Ground-State Physics of light-matter systems in the ultra-strong coupling regime

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In collaboration with:

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(Southampton)

I would like to

thank the organizers for

their kind invitation

(this is a Haiku)

Haikus are very short Japanese poems
often with 5-7-5 syllables.

Another example:

PDFs of our papers
are available 24/7
in our web site

(once I gave an entire talk using Haikus)

Ground-State Physics of light-matter systems in the ultra-strong coupling regime

Summary

Brief Introduction to Cavity Quantum Electrodynamics

Ground State Electroluminescence

Phys. Rev. Lett. **116**, 113601 (2016)

Opto-mechanical transduction of virtual radiation pressure

Phys. Rev. Lett. **119**, 053601 (2017)

Ground-State Physics of light-matter systems in the ultra-strong coupling regime

Recent results (2018)

Many-Body Ground State Electroluminescence

arXiv:1811.08682 [pdf, other]

Authors: Mauro Cirio, Nathan Shammah, Neill Lambert, Simone De Liberato, Franco Nori

Comments: 27 pages (9+19), 8 figures (3+5)

Pedagogical review published January 2019:

A.F. Kockum, A. Miranowicz, S.D. Liberato, S. Savasta, F. Nori,

Ultrastrong coupling between light and matter

Nature Reviews Physics **1**, pp. 19–40 (2019).

Free Open Access for one year (for the first issue only).

Cavity Quantum Electrodynamics

Cavity Quantum Electrodynamics studies the interaction between **matter** (here represented by an atom) and the **electromagnetic field** confined in a cavity.

If the dimension of the atom is small compared to the wavelength of the light, we can model the light-matter coupling as a **dipole interaction**.

Cavity Quantum Electrodynamics

$$H = H_0 - \mathbf{d} \cdot \mathbf{E}$$

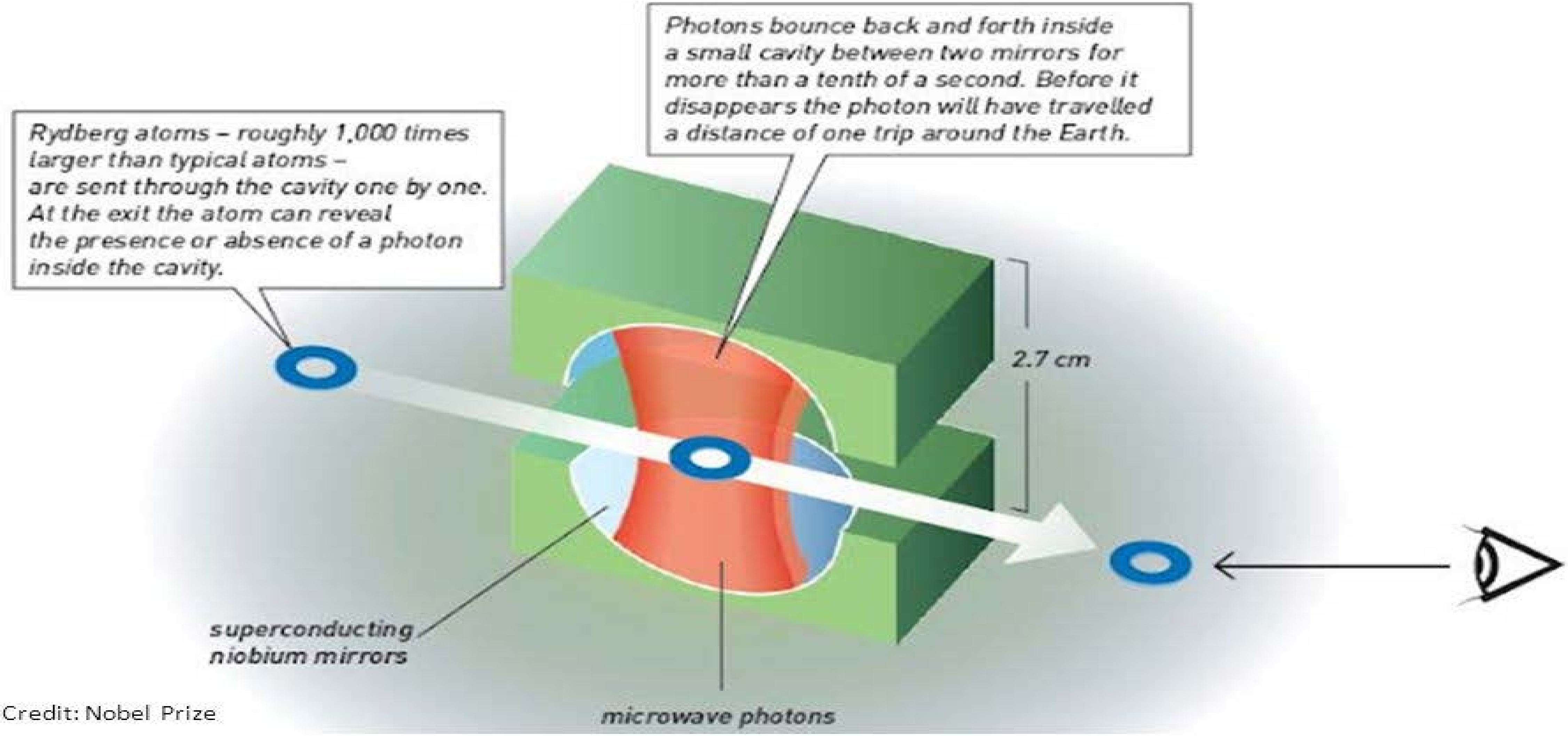
Free Hamiltonian

Dipole moment of the atom

Electric field at the position of the atom

The Nobel Prize in Physics 2012

Quantum non-demolition measurement



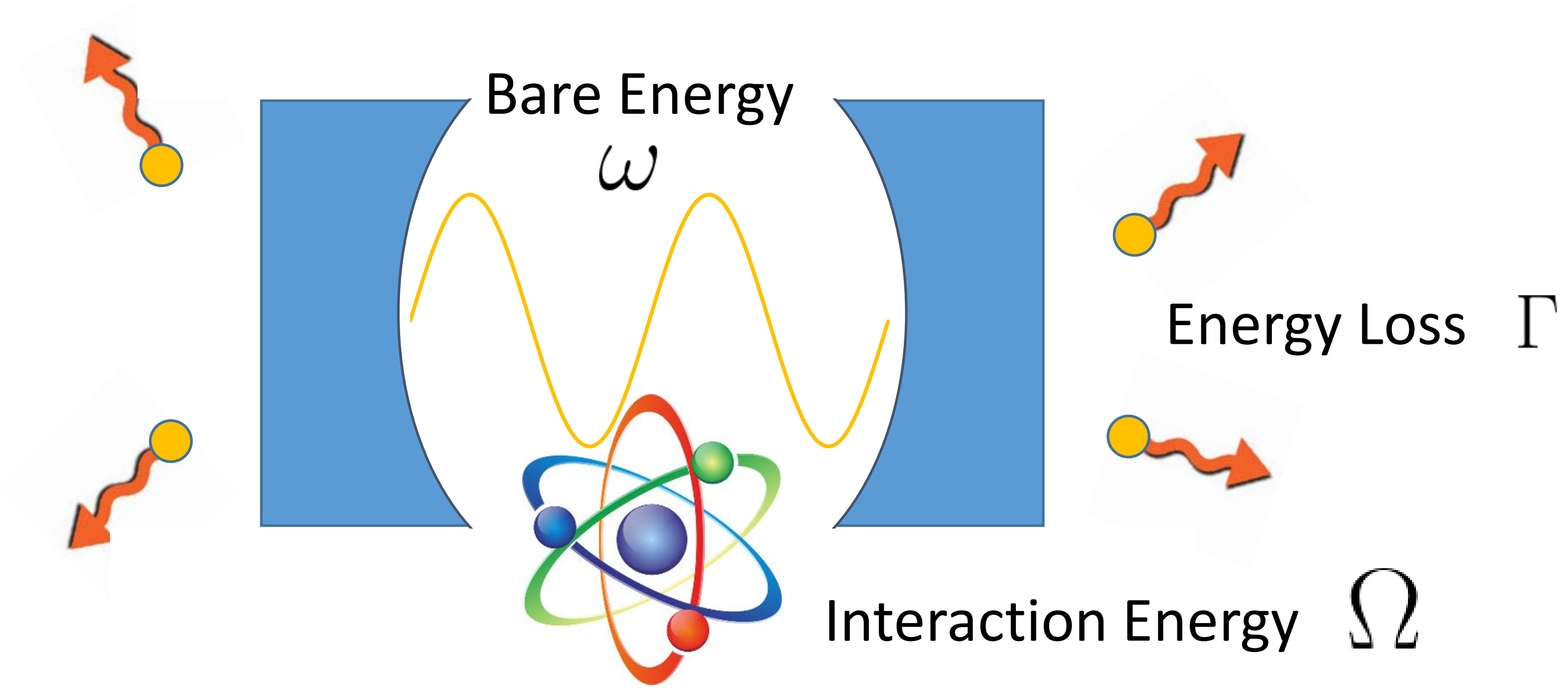
Credit: Nobel Prize

Cavity Quantum Electrodynamics

If we consider only one electromagnetic mode inside the cavity and if we model the atom as a two level system, then the Physics is described the **Rabi Hamiltonian.**

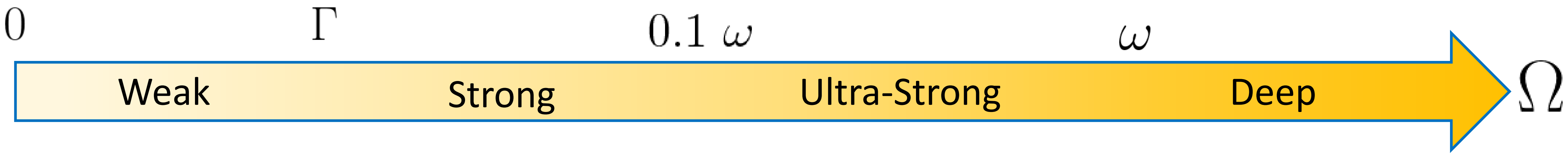
Cavity Quantum Electrodynamics

$$H_{\mathrm{Rabi}} = \omega\, a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-)$$

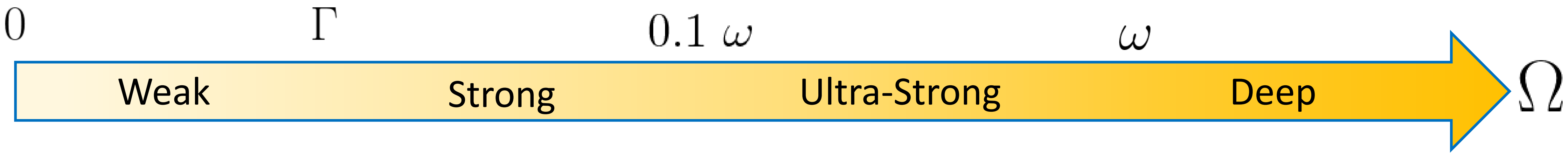


The Physics depends on three energy scales: the **bare energy** ω of the light and the atom (here on resonance), the **dipole interaction energy** Ω (Rabi Frequency) and the **Energy losses** Γ
The interplay between these energy scales gives rise to different physical regimes.

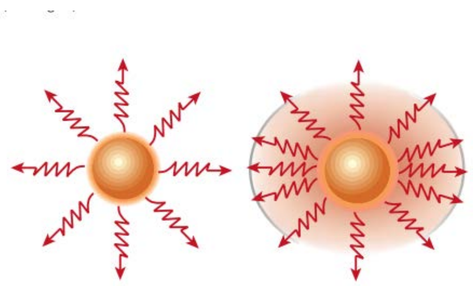
Cavity Quantum Electrodynamics



Cavity Quantum Electrodynamics



Purcell Effect



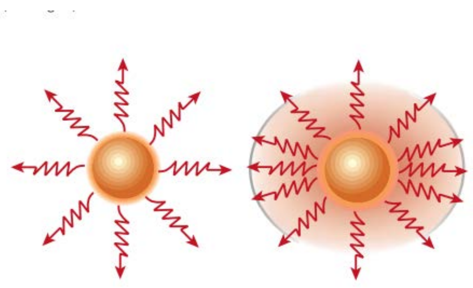
E. M. Purcell et al.,
Phys. Rev. **69**, 37 (1946)

Cavity Quantum Electrodynamics

0 — Γ — $0.1\ \omega$ — ω — Ω

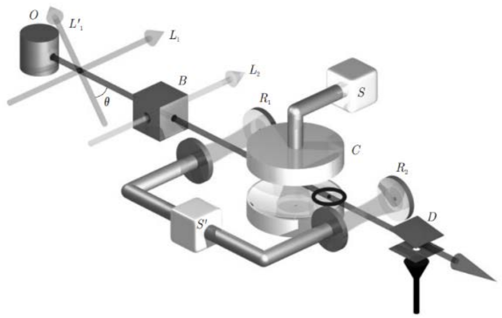
Weak | Strong | Ultra-Strong | Deep

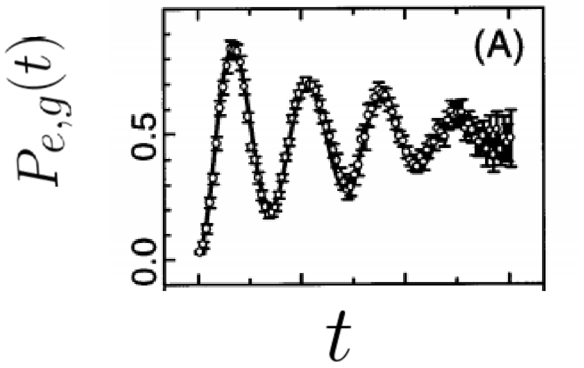
Purcell Effect



E. M. Purcell et al.,
Phys. Rev. **69**, 37 (1946)

Rabi oscillations





S. Haroche group

Cavity Quantum Electrodynamics

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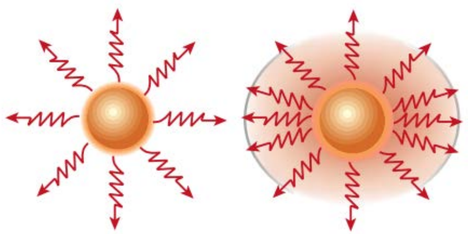

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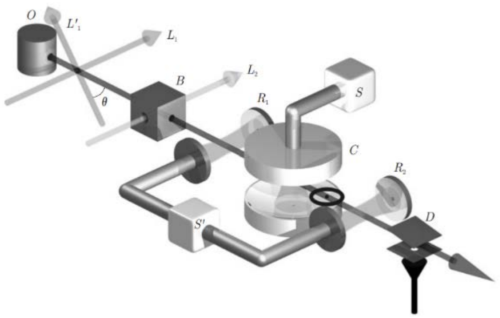
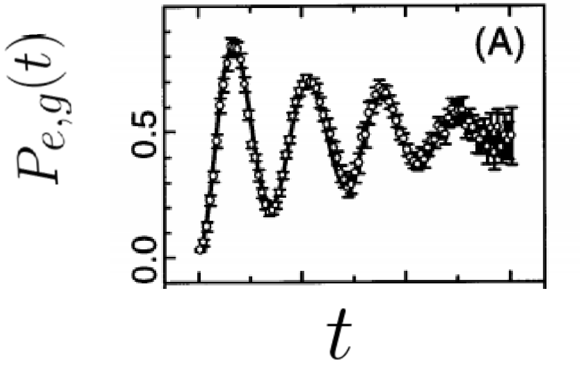
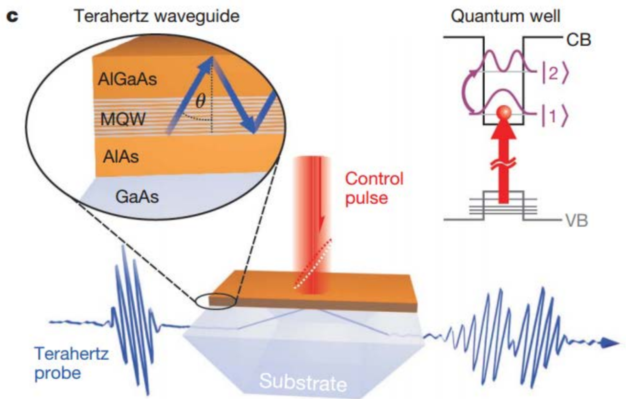


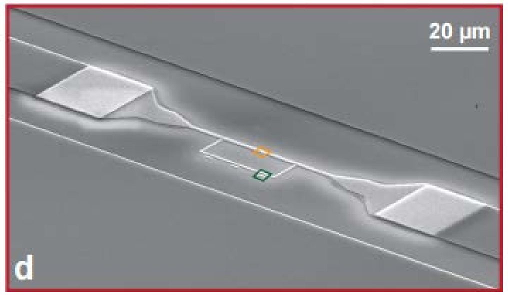


S. Haroche group

Virtual photons
dressing the Ground state



Nature **458**, (2009)



Nat. Phys. **6** (2010)

Cavity Quantum Electrodynamics

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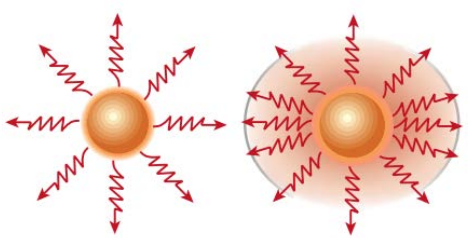
Rabi oscillations

Virtual photons dressing the Ground state

Entangled Ground State



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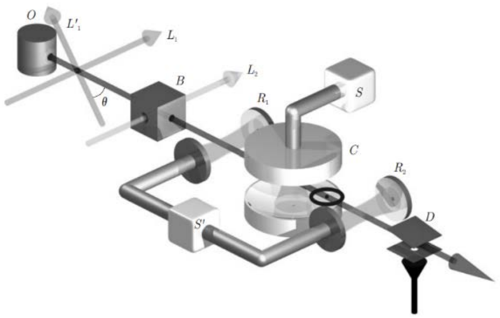
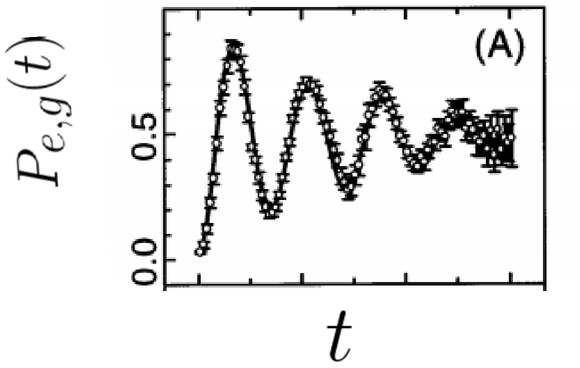
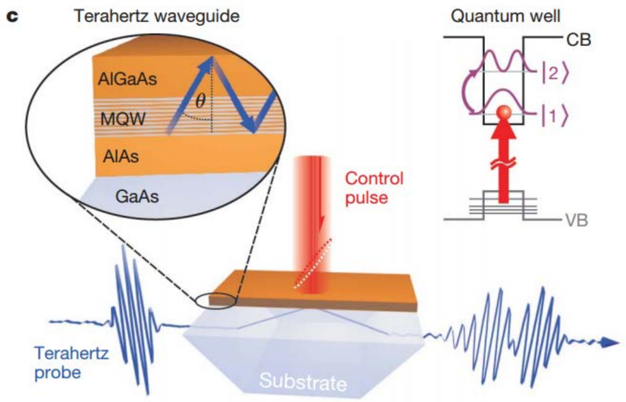
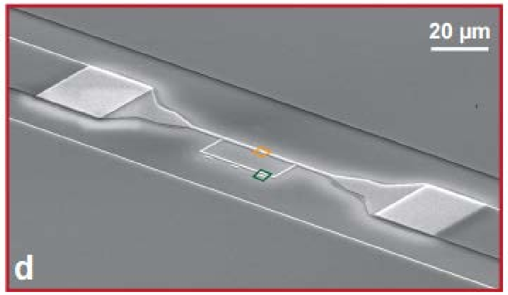




