*, Nature Physics **6**, 772 (2010)
A. Baust *et al.*, arXiv:1412.7372 (2014)
F. Yoshihara *et al.*, arXiv:1602.00415 (2016)
P. Forn-Diaz *et al.*, arXiv:1602.00416 (2016)
Z. Chen *et al.*, arXiv:1602.01584 (2016)



Neither $\hat{N}$ nor $\hat{P}$ conserved

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  A single photon can simultaneously excite two or more atoms

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A single photon can simultaneously excite two or more atoms

- We consider two separate atoms interacting with a single-mode optical or microwave resonator.

- When the frequency of the resonator field is twice the atomic transition frequency, we show that there exists a resonant coupling between *one* photon and *two* atoms, via intermediate virtual states connected by counter-rotating processes.

- If the resonator is prepared in its one-photon state, the photon can be jointly absorbed by the two atoms in their ground state which will both reach their excited state with a probability ~ 1.

A single photon can simultaneously excite two or more atoms

- Like ordinary quantum Rabi oscillations, this process is coherent and reversible, so that **two atoms** in their excited state will undergo a downward transition **jointly emitting a single cavity photon.**

- This joint absorption and emission process can also occur with three atoms.

- The parameters used to investigate this process correspond to experimentally demonstrated values in circuit QED.

Last sentence of our preprint:

We hope that this work (on one photon simultaneously exciting two or more atoms) could simultaneously excite two or more referees.

Indeed, it simultaneously excited the referees.

PRL **117,** 043601 (2016) PHYSICAL REVIEW LETTERS week ending 22 JULY 2016

One Photon Can Simultaneously Excite Two or More Atoms

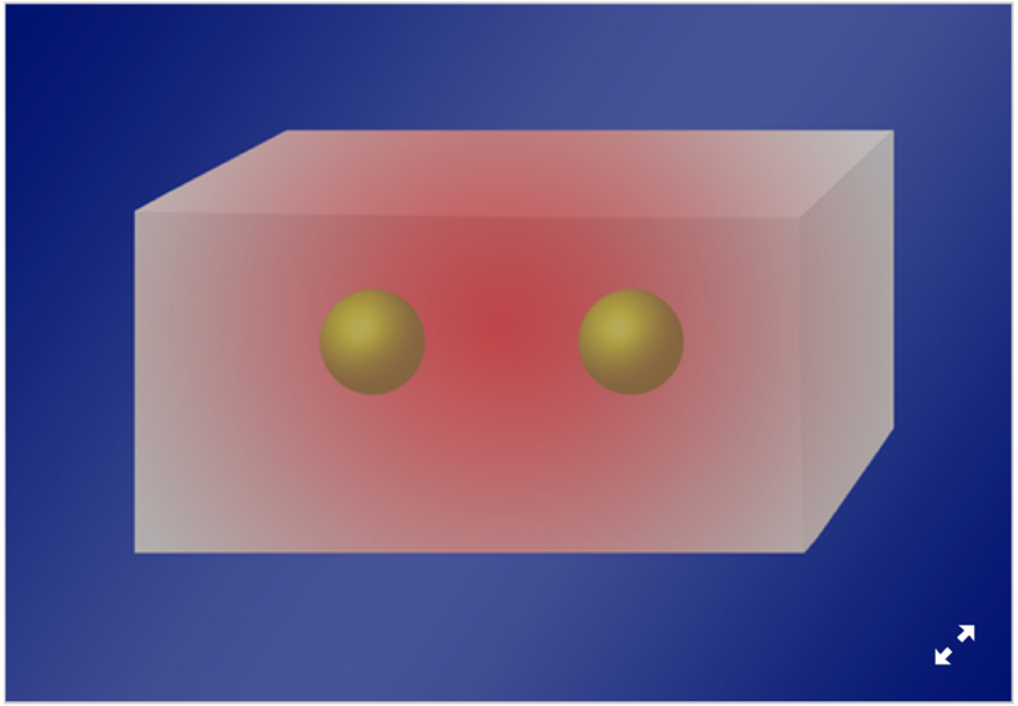
... a nice summary by Philipp Ball

Physics

Focus: Two Atoms Can Jointly Absorb One Photon

July 22, 2016 • *Physics* 9, 83

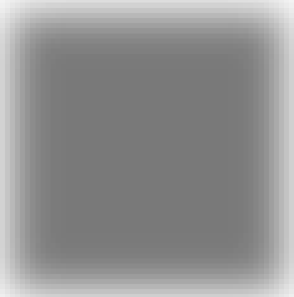
Theorists show that two atoms in an optical cavity can absorb the same photon.



APS/Joan Tycko

Sharing the limelight. Two or more atoms in an optical cavity can absorb a single photon, according to theory. The cavity allows standing light waves of a single frequency (red glow), which can be limited to one photon.

In the top 5% of all research outputs scored by Altmetric



The system

Two or more (identical) flux qubits strongly coupled to a superconducting resonator

$$\hat{H}_0 = \hat{H}_{\mathrm{q}} + \hat{H}_{\mathrm{c}} + \lambda \hat{X} \sum_i \left(\cos\theta\, \hat{\sigma}_x^{(i)} + \sin\theta\, \hat{\sigma}_z^{(i)} \right)$$

Parity symmetry-breaking

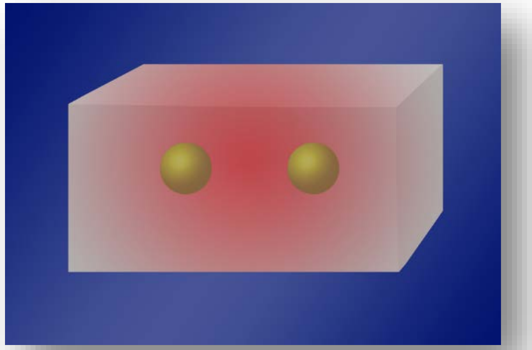
coupling strength

$\hat{H}_{\mathrm{c}} = \omega_{\mathrm{c}} \hat{a}^\dagger \hat{a}$ Resonator-mode Hamiltonian