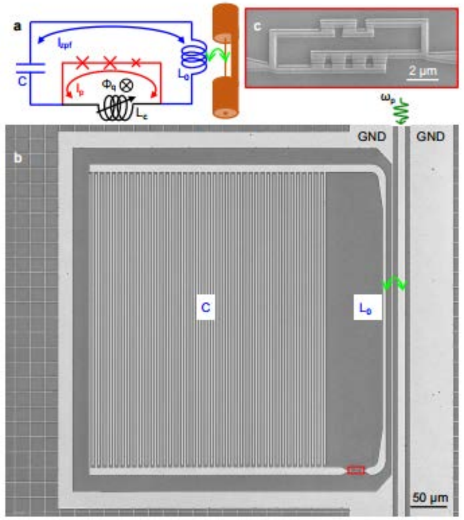
S. Haroche group



Nature **458**, (2009)



Nat. Phys. **6** (2010)



Nat. Phys. **13**, 44 (2017)

Cavity Quantum Electrodynamics

0 | Γ | $0.1\,\omega$ | ω | Ω

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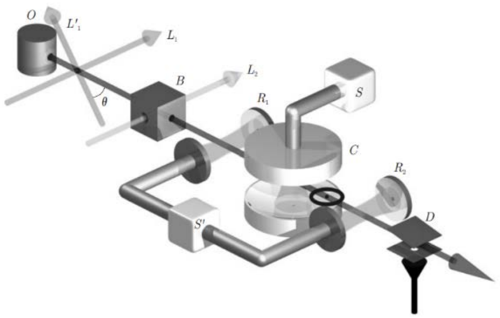
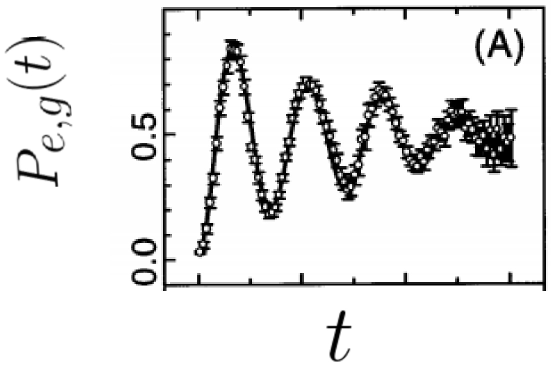
Purcell Effect

E. M. Purcell et al., Phys. Rev. **69**, 37 (1946)

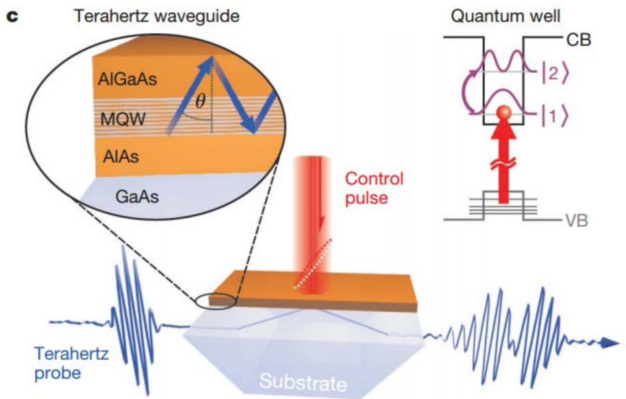
Rabi oscillations



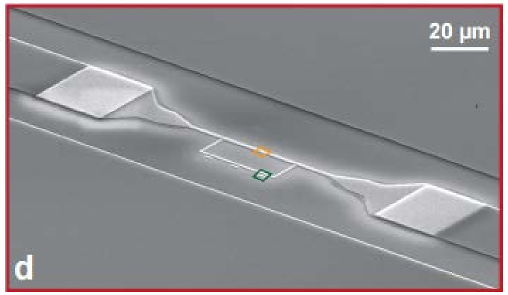


S. Haroche group

Virtual photons dressing the Ground state

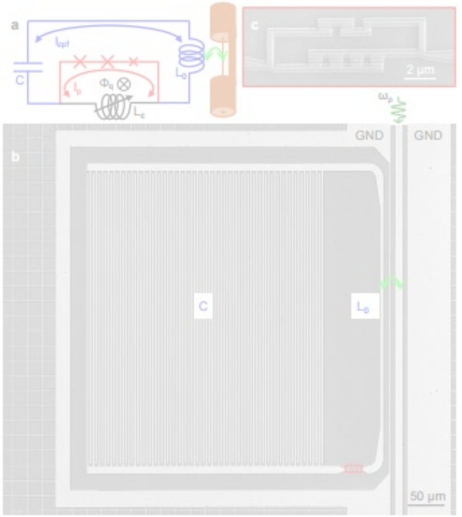


Nature **458**, (2009)



Nat. Phys. **6** (2010)

Entangled Ground State



Nat. Phys. **13**, 44 (2017)

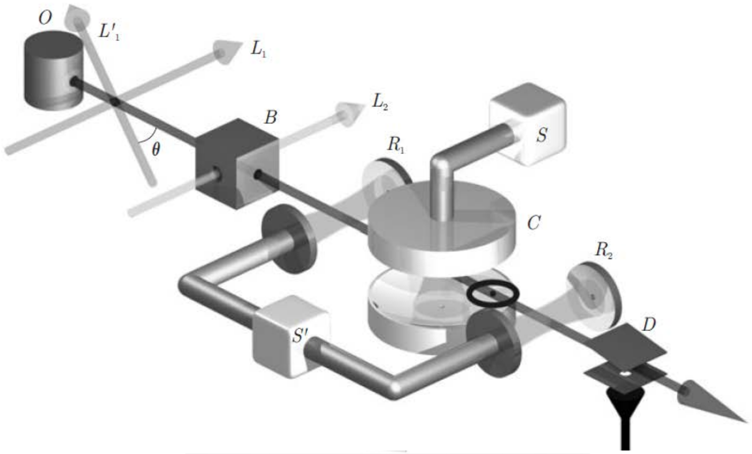
Cavity Quantum Electrodynamics

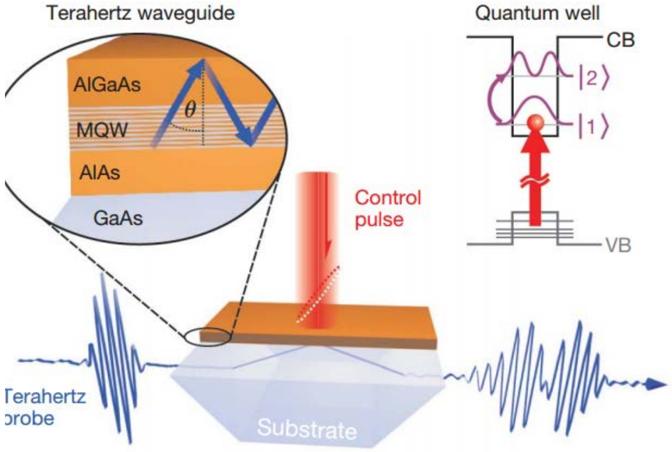
There are different strategies to reach strong/ultrastrong coupling

S. Haroche, J.-M. Raimond, *Exploring the quantum* (2006)

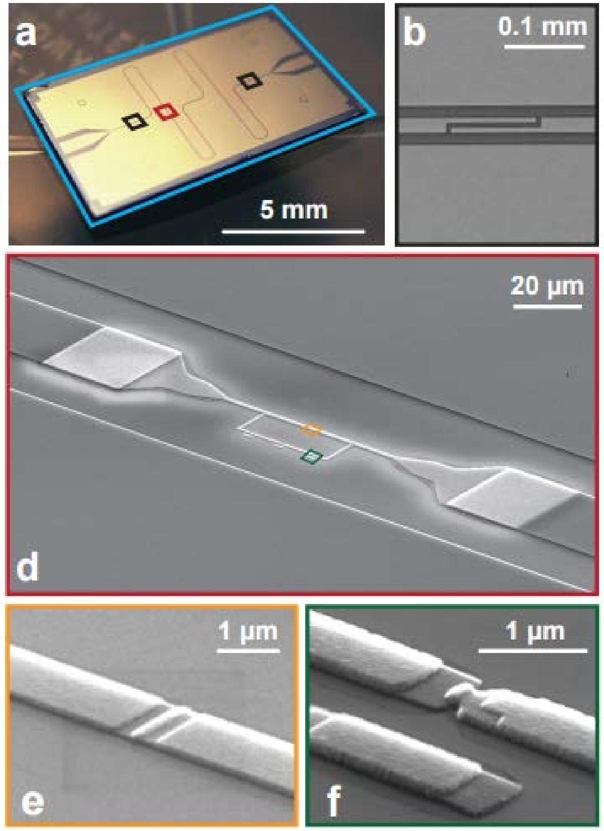
G. Günter et al., Nature **458**, (2009)

T. Niemczyk et al., Nat. Phys. **6** (2010)





Electron gas:
collective enhancement



- Rydberg atoms: high dipole moments
- Good Mirrors: low decay rate

- 1-dimensional resonators: small Volume
- Large Effective dipole (Capacitive coupling)

We studied effects arising when the light-matter coupling is **ultra-strong.**

More specifically, we studied effects related to the dressed structure of the light-matter **ground state** in the ultra-strong coupling regime.

Before proceeding, let us then introduce the properties of the light-matter ground state.

Light-Matter Ground State
In the ultra-strong coupling regime

Light-matter dressed states

Let us go back studying the Rabi Hamiltonian.

$$H_{\mathrm{R}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a^\dagger\sigma^- + a\sigma^+) + \Omega(a^\dagger\sigma^+ + a\sigma^-)$$

Light-matter dressed states

Let us temporarily set the Rabi frequency to zero, i.e., no interaction.
Now, the atom and light are independent.

$$H_{\mathrm{R}} = \omega a^{\dagger} a + \frac{\omega}{2}\sigma_z + \Omega(a^{\dagger}\sigma \quad a\sigma^{+}) + \Omega(a^{\dagger}\sigma \quad a\sigma^{-})$$

E

$|1, e\rangle$
$|2, g\rangle$

$|0, e\rangle$
$|1, g\rangle$

$|0, g\rangle$

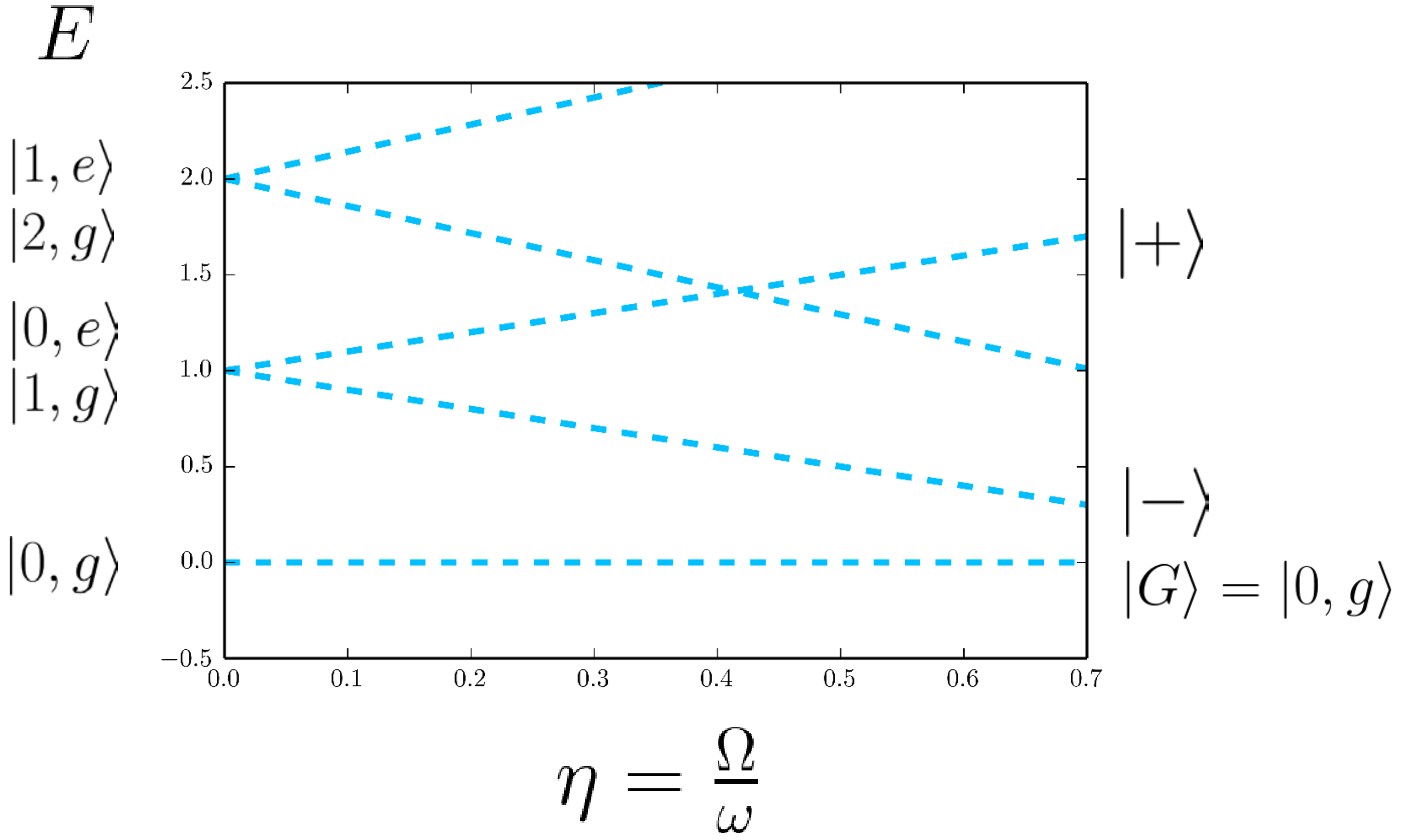
The field can have n photons

The atom can be in the ground |*g*> or excited state |*e*>

Light-matter dressed states: Let us now consider the interaction between light and matter.

When $\Omega \ll \omega$ it is possible to neglect the so-called counter-rotating terms, obtaining the **Jaynes Cummings Hamiltonian**. Now, **excited states hybridize**.

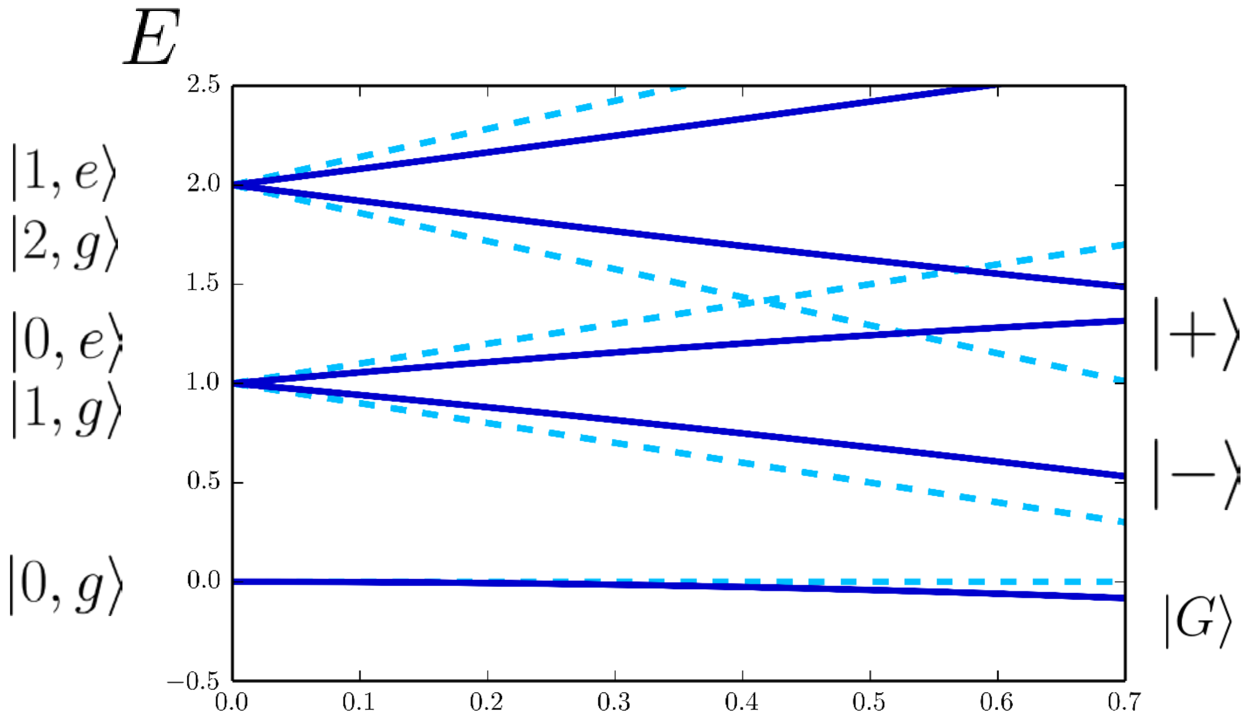
$$H_{\mathrm{R}} = \omega a^{\dagger} a + \frac{\omega}{2}\sigma_z + \Omega(a^{\dagger}\sigma^{-} + a\sigma^{+}) + \Omega(a^{\dagger}\sigma^{+} + a\sigma^{-})$$



Light-matter dressed states: In the **ultra-strong coupling regime** we must consider the full Hamiltonian

In this regime the **ground state is a coherent superposition** of states with different number of bare excitations.

$$H_R = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a^\dagger\sigma^- + a\sigma^+) + \Omega(a^\dagger\sigma^+ + a\sigma^-)$$

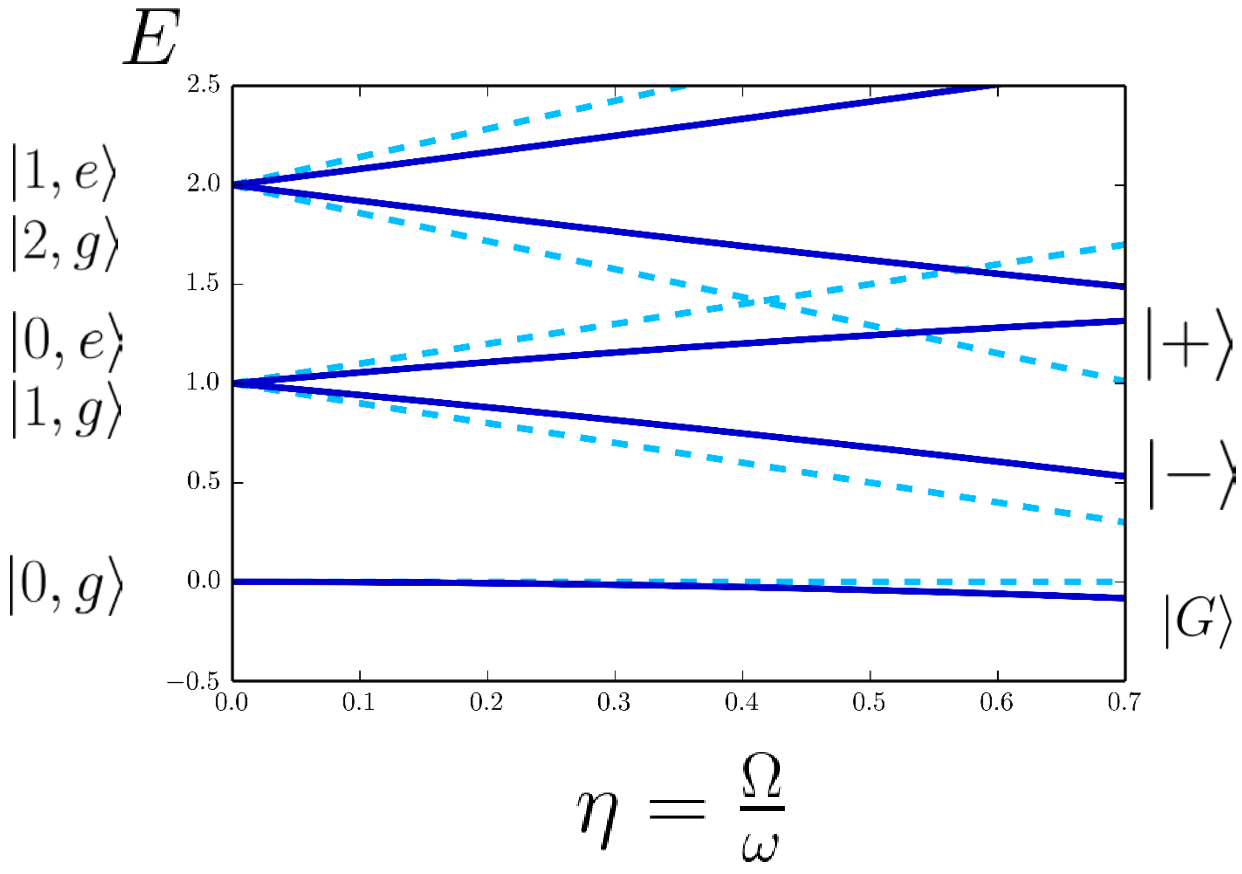


$$|G\rangle = (1-\frac{\eta^2}{8})|0,g\rangle + \frac{\eta}{2}|1,e\rangle + \frac{\eta^2}{2\sqrt{2}}|2,g\rangle$$

$$\eta = \frac{\Omega}{\omega}$$

Light-matter dressed states: In the **ultra-strong coupling regime** the expected number of photons in the ground state is non-zero! These are called **virtual photons** since they cannot be spontaneously emitted (because the ground state has the lowest energy).

$$H_{\mathrm{R}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a^\dagger\sigma^- + a\sigma^+) + \Omega(a^\dagger\sigma^+ + a\sigma^-)$$



Virtual photons

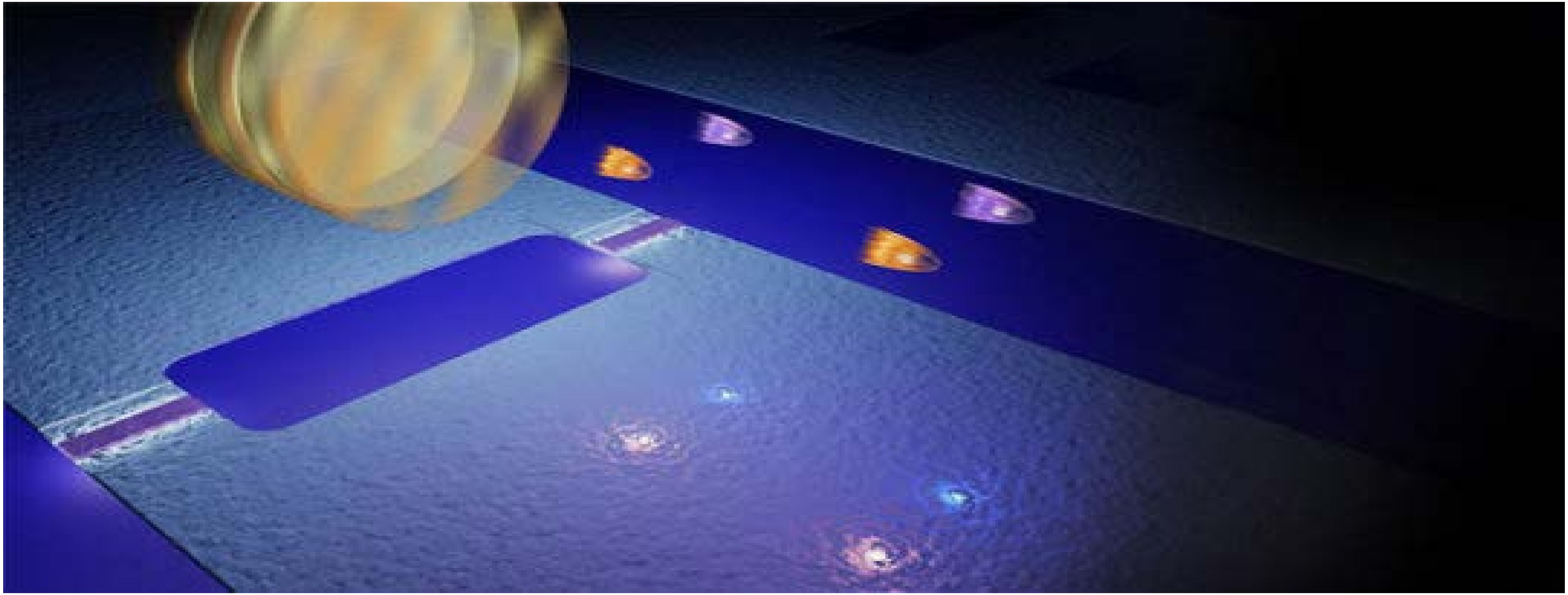
$$\langle G|a^\dagger a|G\rangle = \frac{\eta^2}{4}$$

$$|G\rangle = \left(1-\frac{\eta^2}{8}\right)|0,g\rangle + \frac{\eta}{2}|1,e\rangle + \frac{\eta^2}{2\sqrt{2}}|2,g\rangle$$

Observing virtual photons: Non-adiabatic modulation of the coupling

How to **force** virtual photons to be emitted?

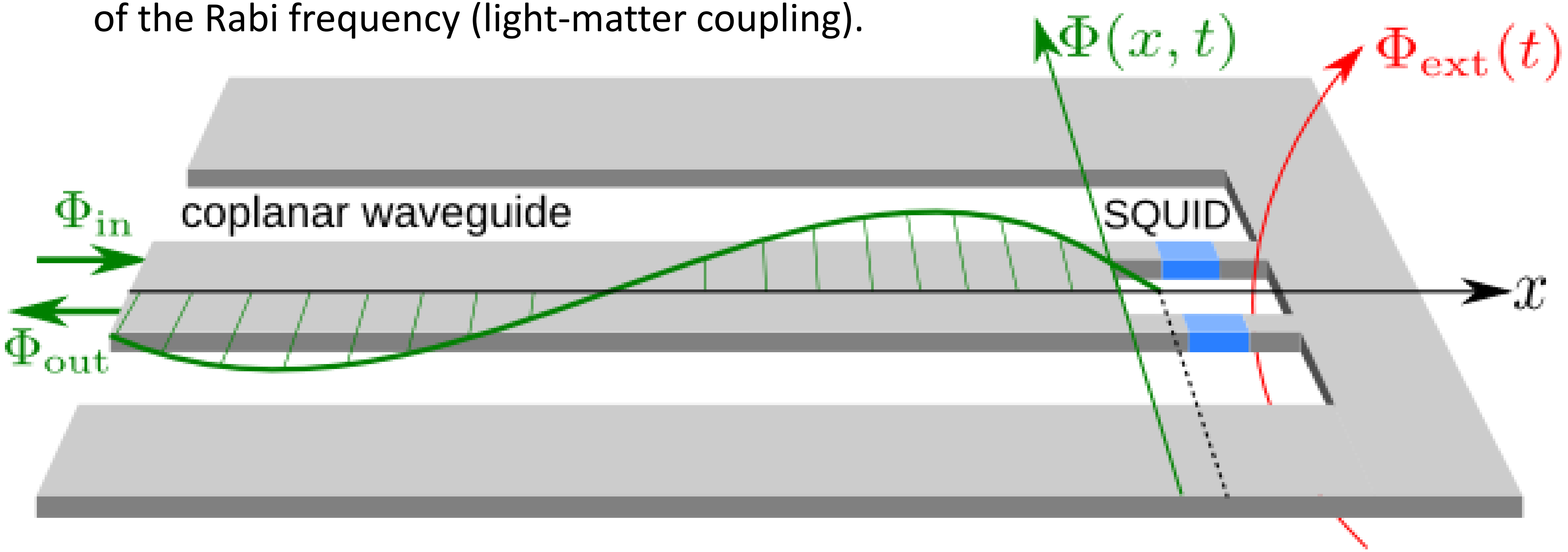
We proposed a **non-adiabatic modulation** of the Rabi frequency (light-matter coupling).



In collaboration with Chalmers: PRL (2009); PRA (2010); RMP (2012); PRA (2013).
Experiments in *Nature* (2011). Top five breakthrough of 2011, according to *Physics World*.

Observing virtual photons: Non-adiabatic modulation of the coupling

How to **force** virtual photons to be emitted?
For the Dynamical Casimir Effect,
we proposed a **non-adiabatic modulation**
of the Rabi frequency (light-matter coupling).



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Observing virtual photons: Non-adiabatic modulation of the coupling

How to **force** the virtual photons to be emitted?
For the DCE, we proposed a **non-adiabatic modulation**
of the Rabi frequency (light-matter coupling).

In this (more recent) work we studied
how **virtual photons in the ground state can be emitted**
due to the **non-adiabatic modulation of the coupling**
induced by the flow of an electric current.

Ground-State Physics of light-matter systems in the ultra-strong coupling regime

Summary

Brief Introduction to Cavity Quantum Electrodynamics

Ground State Electroluminescence

Phys. Rev. Lett. **116**, 113601 (2016)

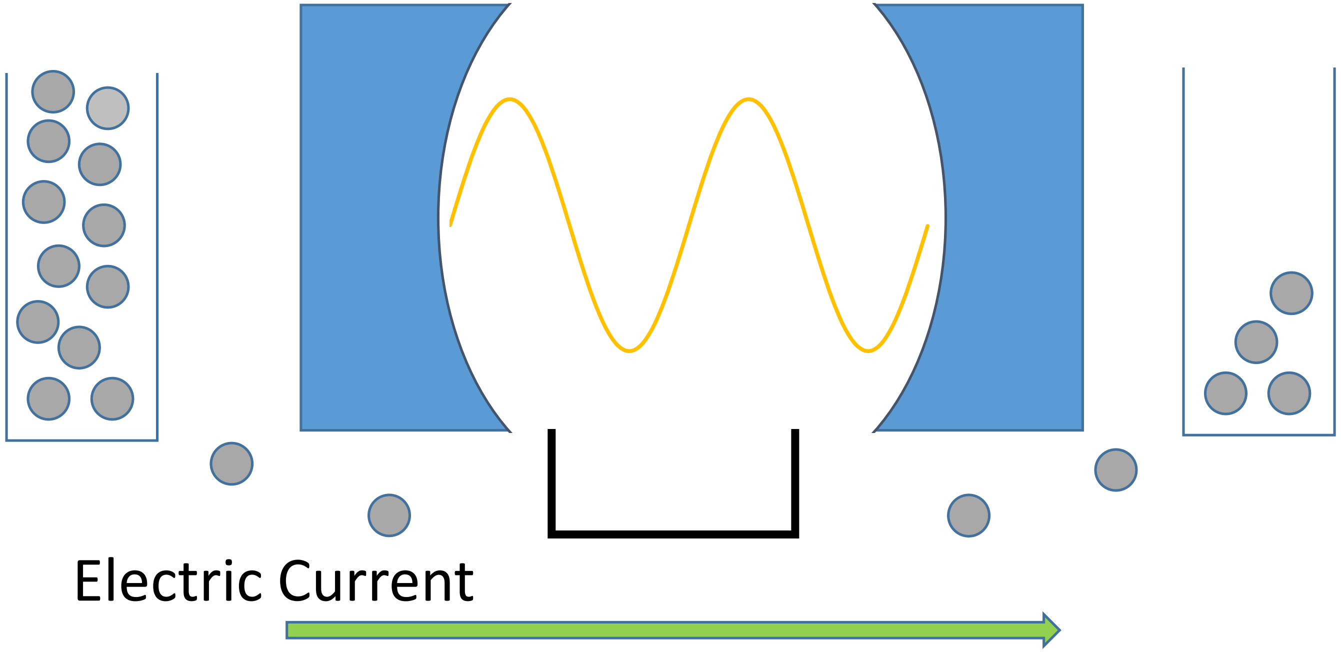
Opto-mechanical transduction of virtual radiation pressure

Phys. Rev. Lett. **119**, 053601 (2017)

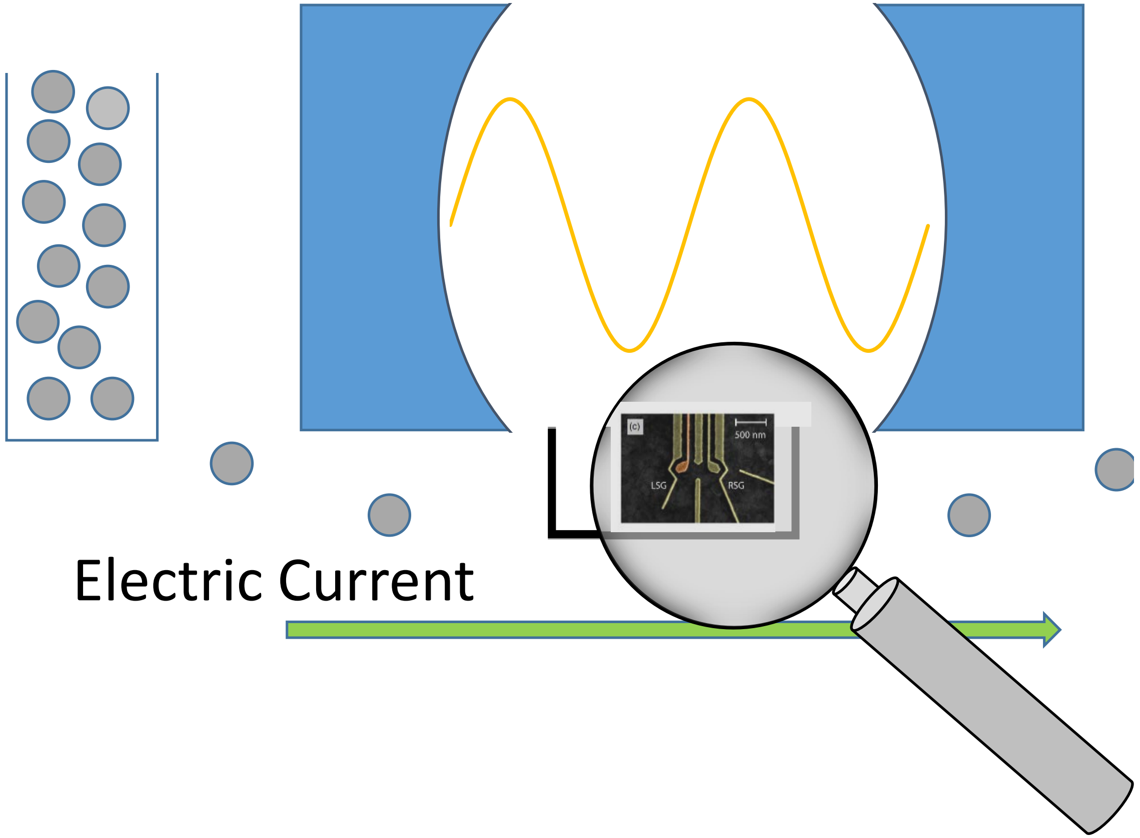
Electroluminescence: the **emission of light as current flows through a system**

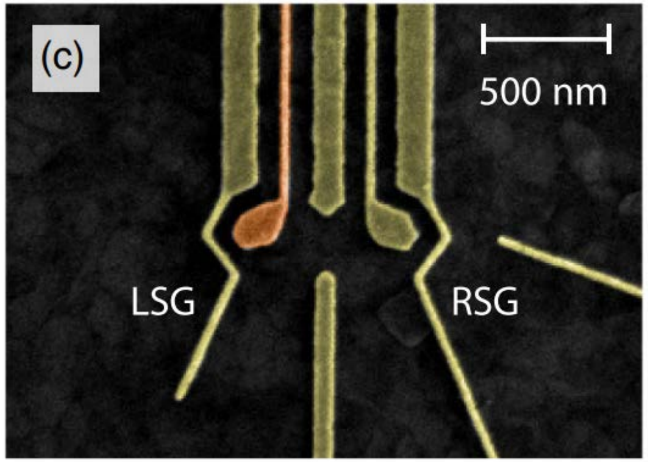
https://en.wikipedia.org/wiki/Electroluminescence

We consider an **electronic trap** where single electrons interact with a cavity mode.
Two **electronic reservoirs** can add or remove electrons from the trap,
allowing the flow of current through the system.



For concreteness, you can think of the trap as a double quantum dot device in the Coulomb blockade regime: which allows **one and only one electron** inside the system at any time.