$\hat{X} = \hat{a} + \hat{a}^\dagger$ Intracavity-field operator

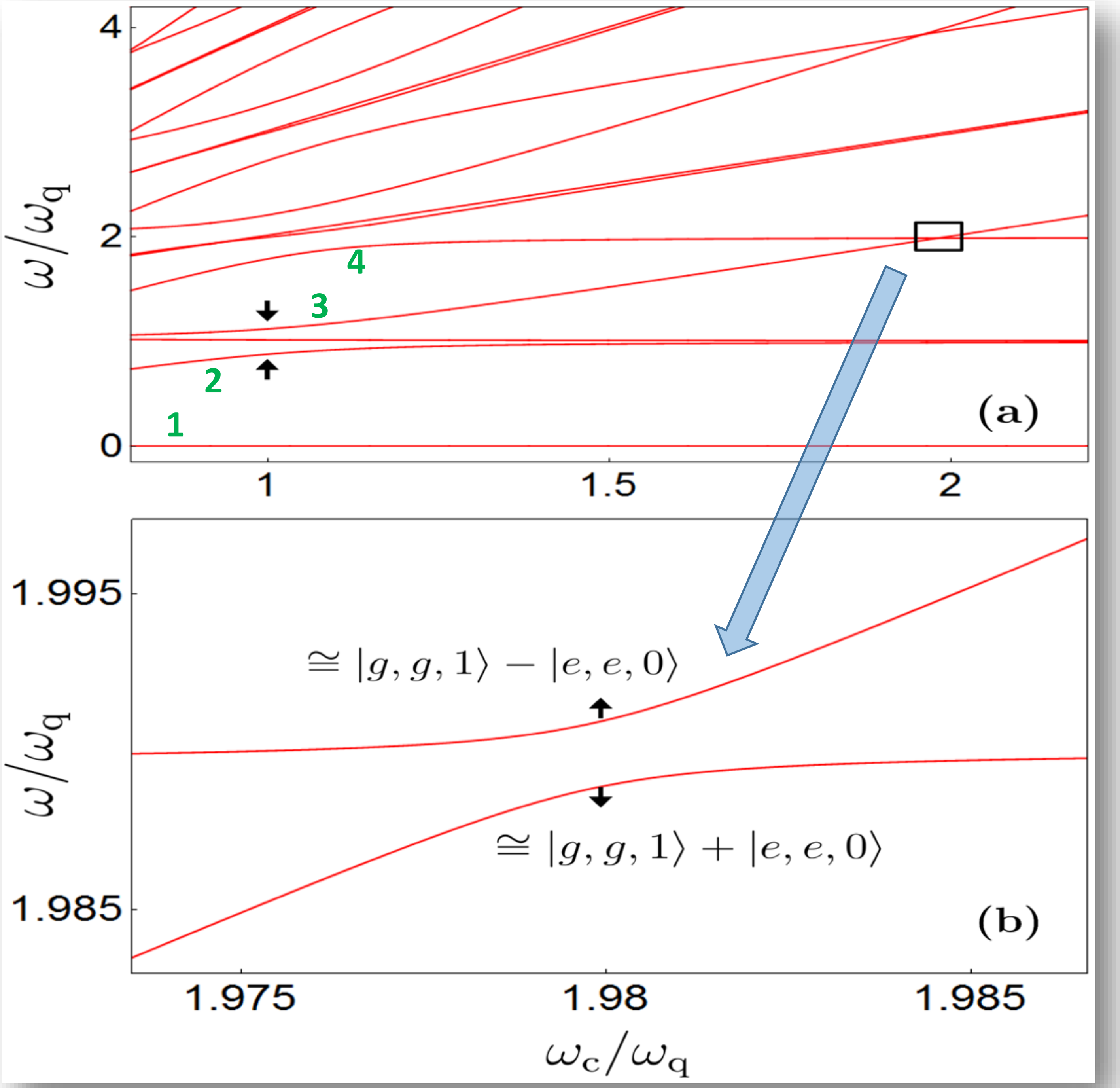
$\hat{H}_{\mathrm{q}} = (\omega_{\mathrm{q}}/2) \sum_i \hat{\sigma}_z^{(i)}$ qubit Hamiltonian

$\hat{\sigma}_x^{(i)}$ and $\hat{\sigma}_z^{(i)}$ Pauli operators for the *i* th qubit



Energy levels

Parameters: $\lambda/\omega_q = 0.1 \quad \theta = \pi/6$



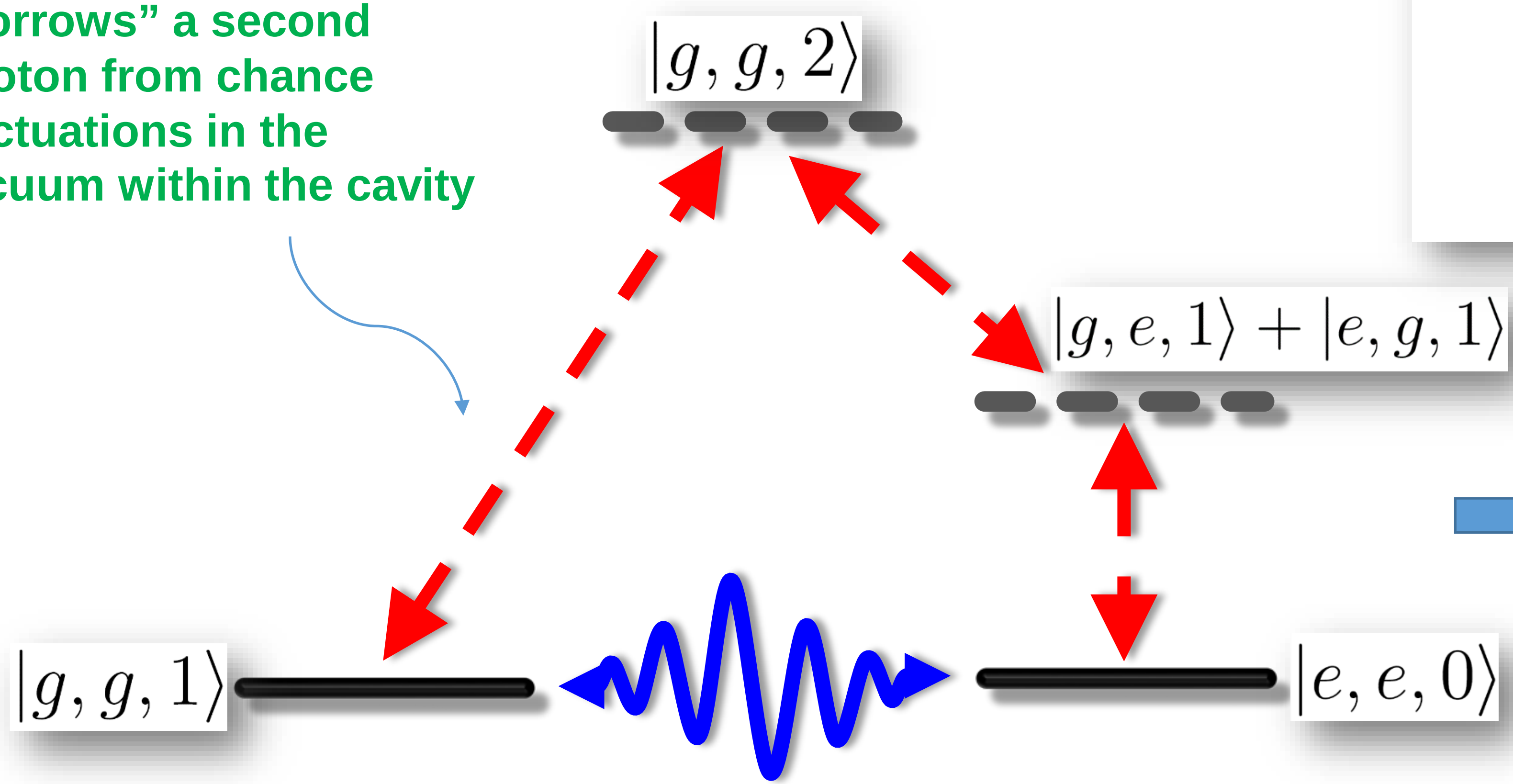
Analogous results occur even if the atoms are nonidentical (different transition energies and couplings)