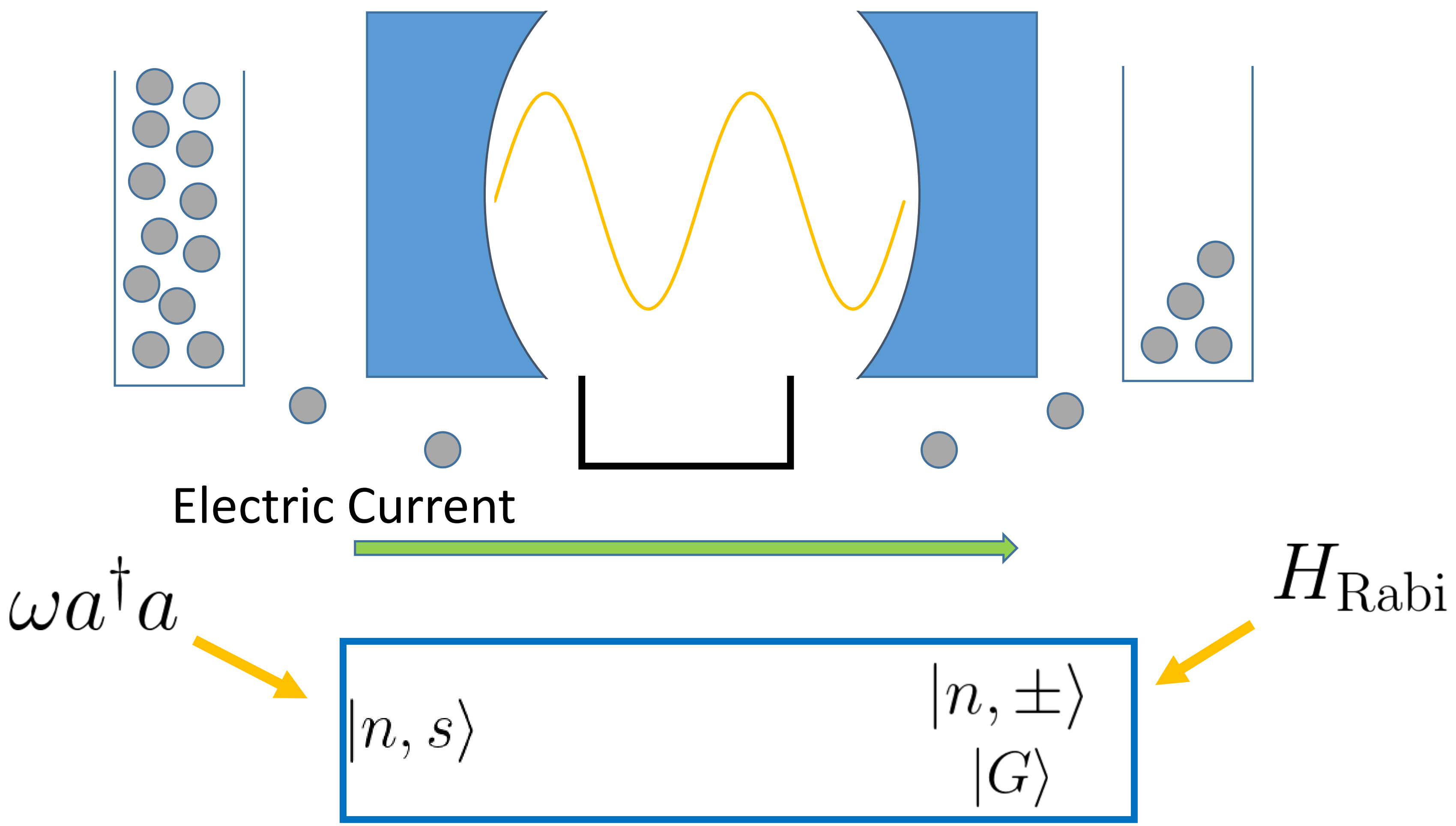




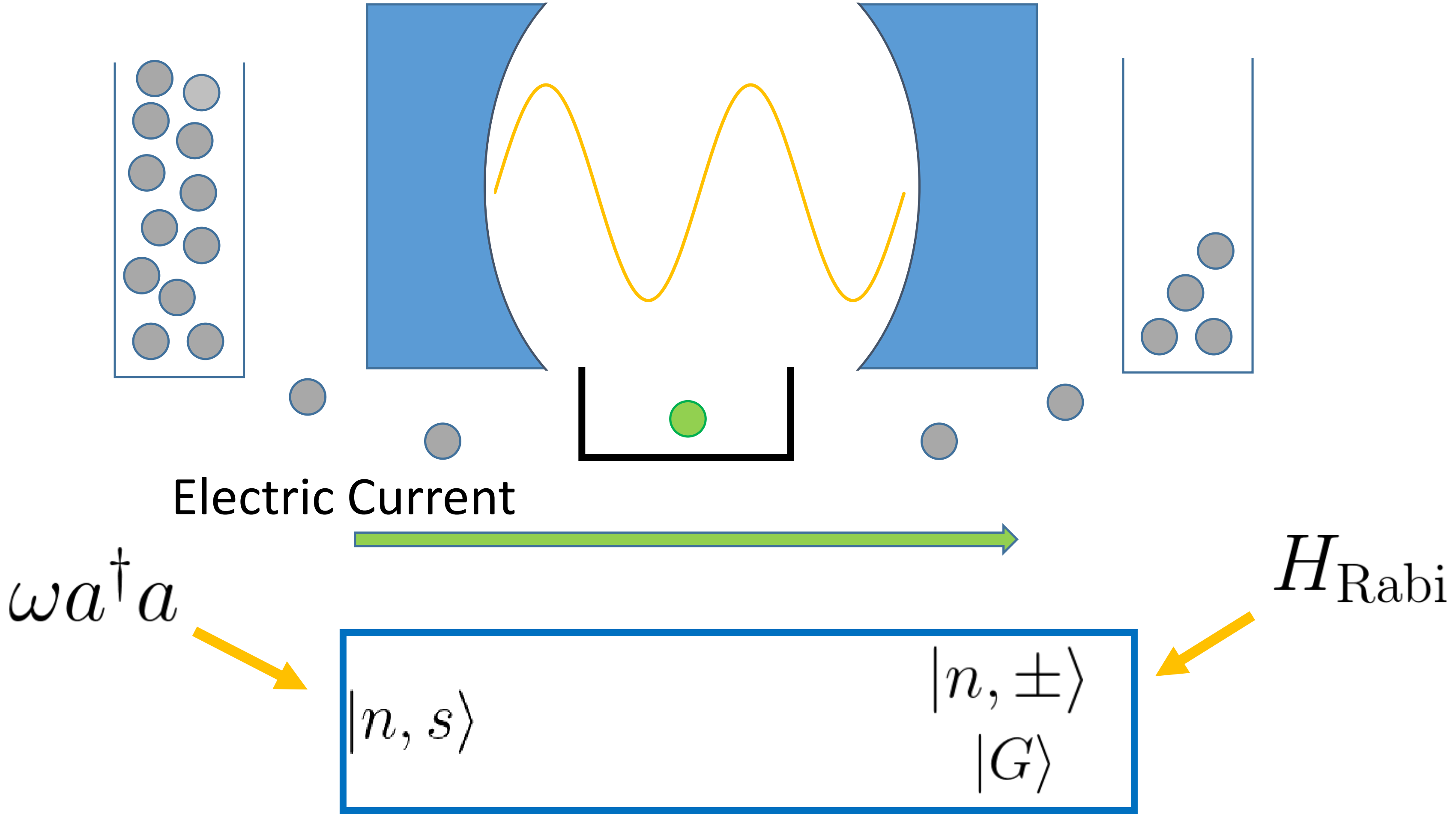
Phys. Rev. X **7**, 011030 (2017)

Let us now study the Hilbert Space of this system

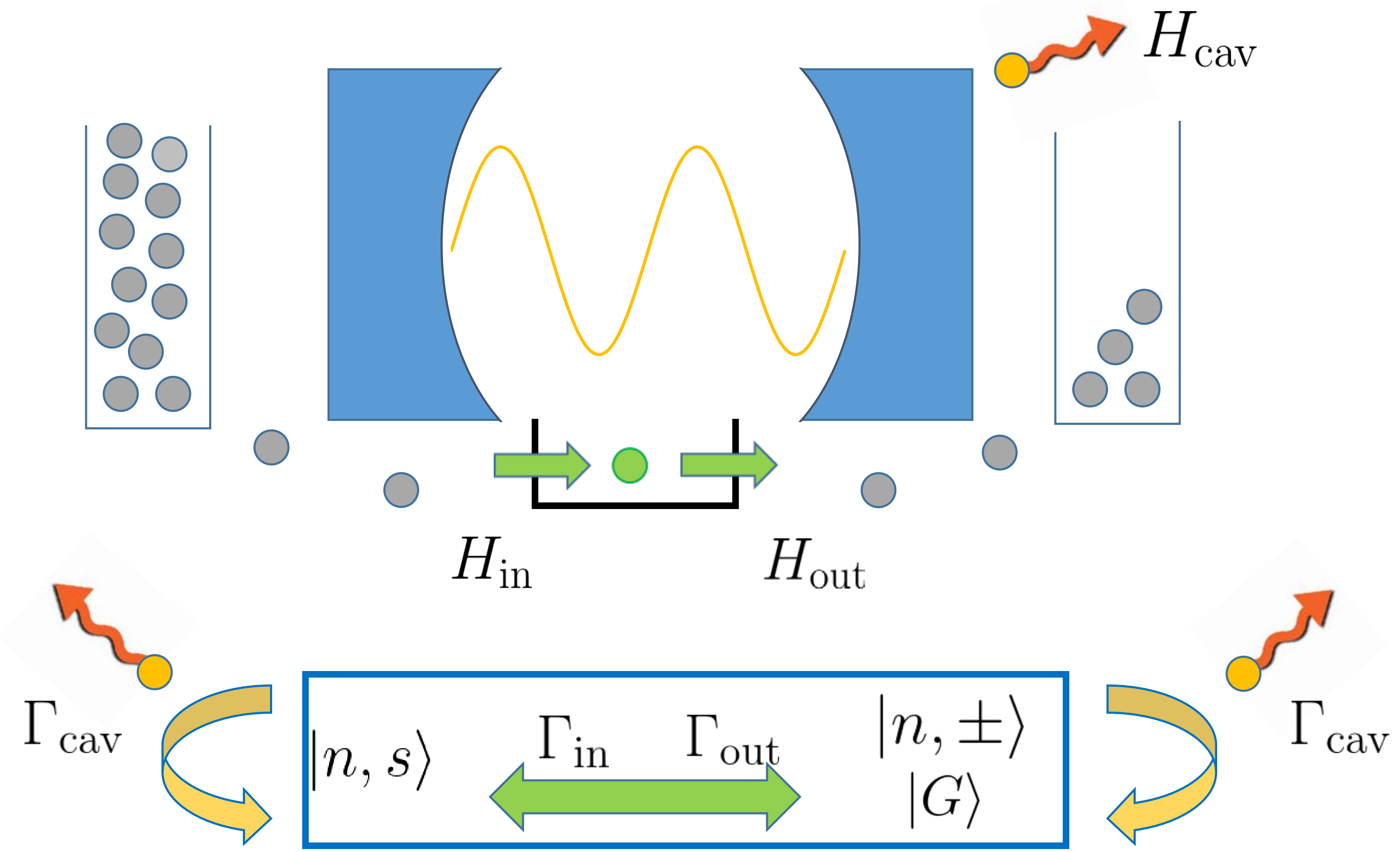
When **no electrons** are inside the trap there is **no light-matter interaction** (since there is no matter there). We have a **free sector** of the Hilbert space whose states are labelled with the letter s , and the number *n* of photons in the cavity. The dynamics describes **free photons** inside the cavity.



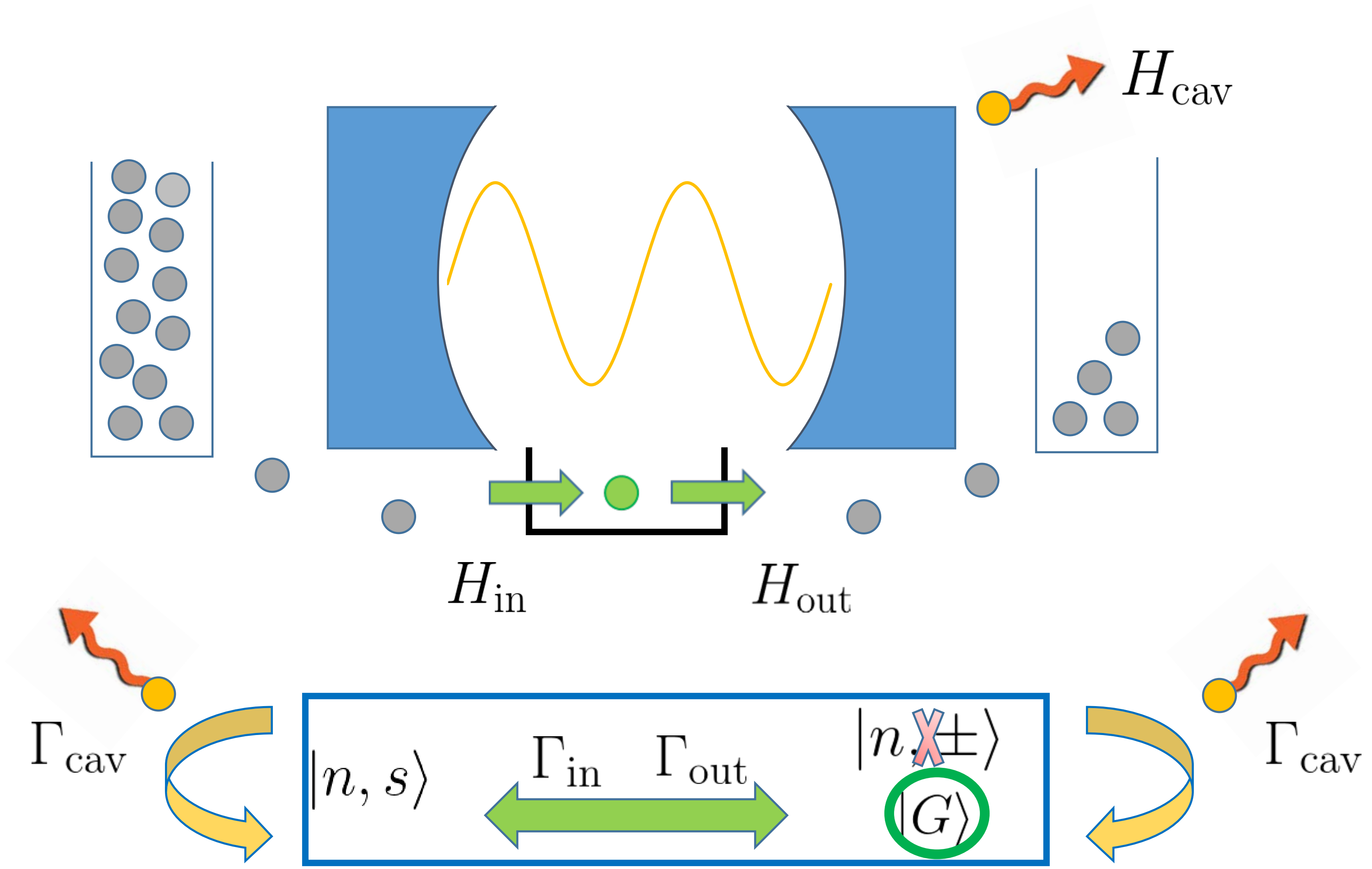
We assume that **one electron** inside the trap can be **in two states** (e.g., left or right dot) which we label as |*g*> or |*e*>. The electron interacts with the light through the **Rabi Hamiltonian** whose eigenstates (ground and higher polaritonic states) can be used to describe the system.



The Environment: **The electronic reservoirs** will cause **one electron** to **enter** or **leave** the system, allowing transitions between the free and interacting sector (green arrows). **Light leaking out** will only cause transitions within each sector (removing one photon. Yellow arrows below).

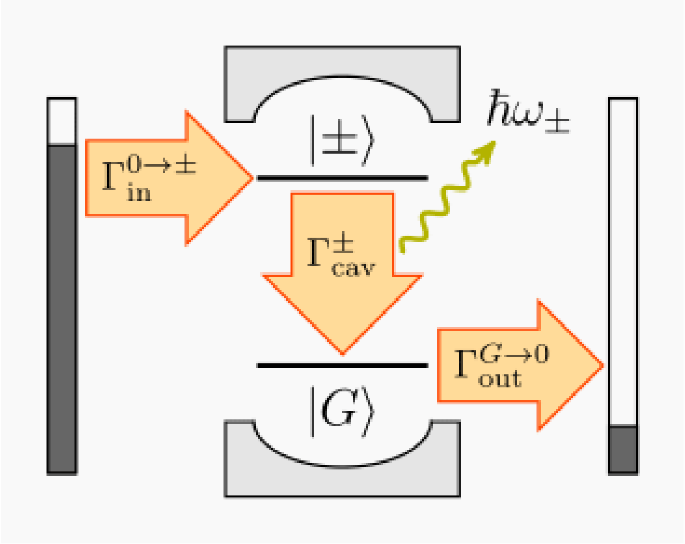


Question: is it possible to have a process which emits extra-cavity radiation without directly populating any excited state of the light-matter system but only its ground state?



Electroluminescence

Regular

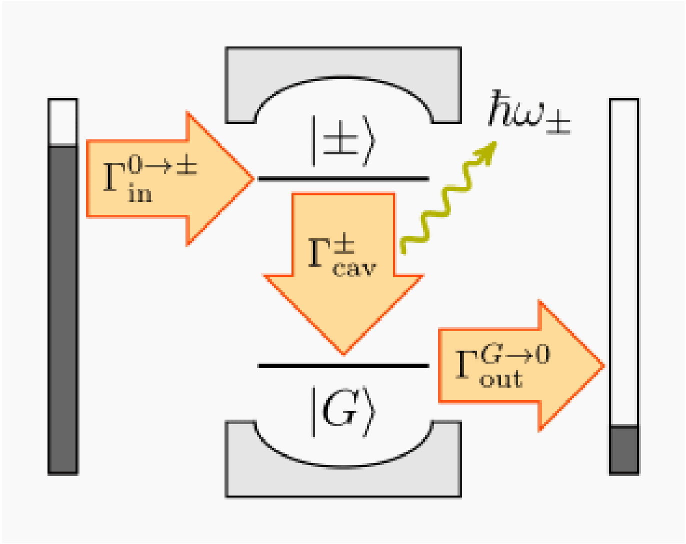


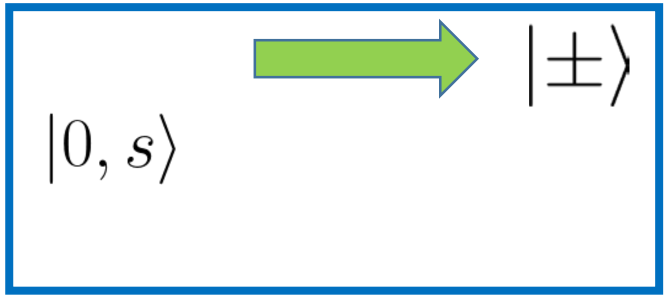
$$|0, s\rangle$$

Let us first consider what happens in regular electroluminescence. We start from **zero photons** in the cavity and **no electron** in the trap. The chemical potential is set high enough to allow electrons **to directly populate excited states**.

Electroluminescence

Regular

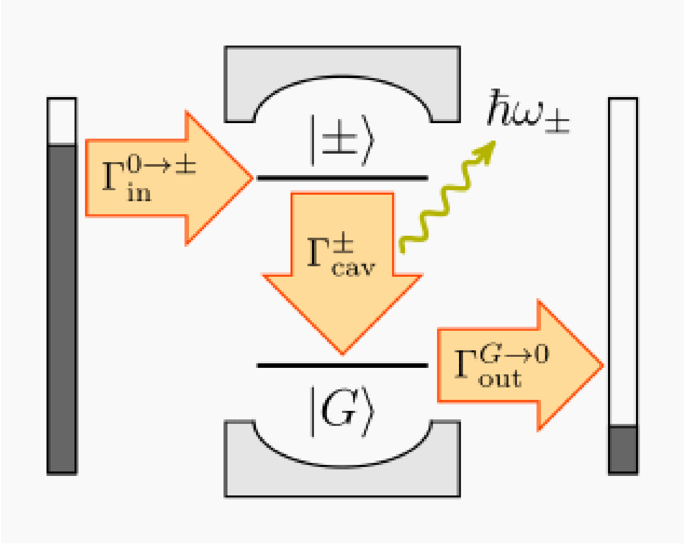


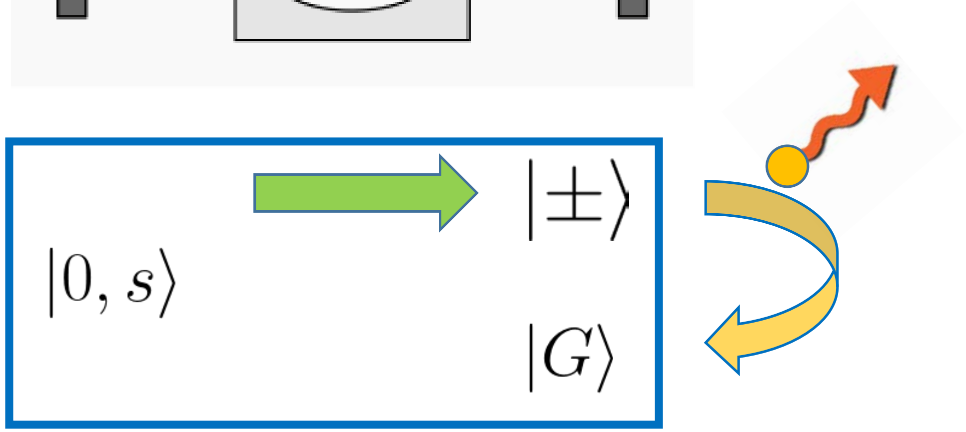


Indeed, one electron enters the system in an excited state.

Electroluminescence

Regular

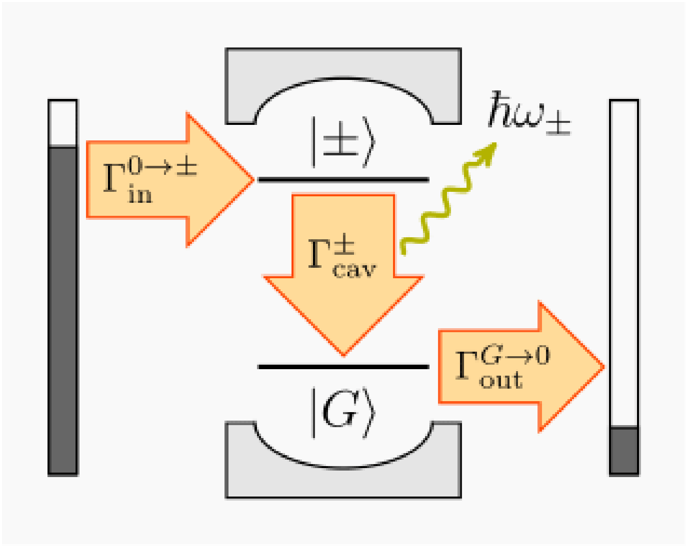


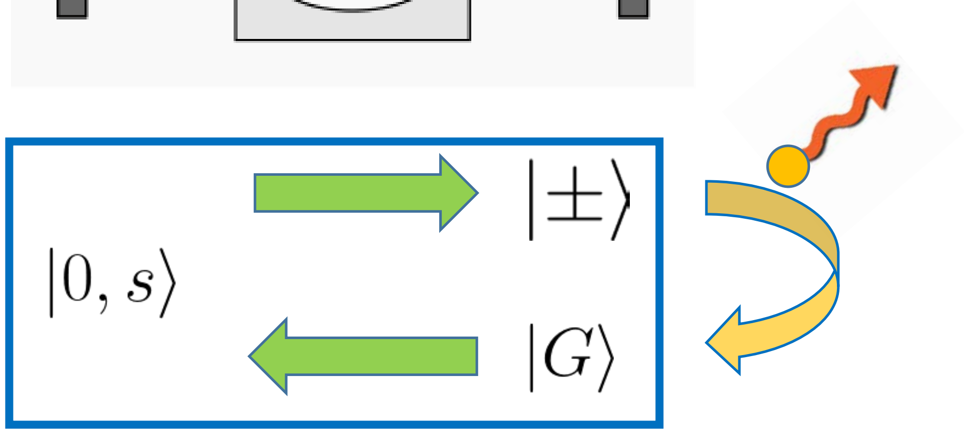


Indeed, one electron enters the system in an excited state. Eventually this state decays to the ground state emitting electroluminescent radiation outside the cavity.

Electroluminescence

Regular

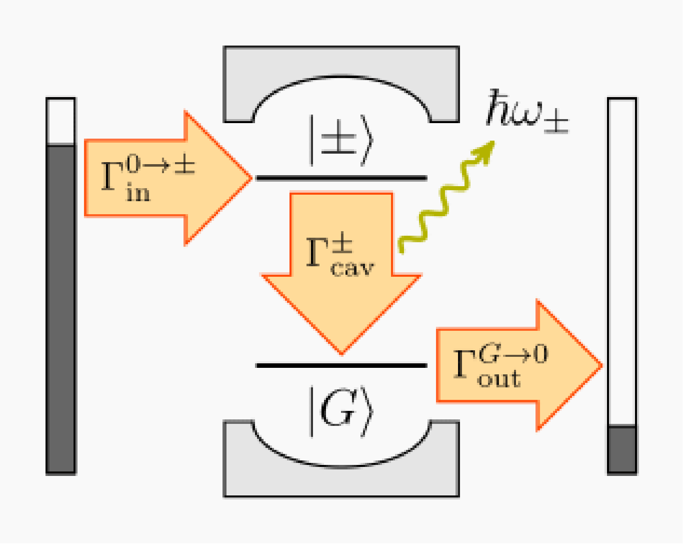


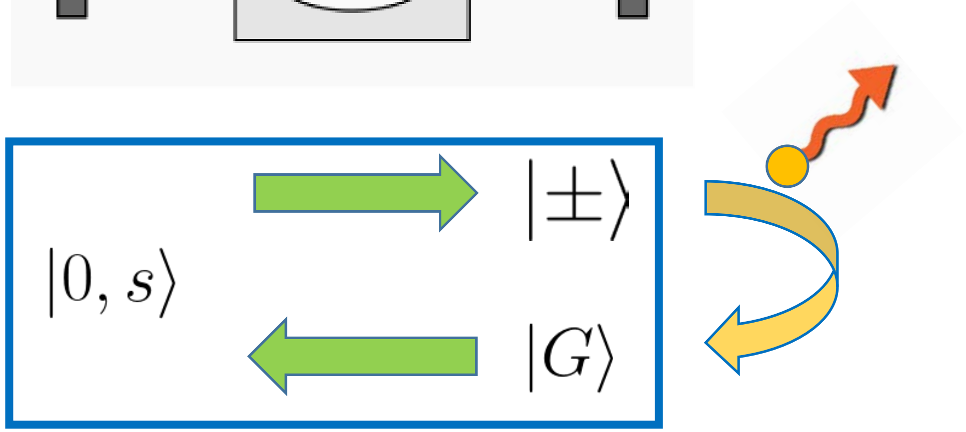


Indeed, one electron enters the system in an **excited state**. Eventually this state decays to the ground state emitting electroluminescent radiation outside the cavity. The electron then finally leaves the system. We have produced radiation after **populating a light-matter excited state**.

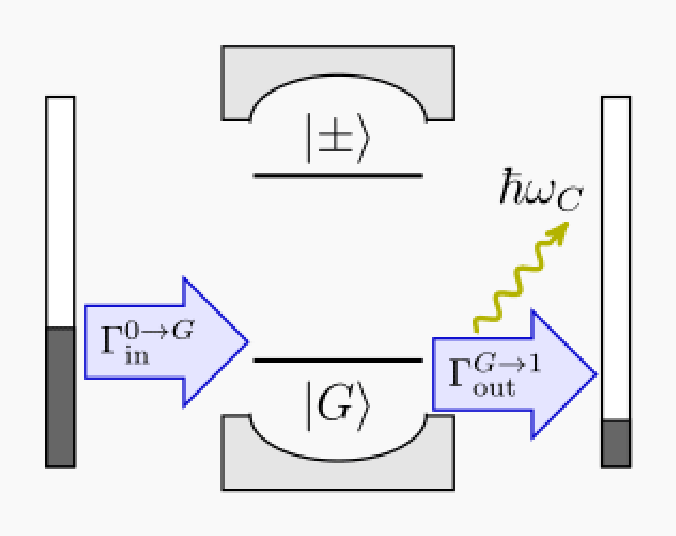
Electroluminescence

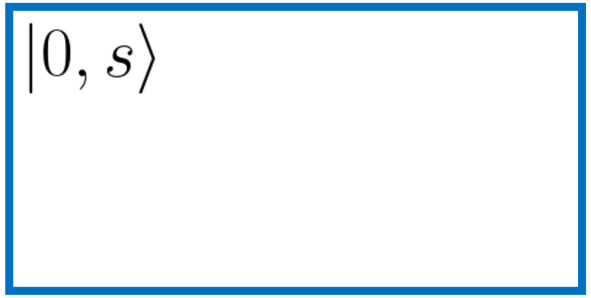
Regular





Ground State

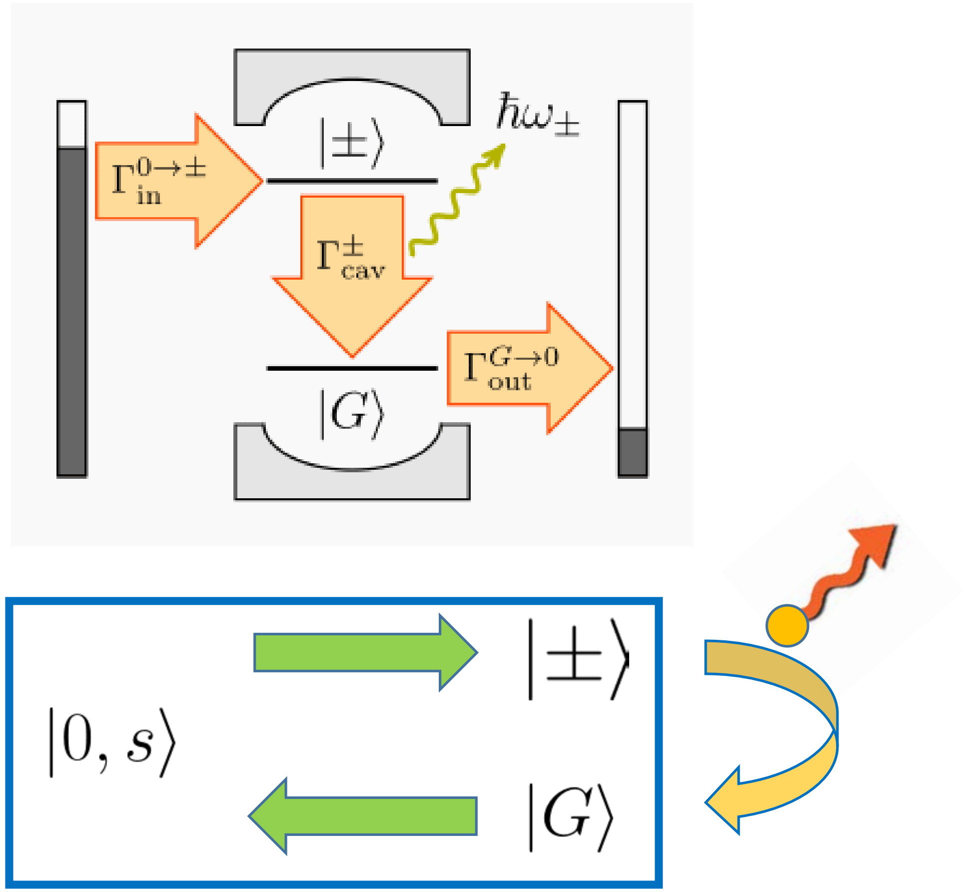




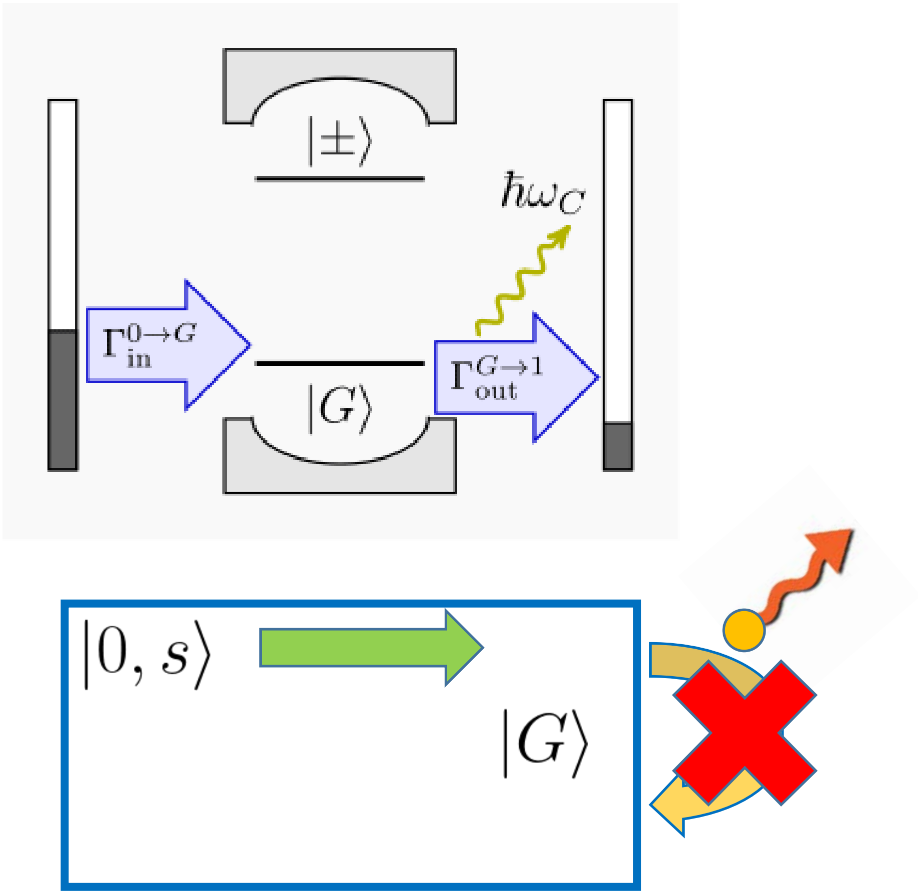
We now start again with **no photons** and **no electrons**. However, we now tune the chemical potential so that electrons can only populate the ground state.

Electroluminescence

Regular



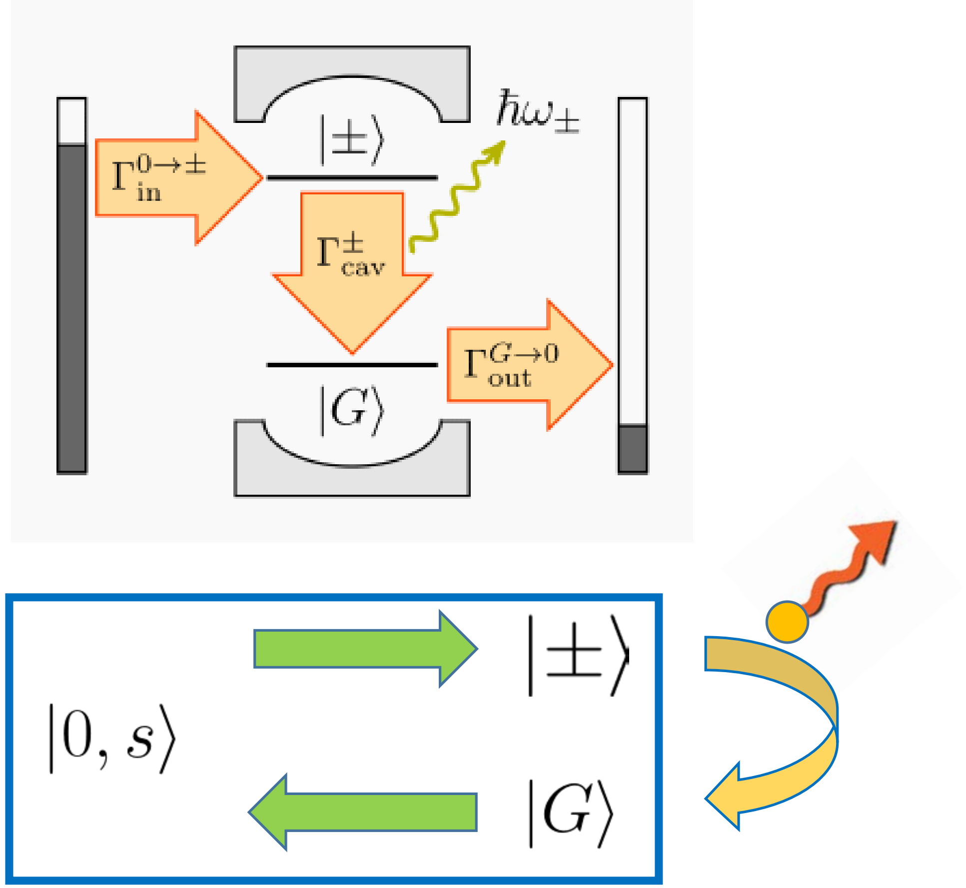
Ground State



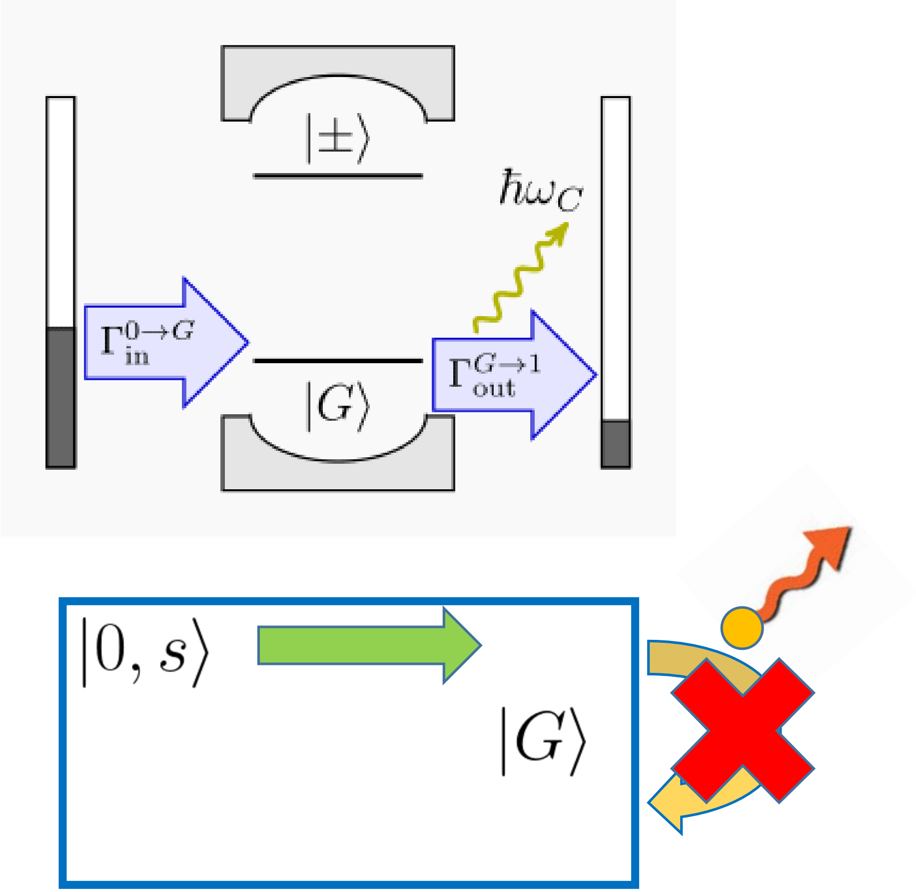
Indeed the electron enters the system in the **ground state**, which cannot decay.