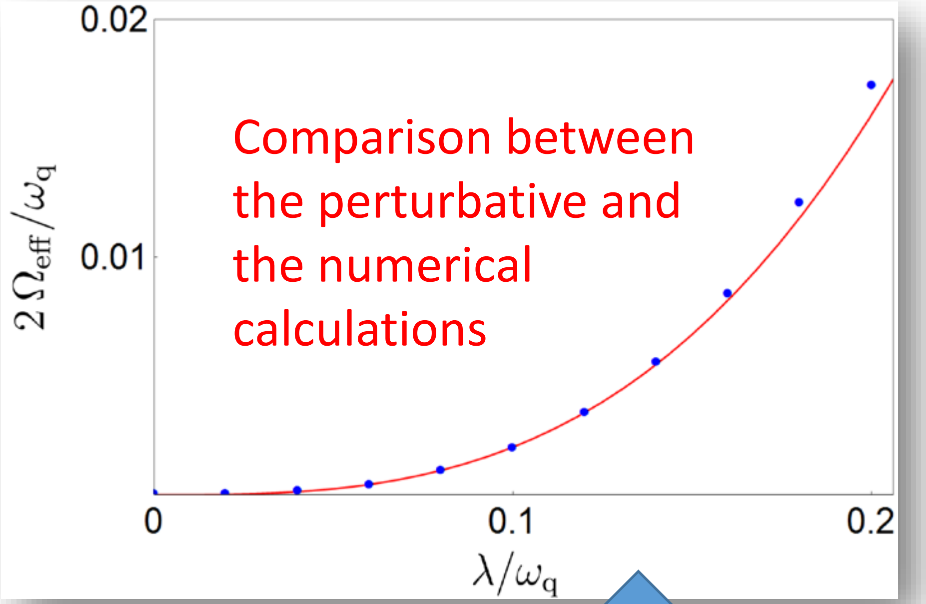
The two split eigenstates, at the minimum splitting, correspond to maximally-entangled 3-particle states.

Perturbation theory

Diagram providing the largest contribution to the coupling strength between the 1-photon & 2-qubits

the system briefly "borrows" a second photon from chance fluctuations in the vacuum within the cavity



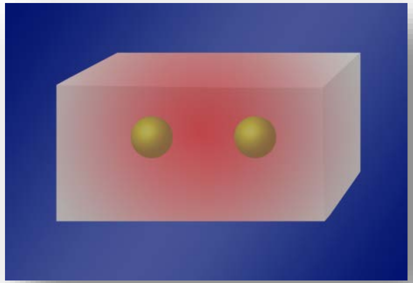


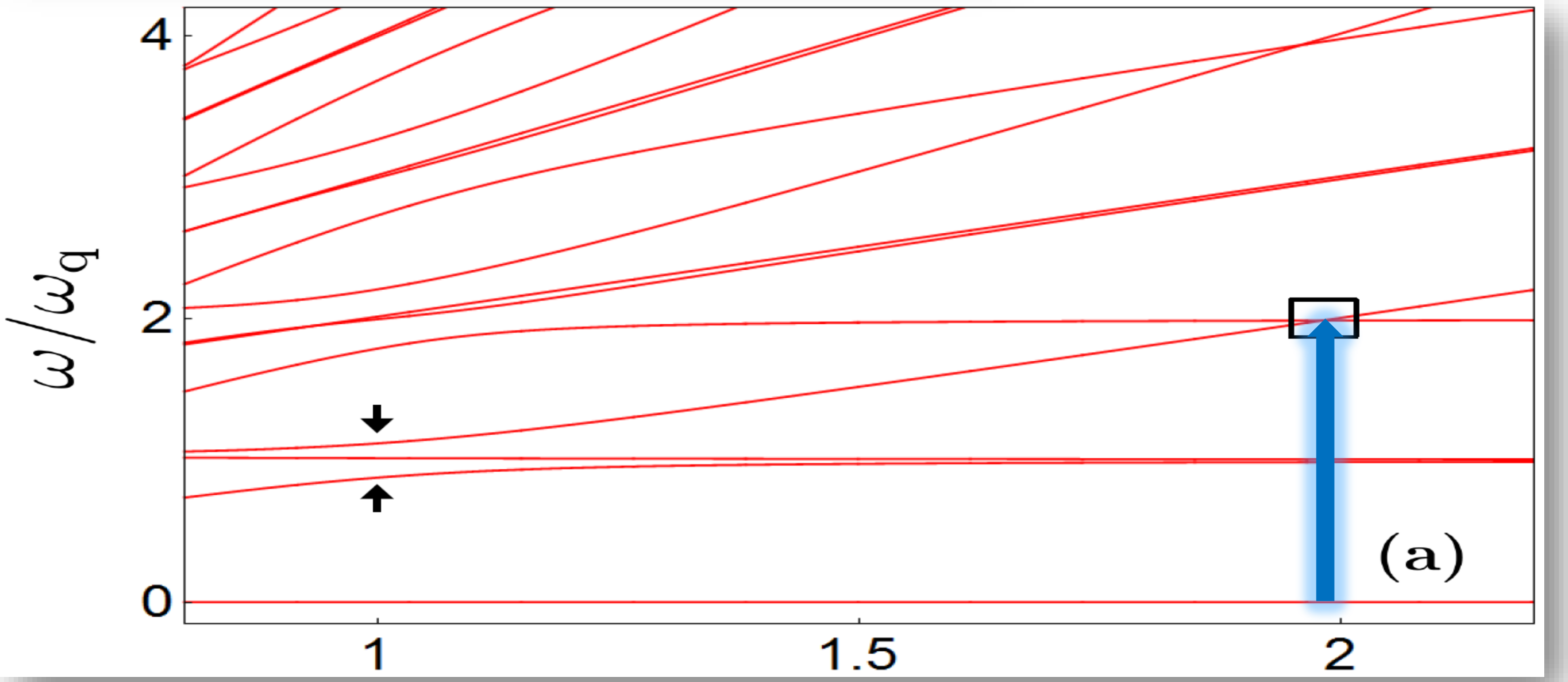
$$\frac{\Omega_{eff}}{\omega_{\mathrm{q}}} = \frac{8}{3}\sin\theta\cos^2\theta\left(\frac{\lambda}{\omega_{\mathrm{q}}}\right)^3$$

Coherent input pulse driving the cavity

$$\hat{H}_{\mathrm{d}}(t) = \mathcal{E}(t)\cos(\omega t)\hat{X}$$

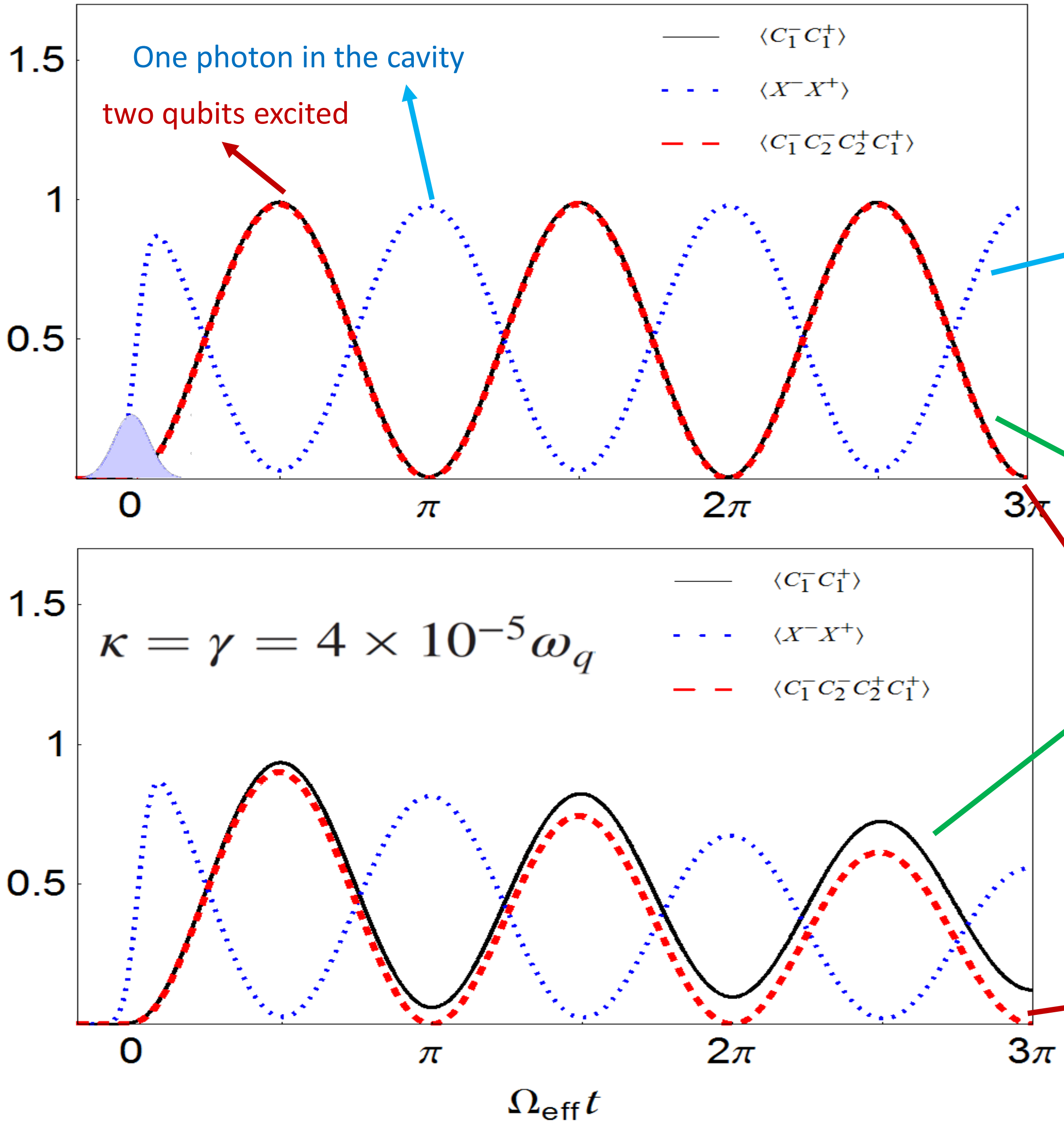
$$\mathcal{E}(t) = A\exp\left[-(t-t_0)^2/(2\tau^2)\right]/(\tau\sqrt{2\pi})$$





Dynamics

no damping



cavity mean photon number

$\langle X^- X^+ \rangle$ $\quad X^+ = \sum_{j,k>j} X_{jk} |j\rangle\langle k|$

$$X_{jk} \equiv \langle j|(a^\dagger + a)|k\rangle$$

atom mean excitation number

$\langle C^- C^+ \rangle$ $\quad C^+ = \sum_{j,k>j} C_{jk} |j\rangle\langle k|$

$$C_{jk} \equiv \langle j|(\sigma_- + \sigma_+)|k\rangle$$

zero-delay two-atom correlation function

$$G_{\mathrm{q}}^{(2)} \equiv \langle \hat{C}_1^- \hat{C}_2^- \hat{C}_2^+ \hat{C}_1^+ \rangle.$$

The processes described here should be observable by placing two superconducting artificial atoms at opposite ends of a superconducting transmission line resonator.

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  A single photon can simultaneously excite two or more atoms

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  1) Description of the system
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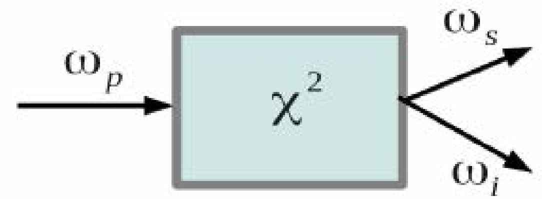
- Conclusions

Quantum Nonlinear Optics without Photons

We propose a physical process, analogous to spontaneous parametric down-conversion, where one excited atom directly transfers its excitation to two spatially-separated atoms with probability approaching one.



- The interaction is mediated by the exchange of virtual rather than real photons.
- This nonlinear atomic process is coherent and reversible, so the pair of excited atoms can transfer the excitation back to the first one: the atomic analogue of sum-frequency generation of light.
- This approach can be expanded to realize other nonlinear inter-atomic processes, such as four-atom mixing, and is an attractive architecture for the realization of quantum devices on a chip.