Electroluminescence

Regular



Ground State

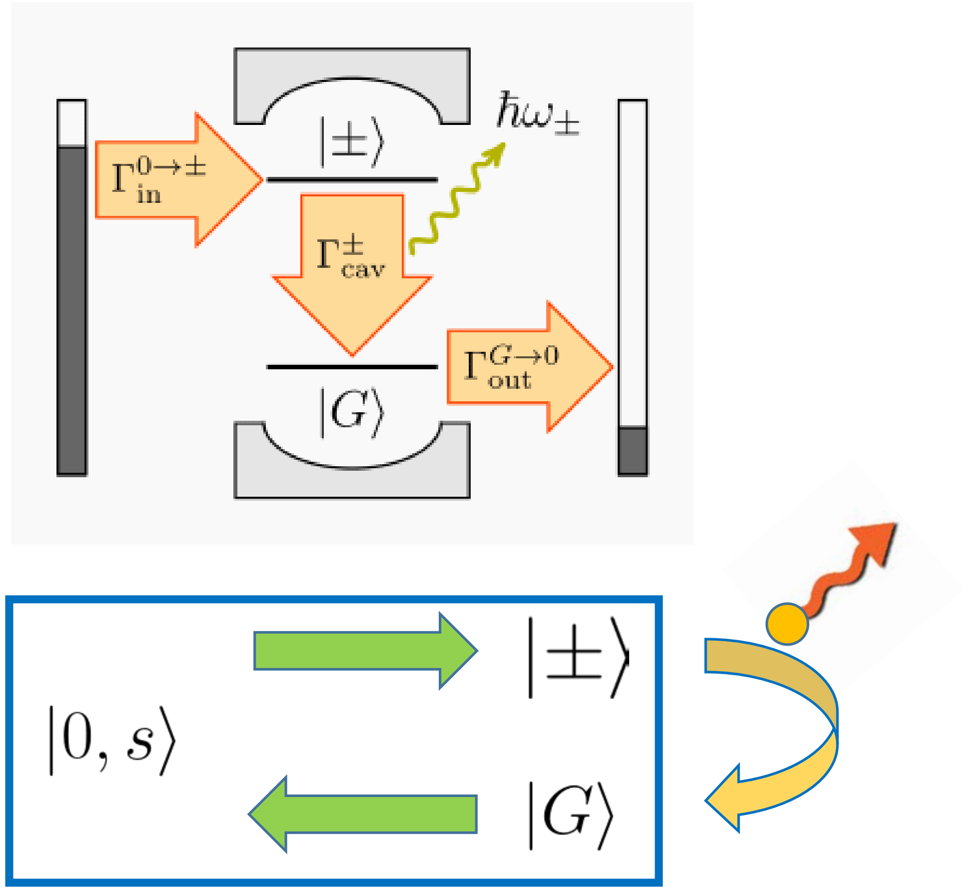


Indeed the electron enters the system in the **ground state**, which cannot decay. However, in the **ultra-strong coupling regime**, the ground state has a component with one photon!

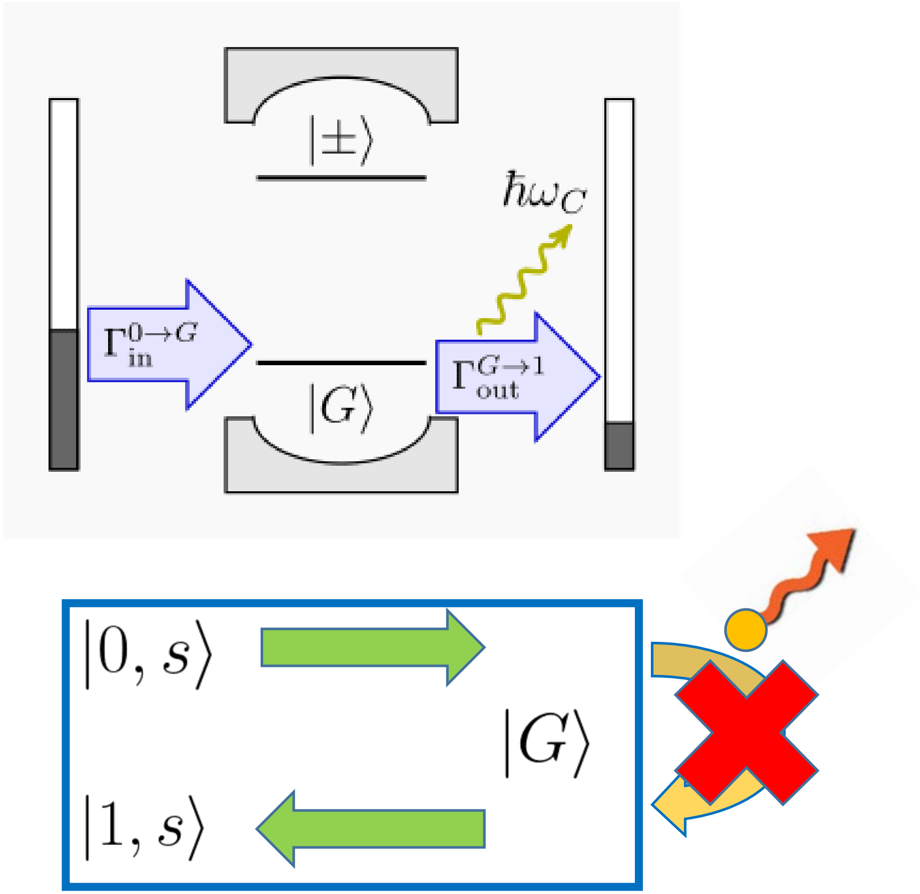
$$|G\rangle = \left(1-\frac{\eta^2}{8}\right)|0,g\rangle + \frac{\eta}{2}|1,e\rangle + \frac{\eta^2}{2\sqrt{2}}|2,g\rangle$$

Electroluminescence

Regular



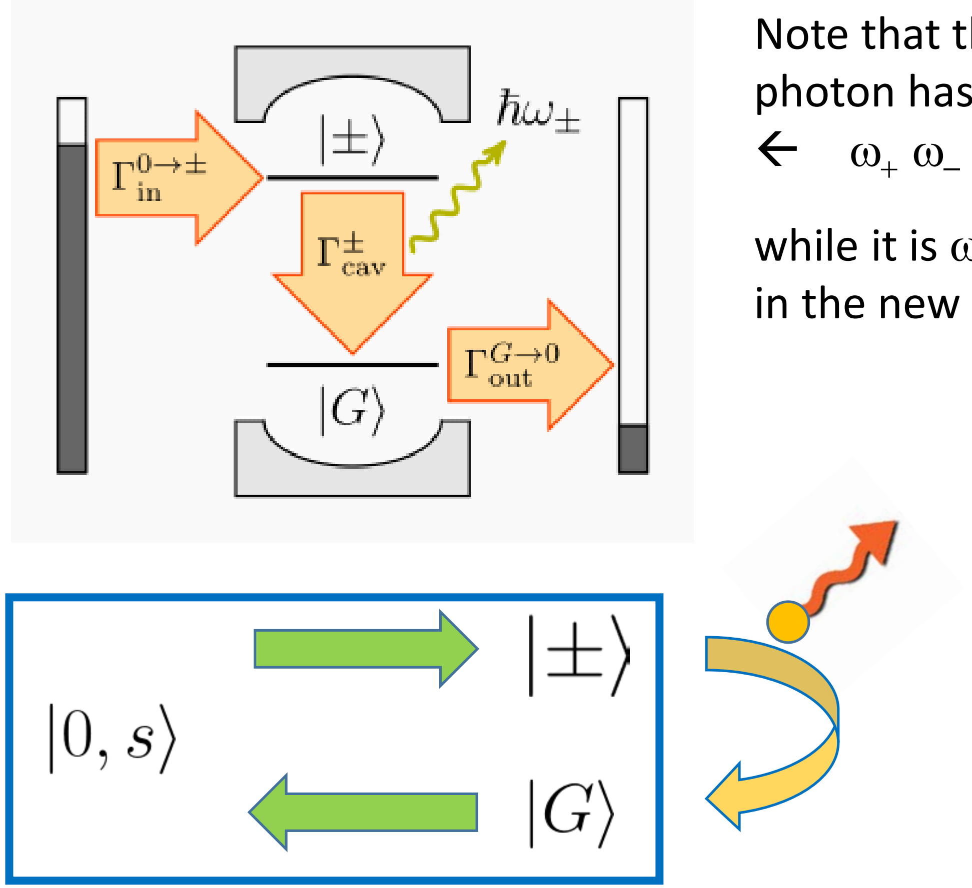
Ground State



In the **ultra-strong coupling regime**, the ground state has a component with one photon! **There is then a finite probability ($\alpha\ \eta^2$) that the electron tunnels out leaving one photon inside the system.** **The $|1,e\rangle$ becomes $|1, s\rangle$ when the electron leaves the system.**

Electroluminescence

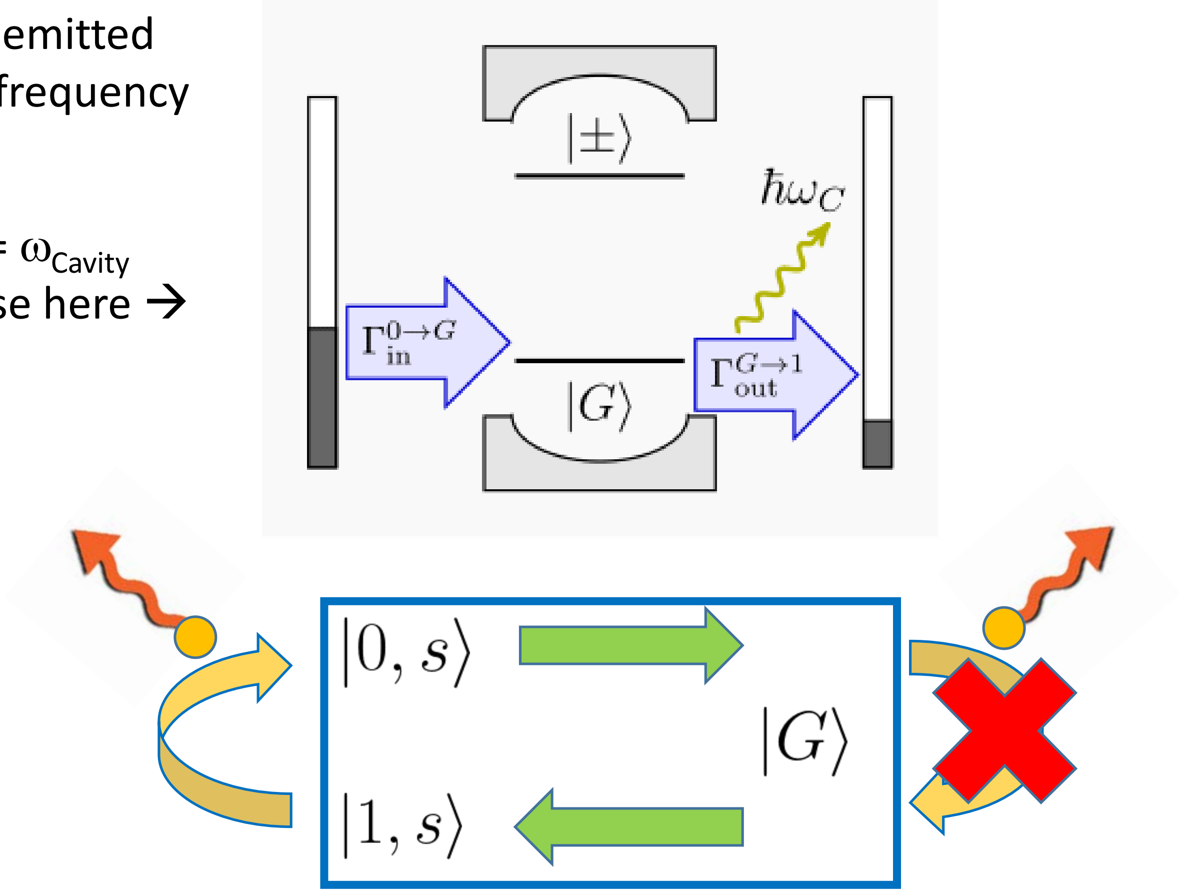
Regular



Note that the emitted photon has a frequency ← ω_+ ω_-

while it is $\omega_C = \omega_{Cavity}$ in the new case here →

Ground State



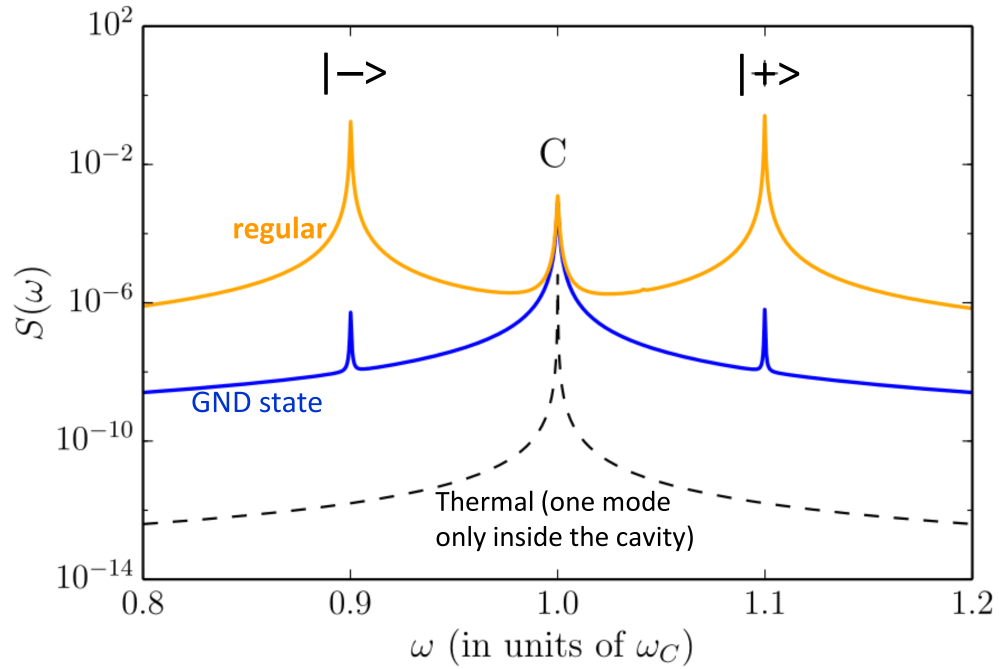
Which can only now decay emitting ground state electroluminescent radiation.

Distinguishing Features:

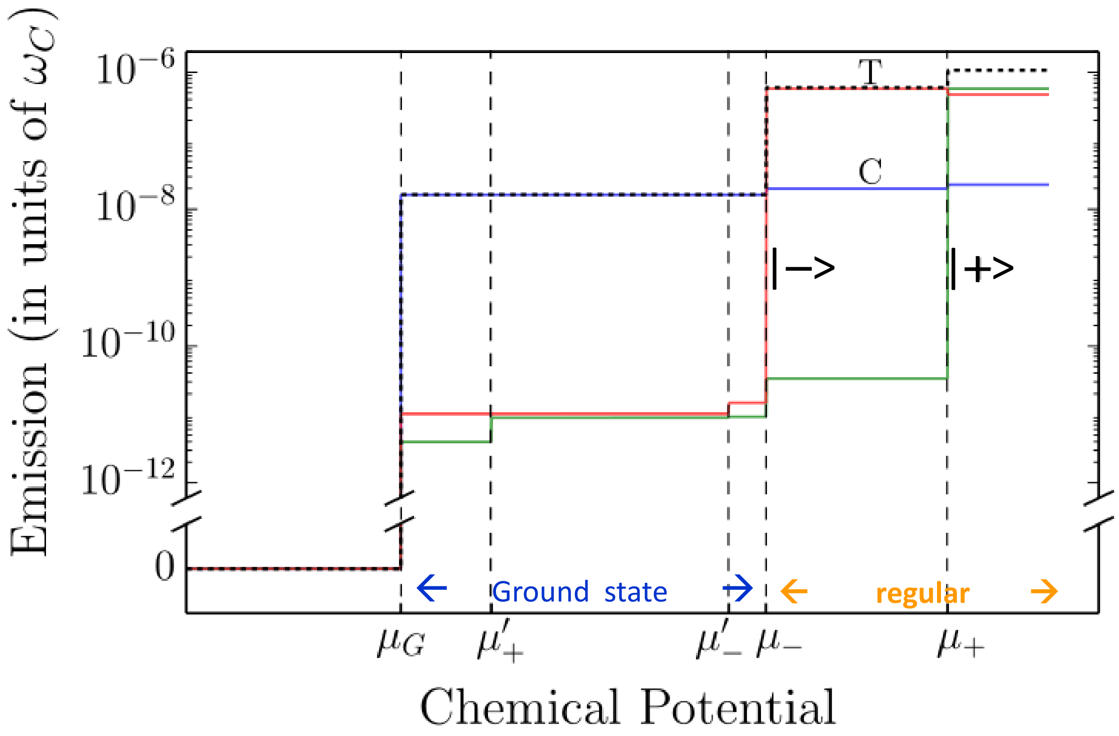
The ground-state extra-cavity radiation has several features which distinguish it from the regular channel.

Emission Properties:

Emission Spectrum



Dependency on Chemical Potential



The emission spectrum is qualitatively different from the regular channel. E.g., in the **regular** (GND-state) channel, electro-luminescence radiation is emitted at the polaritonic |+/–⟩ (cavity) frequency.

There is a very close relationship between emission and electronic transmission properties. T = Total emission.

How to quickly calculate these results numerically?

Numerical results obtained with our software QuTiP

QuTiP = Quantum Toolbox in Python

(our popular software has been mentioned in *The Economist, Nature,* etc.)

To check the possibility for ground state electroluminescence, we performed numerical simulations using QuTiP.

Let us now see portion of the actual code used for this work.

QuTiP

It is straightforward to define the Hilbert space and the relevant operators.

```
a  = tensor(destroy(N), qeye(3))
```

N = Maximum number of photons in the cavity
qeye = identity acting on 3 levels. s-level plus the qubit.