System:

We consider a quantum system of *N* two-level atoms (with possible symmetry-broken potentials) coupled to a single-mode resonator

$$\hat{H}_0 = \hat{H}_{\mathrm{q}} + \hat{H}_{\mathrm{c}} + \hat{V}$$

$$\hat{H}_{\mathrm{q}} = \sum_i (\omega_i/2)\, \hat{\sigma}_z^{(i)}$$

qubits Hamiltonian

$$\hat{H}_{\mathrm{c}} = \omega_{\mathrm{c}} \hat{a}^\dagger \hat{a}$$

Resonator-mode Hamiltonian

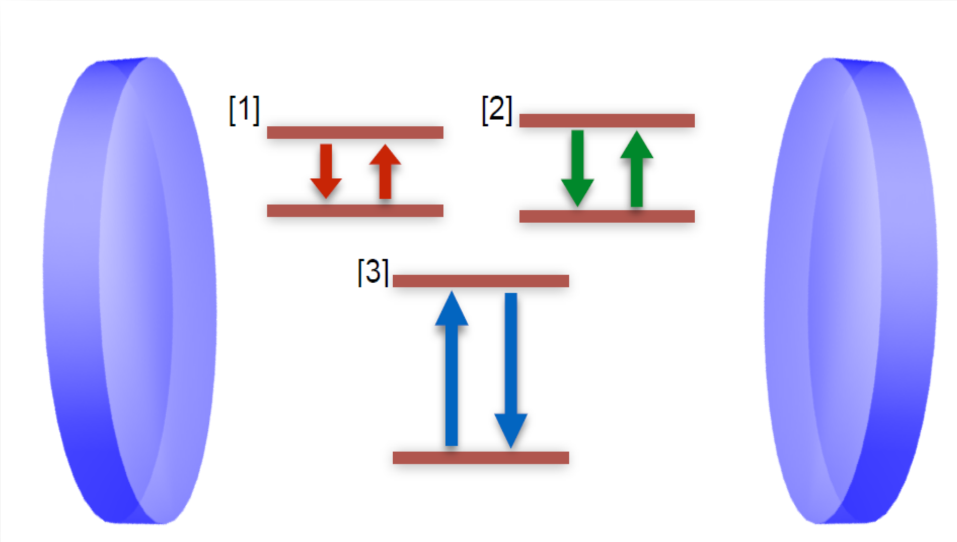
$$\hat{V} = \hat{X} \sum_i \lambda_i (\cos\theta_i\, \hat{\sigma}_x^{(i)} + \sin\theta_i\, \hat{\sigma}_z^{(i)})$$

Interaction Hamiltonian

$\hat{\sigma}_x^{(i)}$ and $\hat{\sigma}_z^{(i)}$

Pauli operators for the *i* th qubit

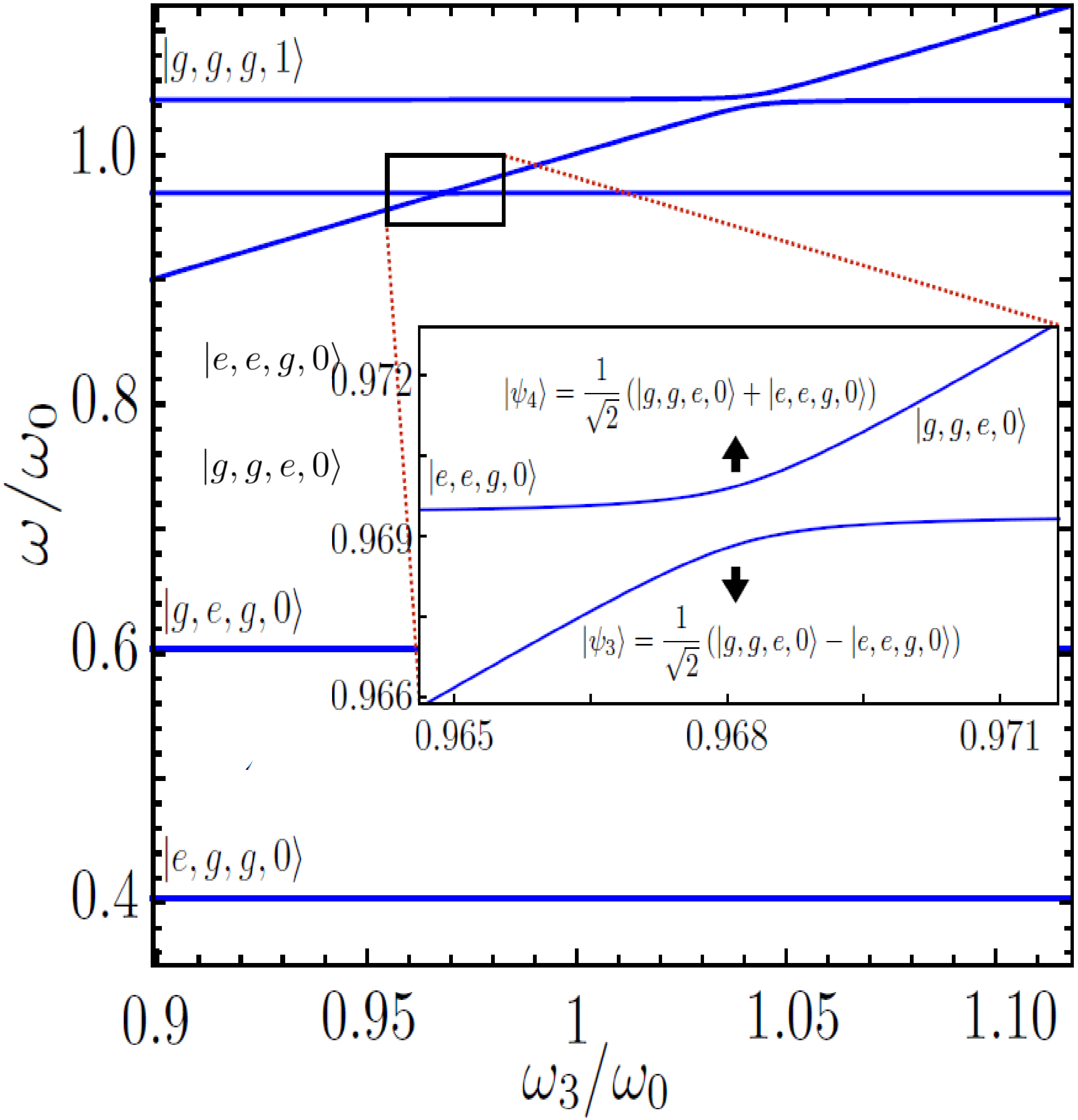
Three-qubit mixing



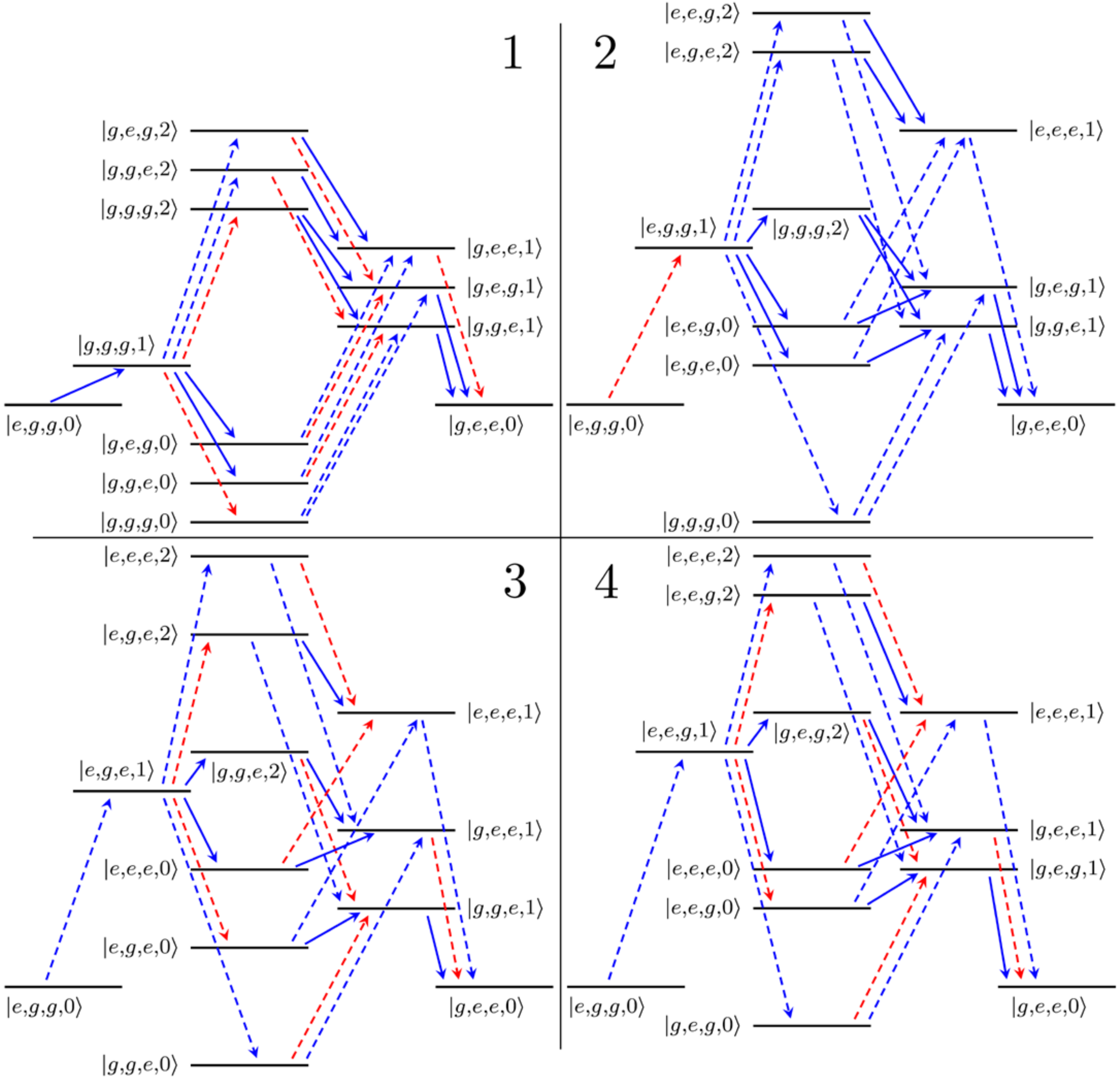
$$\omega_3 \simeq \omega_1 + \omega_2$$

The three-qubit splitting can be described by an effective Hamiltonian:

$$\hat{V}^{(3)} = J^{(3)}\, \hat{\sigma}_+^{(1)} \hat{\sigma}_+^{(2)} \hat{\sigma}_-^{(3)} + \text{H.c.}$$



Perturbation theory



$$\lambda_{\text{eff}} = \sum_{n,m,k} \frac{V_{fn}V_{nm}V_{mk}V_{ki}}{(E_i - E_n)(E_i - E_m)(E_i - E_k)}$$

for

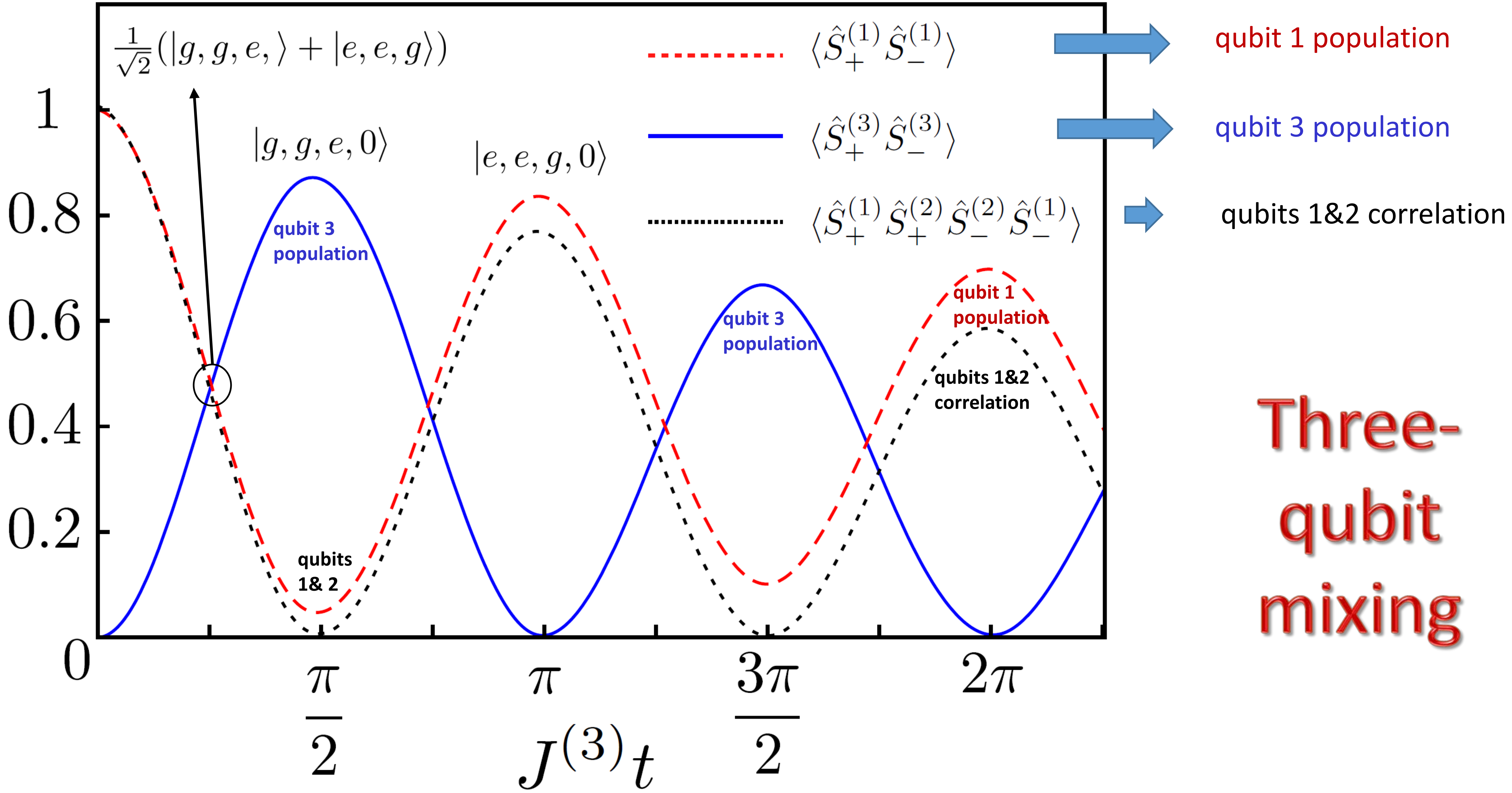
$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda \text{ and } \omega_1 = \omega_2 = \omega_3/2$$

$$J^{(3)} = \frac{64\lambda^4\omega_c^2\left(4\omega_c^2 - 7\omega_3^2\right)\sin\theta\cos^3\theta}{\omega_3\left(\omega_3^2 - \omega_c^2\right)\left(\omega_3^2 - 4\omega_c^2\right)^2}$$

$$\hat{V}^{(3)} = J^{(3)}\,\hat{\sigma}_+^{(1)}\hat{\sigma}_+^{(2)}\hat{\sigma}_-^{(3)} + \text{H.c.}$$