σ_z = |e><e| - |g><g|

```
sigmaZ = tensor(qeye(N),basis(3,2) * basis(3,2).dag()-basis(3,1) * basis(3,1).dag())
sigmaX = tensor(qeye(N),basis(3,2) * basis(3,1).dag()+ basis(3,1) * basis(3,2).dag())
```

σ_x = |e><g| + |g><e|

```
NI = tensor(qeye(N), basis(3,0) * basis(3,0).dag())
```

= Non Interacting = |s>< s|.

```
# decoupled Hamiltonian
def H(g):
    H_0 = w_c * a.dag() * a + t / 2.  * sigmaX + w_ni * NI * NI.dag() - w_ni
    H_I = g * (a + a.dag()) * sigmaZ
    return H_0 + H_I
```

= H(*g*) = H(Ω) = Rabi Hamiltonian (coupling)

Rabi Hamiltonian

$$H_{\rm Rabi} = \omega\, a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-)$$

QuTiP

It is straightforward to define the Hilbert space and the relevant operators.

```
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```

N = Maximum number of photons in the cavity
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$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$

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```

$\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$

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NI = tensor(qeye(N), basis(3,0) * basis(3,0).dag())
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```

= H(g) = H(Ω) = Rabi Hamiltonian (coupling)

Incidentally, several groups are using QuTiP without citing it.

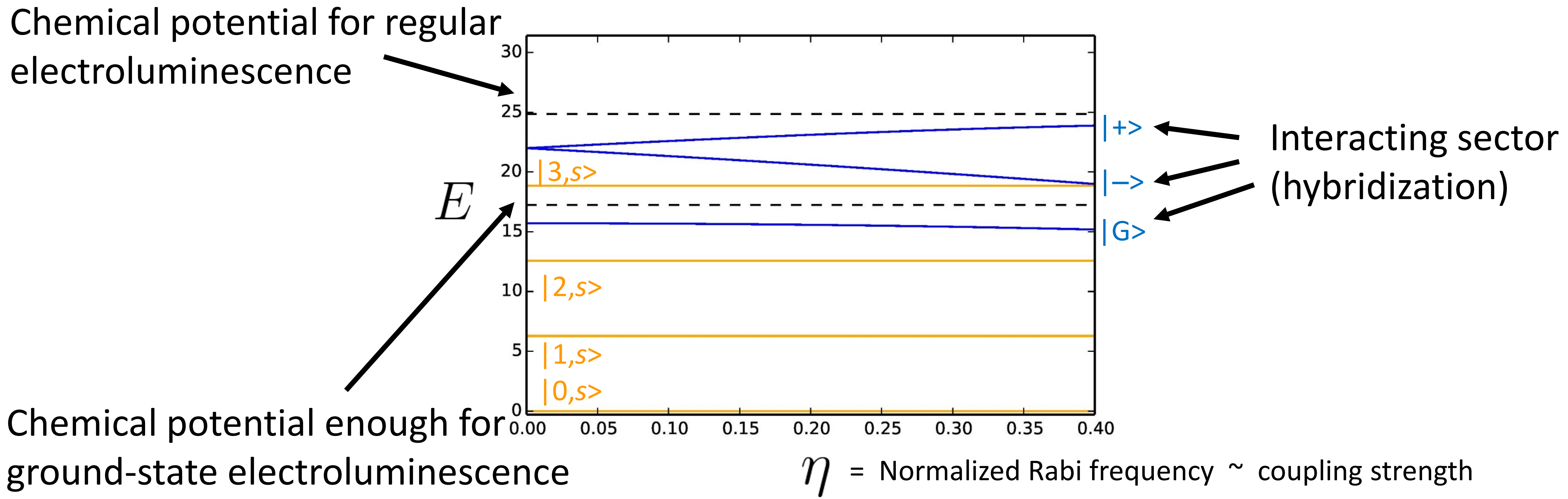
Please cite both QuTiP papers when you use this software for your research.

QuTiP

We can then compute the eigen-spectrum with a single line of code and then plot it.

```
g_list = linspace(0,0.4 * wc,100)
eigen_energies = [H(g).eigenenergies() for g in g_list]
fig, ax = plt.subplots()
ax.set_color_cycle(['orange', 'orange', 'orange','blue','orange','blue','blue'])
plt.plot([g/w_c for g in g_list],[eigen_energies[int(i)][0:7] for i in linspace(0,len(eigen_energie
```

Energy spectrum



η = Normalized Rabi frequency ~ coupling strength

QuTiP

$$S(\omega) \propto \int_{-\infty}^{\infty} dt \, \langle X^{+}(t) X^{-}(0) \rangle e^{-i\omega t}$$

```
spectrum_GS = spectrum(H(g_us),w_list,c_GS,rad.dag(),rad)
spectrum_reg = spectrum(H(g_us),w_list,c_reg,rad.dag(),rad)
```

$$X^{-} = \sum_{j>i} \langle i|X|j\rangle |i\rangle\langle j|$$

$$X = a + a^{\dagger}$$

Lindblad operators in the ultra-strong coupling regime

$$c^{\alpha\beta} \propto (|\langle \alpha | H_{\mathrm{in/out}} | \beta \rangle|) |\alpha\rangle\langle\beta|$$

```
def createX_L(coupling,chemPot,temp):
## Collapse operators for the left lead
    eigen_values , eigen_vec = H(coupling).eigenstates()

    Left = tensor(qeye(N),basis(3,1) * basis(3,0).dag())


    X_L = []
    for i in np.linspace(0,NN-1,NN):
        for j in np.linspace(0,NN-1,NN):

            ## In process
            transition_in = eigen_vec[j-1].dag() * Left * eigen_vec[i-1]

            Fermi_Dirac = FD(eigen_values[j-1]-eigen_values[i-1],chemPot,temp)
            rate = 2 * np.pi * k_L * abs(transition_in.full()[0][0])**2
            op = eigen_vec[j-1]*eigen_vec[i-1].dag()

            X_L.append(np.sqrt(Fermi_Dirac * rate) * op)
```

We can then calculate the emission spectrum by properly defining the Lindblad operators by the Fermi golden rule.

QuTiP

$$S(\omega) \propto \int_{-\infty}^{\infty} dt \, \langle X^{+}(t) X^{-}(0) \rangle e^{-i\omega t}$$

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H_in adds 1 electron and H_out removes it. (3,1) = |g> , (3,0) = |s> , (3,0)_dag = <s|

We can then calculate the emission spectrum by properly defining the Lindblad operators by Fermi golden rule.

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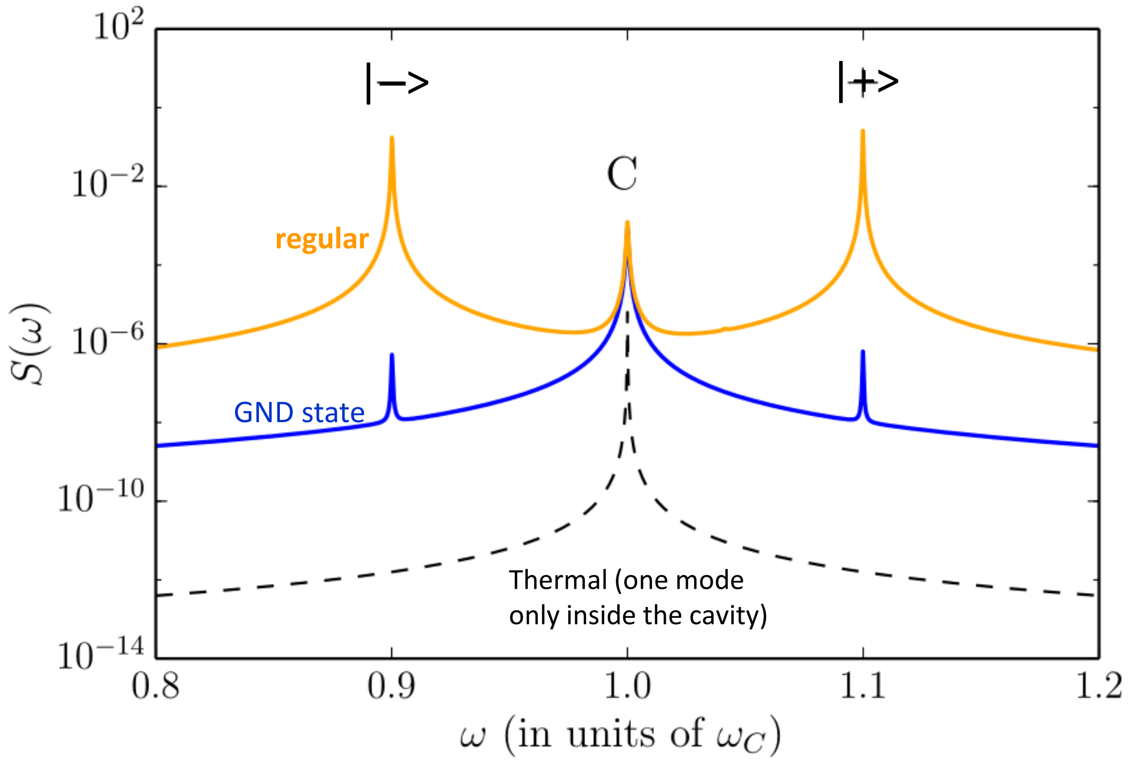
            X_L.append(np.sqrt(Fermi_Dirac * rate) * op)
```

We can then calculate the emission spectrum by properly defining the Lindblad operators by Fermi golden rule.

QuTiP

We can then finally plot the spectrum.

```
spectrum_GS = spectrum(H(g_us),w_list,c_GS,rad.dag(),rad)
spectrum_reg = spectrum(H(g_us),w_list,c_reg,rad.dag(),rad)
```

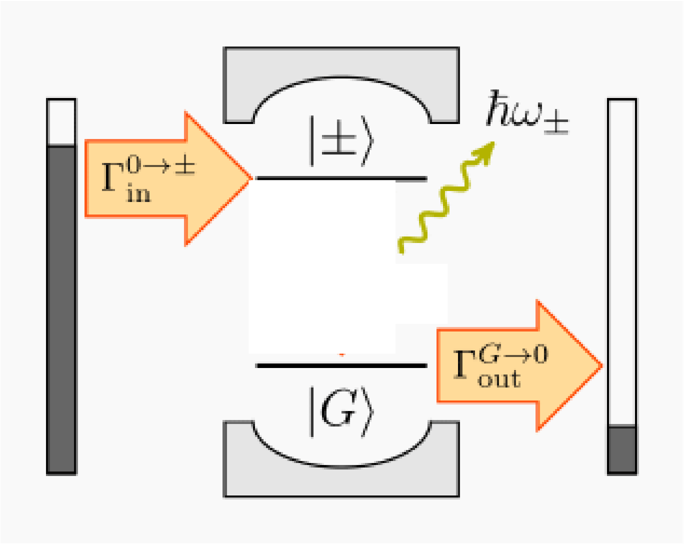


Conclusions

Ground State Electroluminescence

Electroluminescence: the emission of light as current flows through a system

Normal Electroluminescence



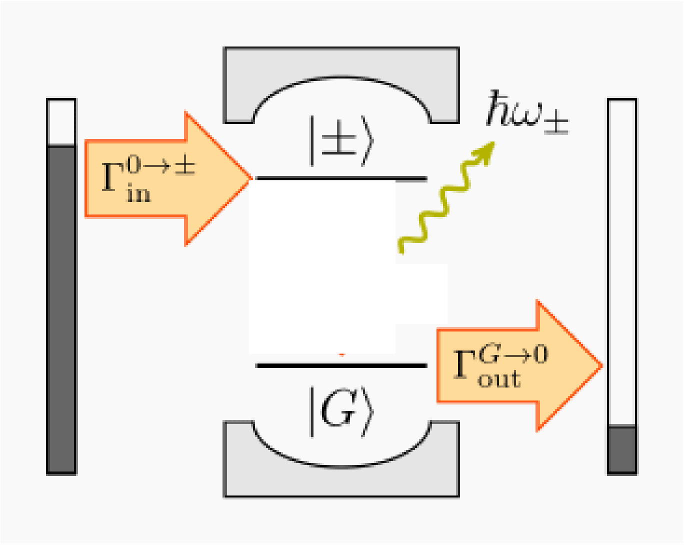
the current populates excited states of the system

Ground State Electroluminescence

Electroluminescence: the emission of light as current flows through a system

Weak → Ultra-Strong

Normal Electroluminescence



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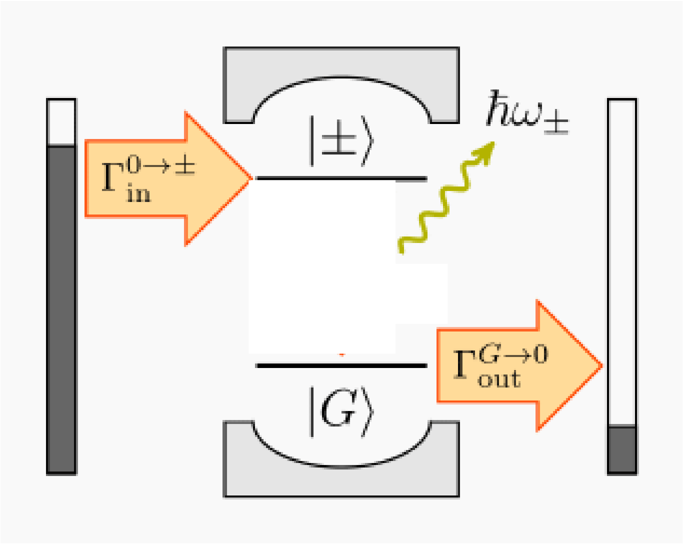

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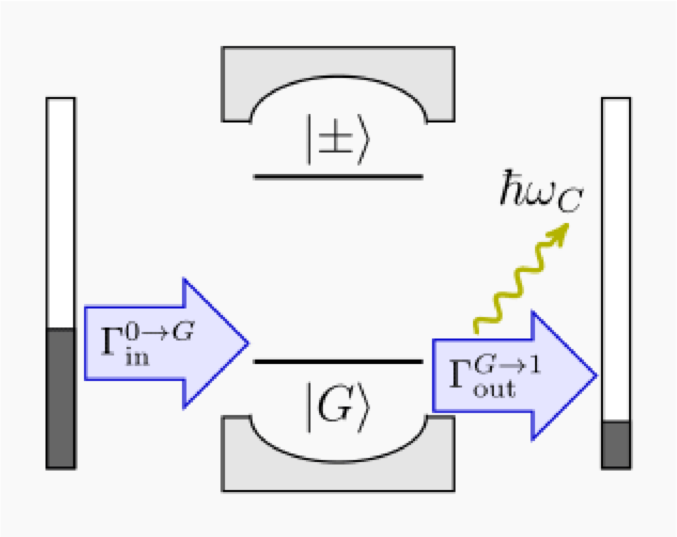
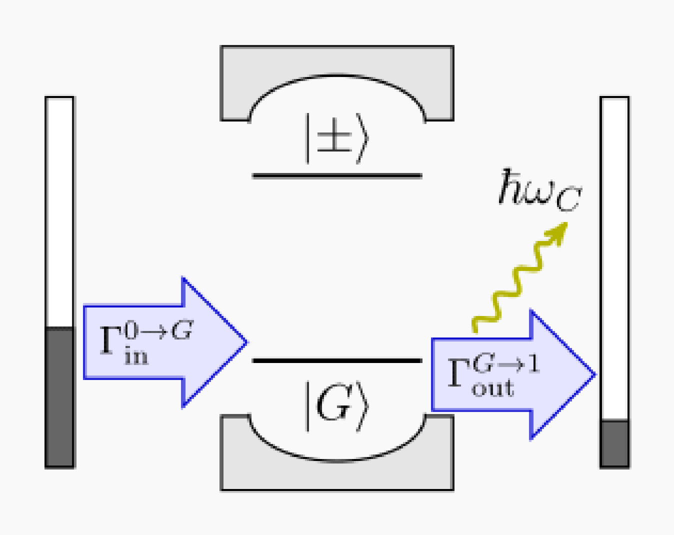
Ground State Electroluminescence



Current populates only the ground state and virtual photons are converted to real photons

Ground state electroluminescence

$$|G\rangle = (1-\frac{\eta^2}{8})|0,g\rangle+\frac{\eta}{2}|1,e\rangle+\frac{\eta^2}{2\sqrt{2}}|2,g\rangle$$



Ground-state electroluminescence can be seen as a way to **probe** the coherent structure of the light-matter ground state.
However, each time a photon is emitted, all the **internal coherences** are **destroyed**.
As other methods based on non-adiabatic modulations of the light-matter coupling, ground state electroluminesence is highly **disruptive**.
As a natural follow up, we asked:

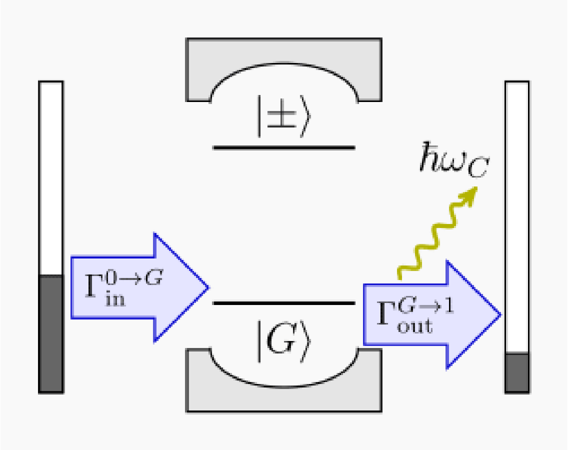
Are there ways to **probe the ground state** structure **without** destroying its internal coherences?

Using a mechanical oscillator as a probe for virtual photons in the dressed ground state

Conclusions

Two methods to **directly probe the virtual photons** in the **light-matter ground state** in the ultra-strong coupling regime.

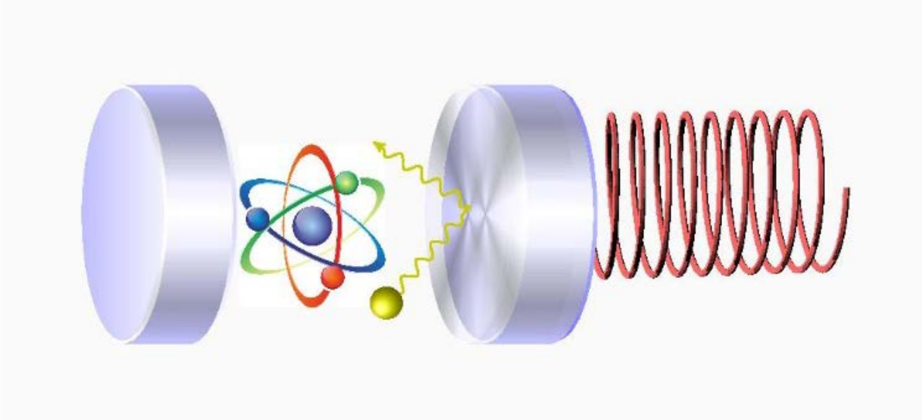
Ground State Electroluminescence



Phys. Rev. Lett. **116**, 113601 (2016)

By allowing a current to modulate the light-matter coupling, virtual photons can be emitted as electroluminescent radiation.

Amplified Transduction of Virtual Radiation Pressure



Phys. Rev. Lett. **119**, 053601 (2017)

By adding a mechanical probe to the system, virtual photons can be detected by the radiation pressure force.