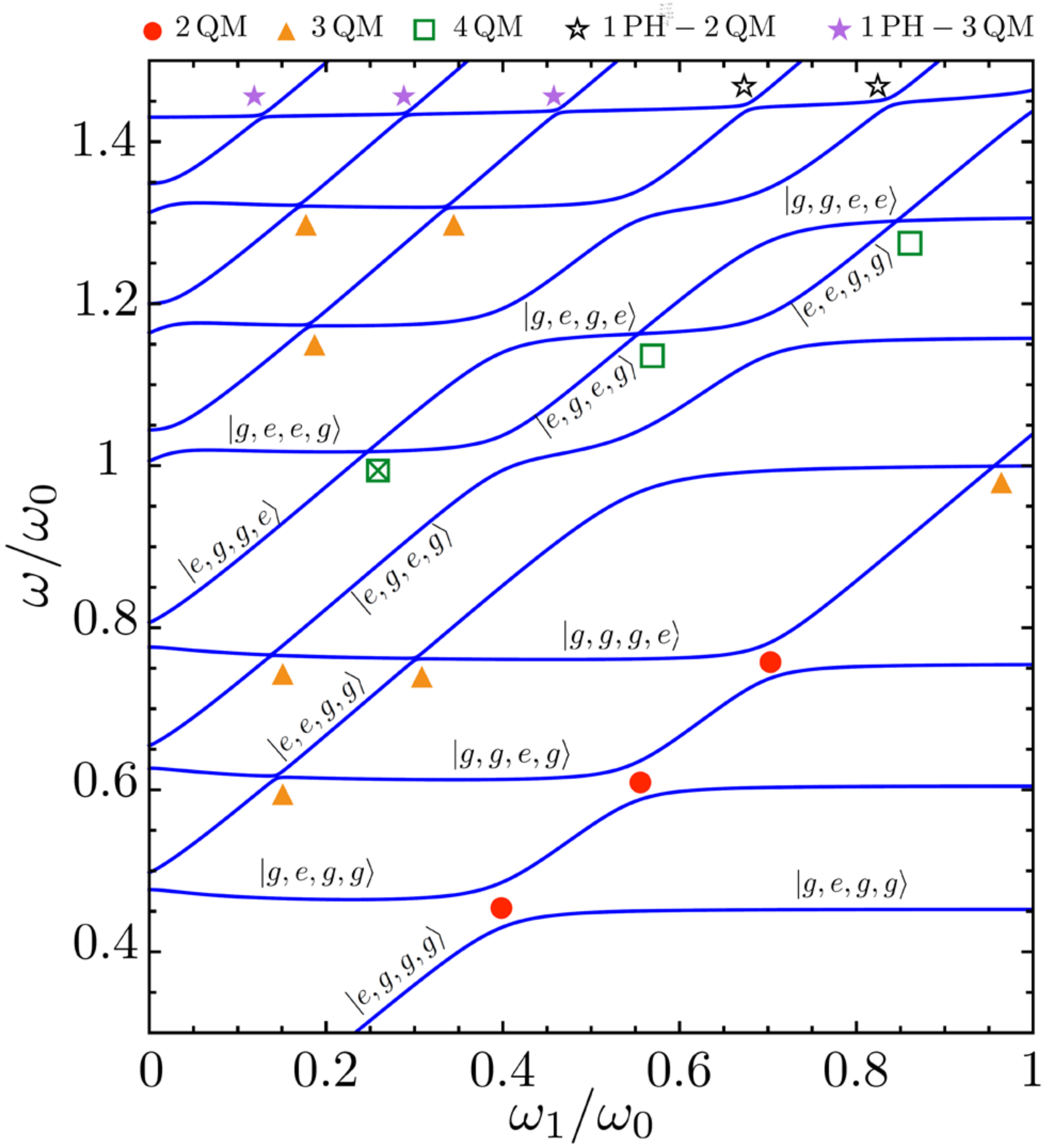
... virtual photons at work!