Ground-State Physics of light-matter systems in the ultra-strong coupling regime

Summary

Brief Introduction to Cavity Quantum Electrodynamics

Ground State Electroluminescence

Phys. Rev. Lett. **116**, 113601 (2016)

Opto-mechanical transduction of virtual radiation pressure

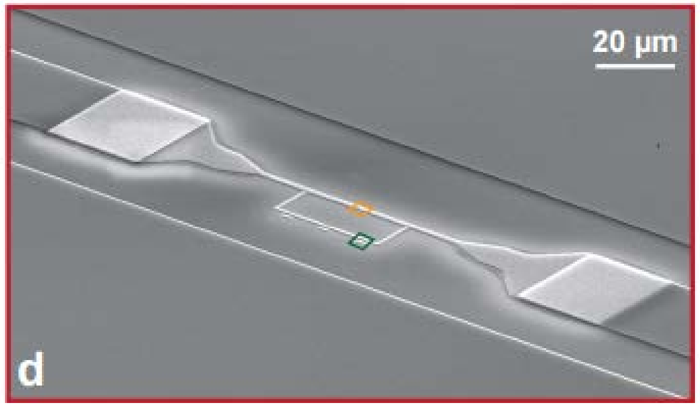
Phys. Rev. Lett. **119**, 053601 (2017)

Hybrid systems

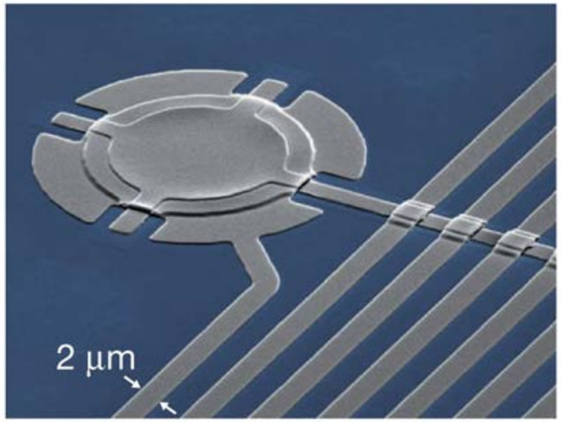
Signal → Primary Transducer

Cavity Quantum Electrodynamics

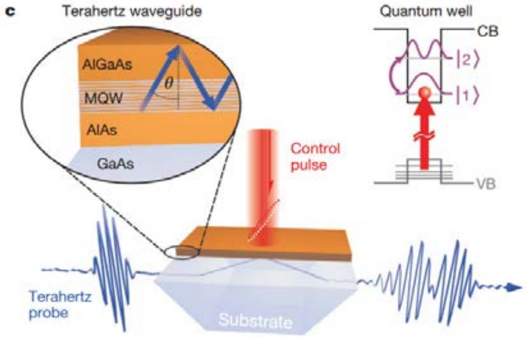
Opto-mechanics



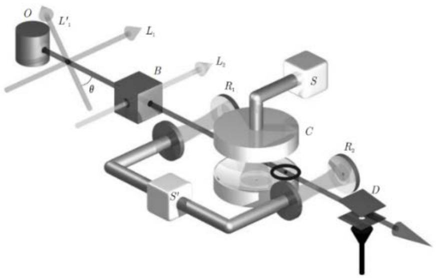
Nat. Phys. **6** (2010)



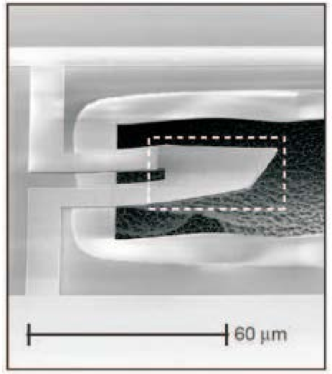
Nature **475**, 359 (2011)



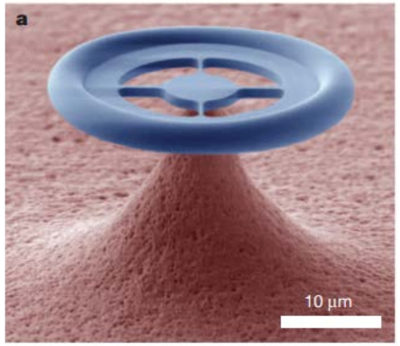
Nature **458**, (2009)



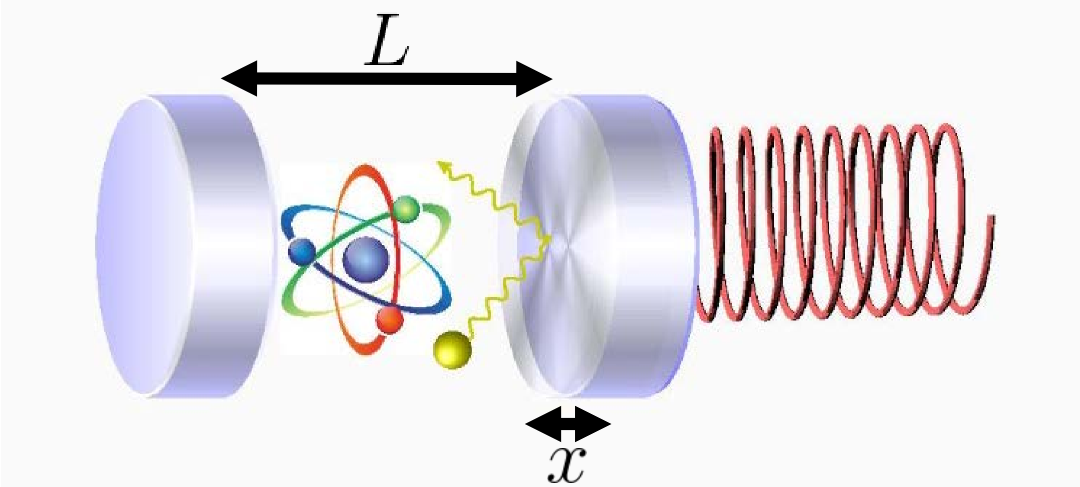
S. Haroche group



Nature **464**, 697-703, 2010



Nature **482**, 63-67, 2012



$$H = \omega(x)a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-) + \omega_m b^\dagger b$$

$$\omega(x) = \frac{2\pi c}{L - x} = \omega + g_0(b + b^\dagger) \quad \text{with} \quad g_0 = \frac{\omega}{L} x_{zp}$$

We considered the same cavity QED setting as before (one atom inside a cavity). However, we now let one of the cavity mirrors free to move in a harmonic potential. The position x of the mirror modulates the length of the cavity which, in turn, modulates its frequency. Expanding at first order we obtain the optomechanical coupling g_0 between light and mechanics which describes radiation pressure.

"Virtual" Radiation Pressure

$$H = \omega\, a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-) + \omega_m b^\dagger b + g_0\, a^\dagger a(b + b^\dagger)$$

Radiation pressure

“Virtual” Radiation Pressure

$$H = \omega\, a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-) + \omega_m b^\dagger b + g_0\, \underbrace{a^\dagger a}_{n}\, \underbrace{(b + b^\dagger)}_{x/x_{zp}}$$

Radiation pressure

n photons in the cavity exert a **radiation pressure force** $P_n = n\frac{g_0}{x_{zp}}$

This force induces a mechanical **displacement** $|\langle x \rangle_n| \propto n\, \frac{g_0}{\omega_m}$

“Virtual” Radiation Pressure

$$H = \omega\, a^\dagger a + \tfrac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-) + \omega_m b^\dagger b + \underbrace{g_0\, \overbrace{a^\dagger a}^{n}\overbrace{(b + b^\dagger)}^{x/x_{zp}}}_{\text{Radiation pressure}}$$

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At zero temperature

For weak light-matter couplings

the cavity contains no photons $\Rightarrow$ $\cancel{P_0}$ $\quad |\cancel{\langle x \rangle_0}|$

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Signal:
Zero temperature/Ground-state Displacement

At zero temperature

In the ultra-strong coupling regime

light-matter system has virtual photons

$$n_G = \langle G|a^\dagger a|G\rangle = \frac{\eta^2}{4}$$

$$\langle x\rangle_{\mathrm{GS}} \propto \eta^2\ \frac{g_0}{\omega_m}$$

Probing "Virtual" Radiation Pressure?

The signal is non-zero which is encouraging. However, when we compare it with the zero point motion of the oscillator we notice that the signal is quite **weak**. A signal bigger than the zero point motion would require a **ultra-strong/deep opto-mechanical coupling**.

$$\langle x \rangle_{\mathrm{GS}} \propto \eta^2 \, \frac{g_0}{\omega_m}$$

Encouraging, but $\langle x \rangle_{\mathrm{GS}} > x_{\mathrm{zp}}$ requires $\frac{g_0}{\omega_m} > \frac{2}{\eta^2}$

- A regime not currently reachable

Amplification

Amplification

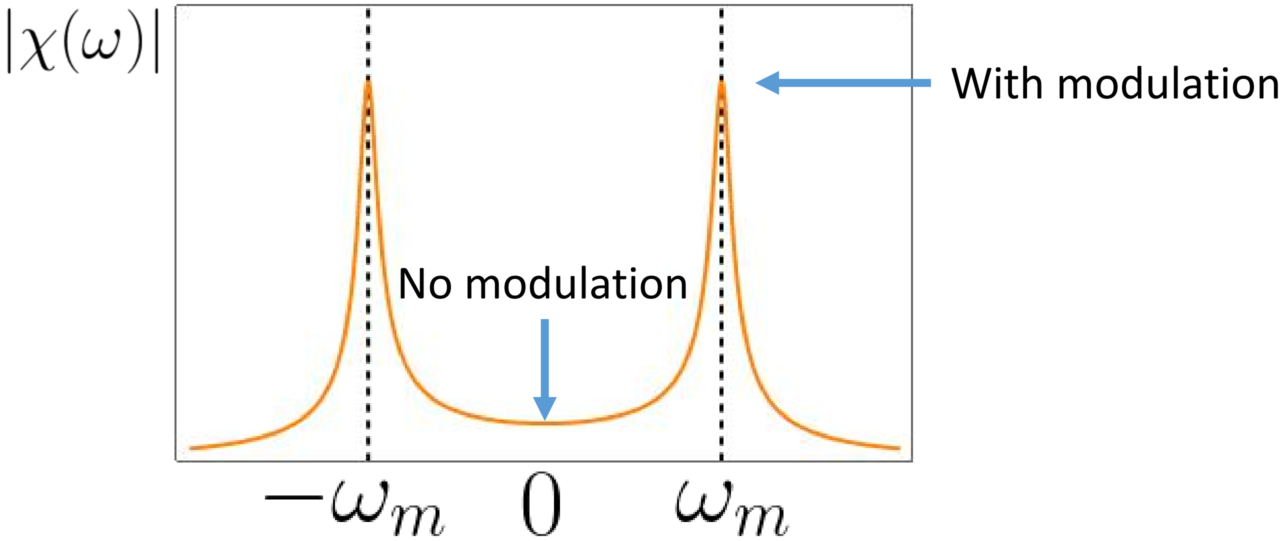
$$H = \omega\, a^\dagger a + \frac{\omega}{2}\sigma_z + \Omega(a + a^\dagger)(\sigma_+ + \sigma_-) + \omega_m b^\dagger b + g_0\, a^\dagger a(b + b^\dagger)$$

We considered to **parametrically modulate the light-matter coupling** at the mechanical frequency.

$$g_0 \mapsto g_0 \cos \omega_m t$$

Intuitively, this parametric dependence **turns the virtual radiation pressure into a driving force**.

Since any response to a drive is proportional to the mechanical susceptibility, we expect an amplification of the signal given by the mechanical **quality factor**



$$Q = \left|\frac{\chi(\omega_m)}{\chi(0)}\right| = \frac{\omega_m}{\Gamma_m}$$

Amplification

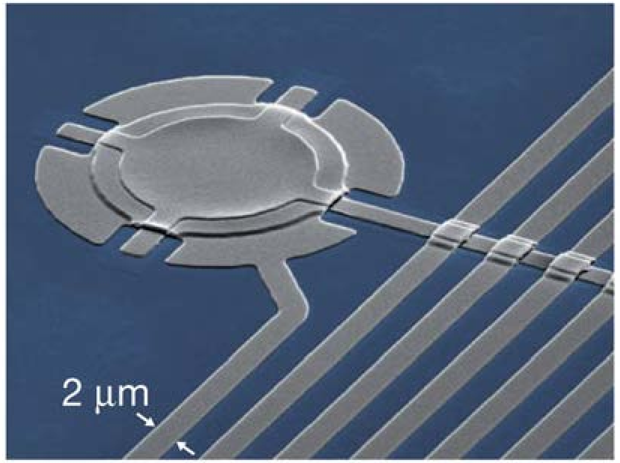
In this way, we get a **signal amplified by the mechanical quality factor**, making it, in principle, feasible to be measured in state-of-the-art experiments.

$$Q = \frac{\omega_m}{\Gamma_m}$$

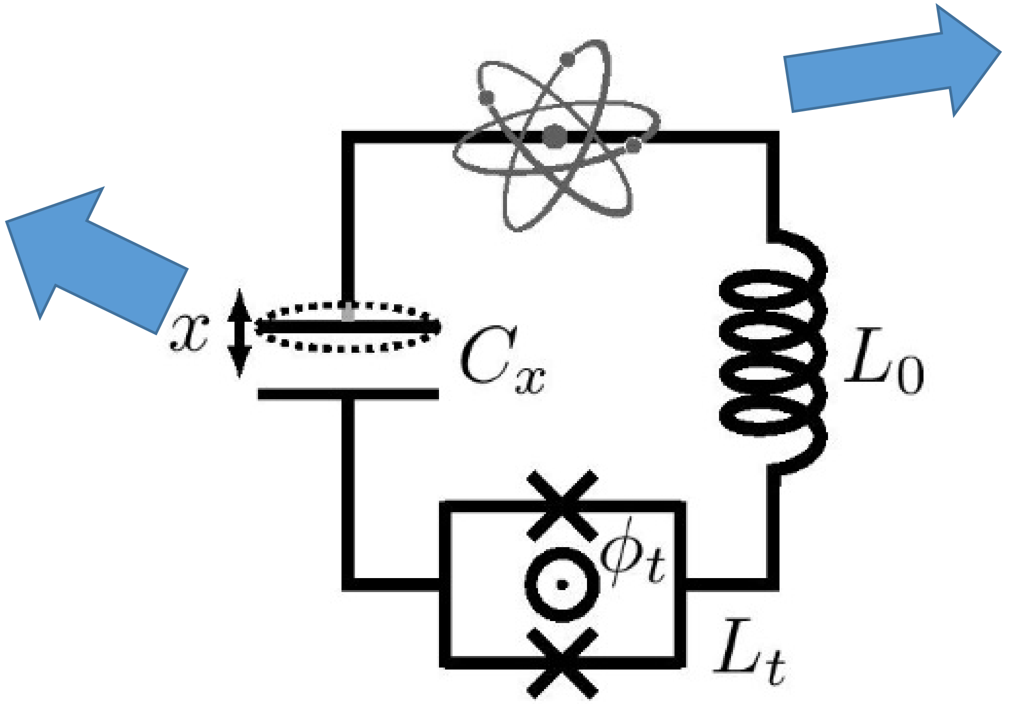
$$\langle x \rangle_{\mathrm{GS}} \propto \eta^2 \, \frac{g_0}{\omega_m} \quad \longrightarrow \quad \langle x \rangle_{\mathrm{GS}} \propto \eta^2 \, \frac{g_0}{\Gamma_m}$$

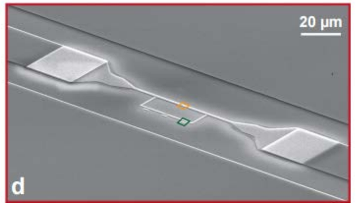
Signal amplified by the Quality Factor

(Possible) Physical Implementations



J. D. Teufel,
Nature **475**, 359 (2011)





Nat. Phys. **6** (2010)

Modulation of the opto-mechanical coupling

Opto-mechanical coupling

$$H = \omega a^\dagger a + \cdots \quad \text{with} \quad \omega = \frac{1}{\sqrt{L(t)C(x)}}$$

In circuit-QED, we can consider the harmonic mode of an **LC circuit** interacting with an **artificial atom**. By modulating the capacitance with a **mechanical membrane**, it is possible to achieve mechanics/light coupling. By further **modulating the kinetic inductance** of the circuit, it can be possible to further modulate the opto-mechanical coupling.

Conclusions

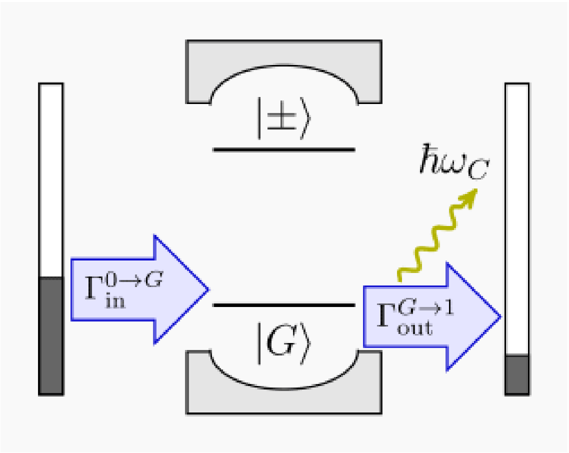
- Hybrid system that probes virtual photons with a mechanical transducer.
- This signal can be amplified by modulating the opto-mechanical coupling.
- The measurement minimally disturbs the light-matter system.

- Extensions of this protocol could be used to directly probe the light-matter ground state in the deep-coupling regime.
- This amplification technique can be used in other contexts to detect weak constant forces.

Conclusions

Two methods to **directly probe the virtual photons** in the **light-matter ground state** in the ultra-strong coupling regime.

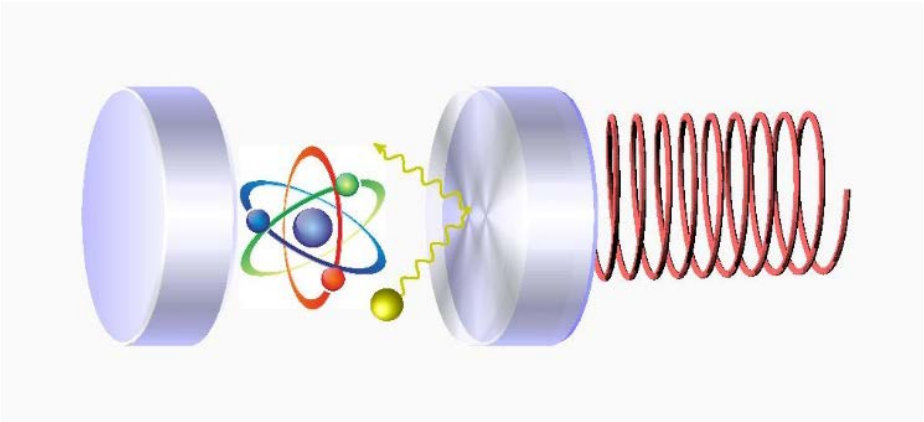
Ground State Electroluminescence



Phys. Rev. Lett. **116**, 113601 (2016)

By allowing a current to modulate the light-matter coupling, virtual photons can be emitted as electroluminescent radiation.

Amplified Transduction of Virtual Radiation Pressure



Phys. Rev. Lett. **119**, 053601 (2017)

By adding a mechanical probe to the system, virtual photons can be detected by the radiation pressure force.

THANK YOU FOR YOUR ATTENTION

Below is a photo of a sunset behind Mount Fuji, as seen from our office building.