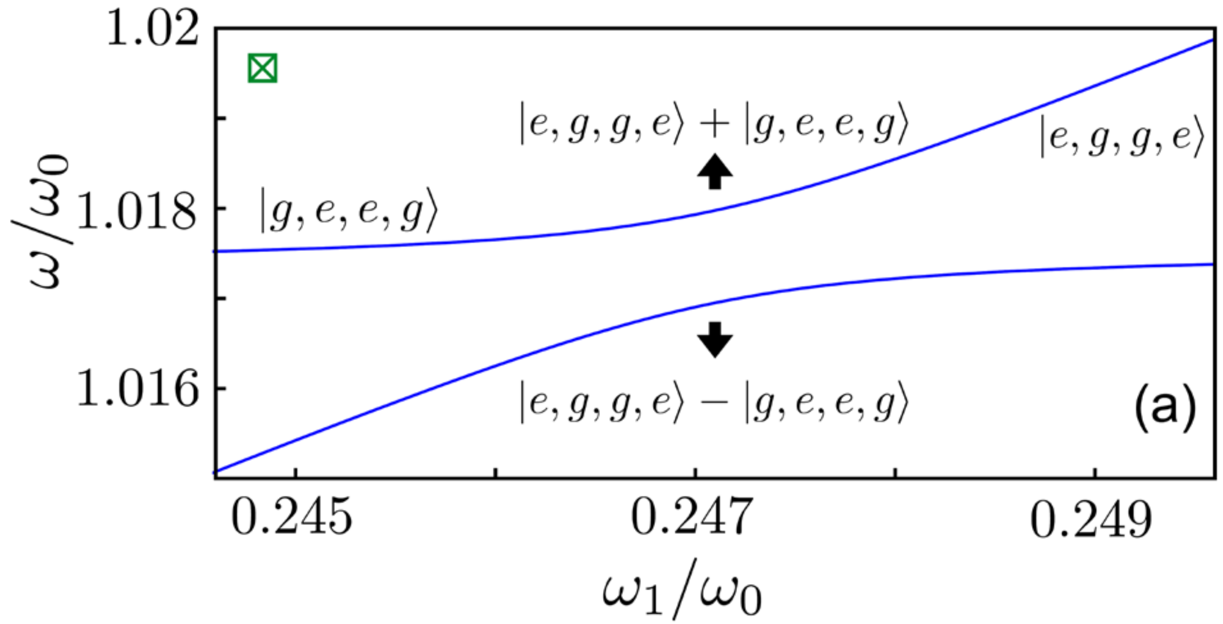
Dynamics



Three-qubit mixing

Four-qubit mixing



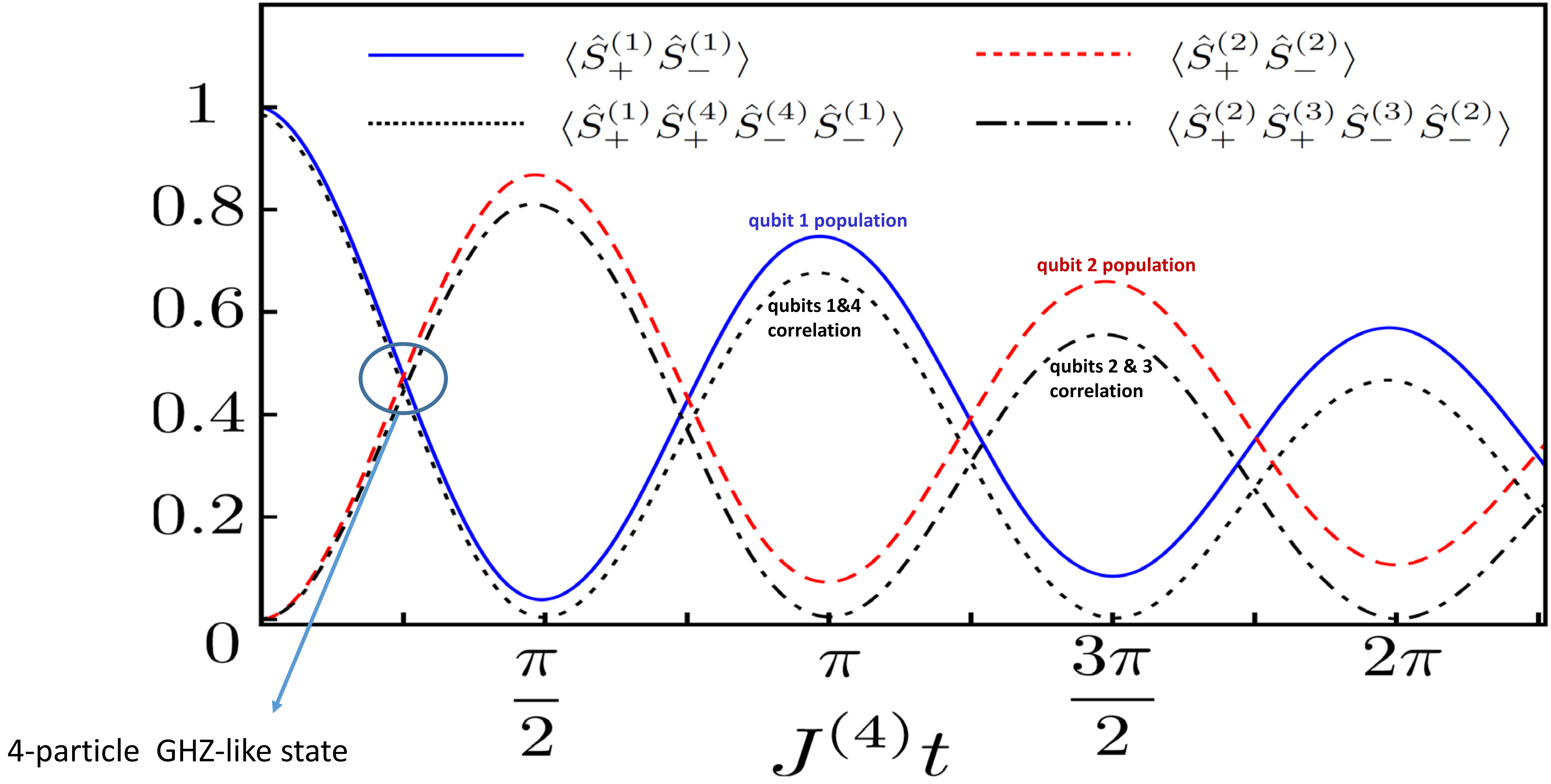


The four-qubit splitting can be described by an effective Hamiltonian:

$$\hat{V}^{(4)} = J^{(4)}\,\hat{\sigma}_{-}^{(1)}\hat{\sigma}_{-}^{(2)}\hat{\sigma}_{+}^{(3)}\hat{\sigma}_{+}^{(4)} + \mathrm{H.c.}$$

Dynamics

Four-qubit mixing



Applications of 4-qubit mixing (4QM) for entanglement transfer

This 4QM process can be used to transfer the entanglement from a pair of qubits to another spatially-separated pair, initially in a factorized state:

$$(a|g,g\rangle + b|e,e\rangle)|g,g\rangle \rightarrow |g,g\rangle(a|g,g\rangle + b|e,e\rangle)$$

$$t' = \pi/2J^{(4)}$$

Application of 4QM for error-correction codes

Adjusting the transition frequencies of the qubits, a four-qubit down-conversion analogous to that studied above for three qubits can also occur. This process is enabled by the resonant coupling between the states

$$|e, g, g, g, 0\rangle \leftrightarrow |g, e, e, e, 0\rangle$$

and can be described by the effective Hamiltonian

$$\hat{V}'^{(4)} = J'^{(4)}\, \hat{\sigma}_-^{(1)} \hat{\sigma}_+^{(2)} \hat{\sigma}_+^{(3)} \hat{\sigma}_+^{(4)} + \mathrm{H.c.}$$

This coupling offers the possibility to encode an arbitrary qubit state into a three-qubit entangled state:

three-qubit repetition code

$$U_{t'}(a|0\rangle + b|1\rangle)|000\rangle = |0\rangle(a|000\rangle + b|111\rangle)$$

$$t' = \pi/2J^{(4)}$$

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Conclusions

- Two separate atoms can be jointly excited by a **single** photon and *viceversa*. This joint absorption and emission processes can also occur with three or more atoms [and with atoms in separate cavities].
- We described nonlinear optical processes with qubits, where only ***virtual photons*** are involved. The results presented here show that N spatially-separated and non-degenerate qubits can coherently exchange energy in analogy with light modes in nonlinear optics. These processes can produce multiparticle entanglement simply starting from one or more qubits in their excited state and letting the system evolve spontaneously.
- For nonlinear optical processes where *both* virtual and real photons are involved.
- The spontaneous time evolution is also able to transfer the entanglement from a pair of qubits to a different one.
- The processes proposed here extend further the broad field of nonlinear optics. This architecture can be extended to consider qubits in different coupled cavities and may open new possibilities for quantum information processing on a chip. Maximally-entangled multiparticle states can be obtained by free evolution.
- These effects arise from terms that can change the number of excitations in the system, enabling higher-order processes via virtual photons. We have developed a unified picture of this type of processes and their relation to nonlinear optics.