Interaction of Mechanical Oscillators Mediated by the Exchange of Virtual Photon Pairs

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Two close parallel mirrors attract due to a small force (Casimir effect) originating from the quantum vacuum fluctuations of the electromagnetic field. These vacuum fluctuations can also induce motional forces exerted upon one mirror when the other one moves. Here, we consider an optomechanical system consisting of two vibrating mirrors constituting an optical resonator. We find that motional forces can determine noticeable coupling rates between the two spatially separated vibrating mirrors. We show that, by tuning the two mechanical oscillators into resonance, energy is exchanged between them at the quantum level. This coherent motional coupling is enabled by the exchange of virtual photon pairs, originating from the dynamical Casimir effect. The process proposed here shows that the electromagnetic quantum vacuum is able to transfer mechanical energy somewhat like an ordinary fluid. We show that this system can also operate as a mechanical parametric down-converter even at very weak excitations. These results demonstrate that vacuum-induced motional forces open up new possibilities for the development of optomechanical quantum technologies.

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Effective interactions able to coherently couple spatially separated qubits [1] are highly desirable for any quantum computer architecture. Efficient cavity-QED schemes, where the effective long-range interaction is mediated by the vacuum field, have been proposed [2–4] and realized [1,5,6]. In these schemes, the cavity is only virtually excited and thus the requirement on its quality factor is greatly loosened. Based on these interactions mediated by vacuum fluctuations, a two-qubit gate has been realized [7] and two-qubit entanglement has been demonstrated [1]. Creation of multiqubit entanglement [8] has also been demonstrated in circuit-QED systems. Very recently, it has been shown that the exchange of virtual photons between artificial atoms can give rise to effective interactions of multiple spatially separated atoms [9,10], opening the way to vacuum nonlinear optics. Moreover, it has been shown that systems where virtual photons can be created and annihilated can be used to realize many nonlinear optical processes with qubits [11,12]. Multiparticle entanglement and quantum logic gates, via virtual vibrational excitations in an ion trap, have also been implemented [13,14]. A recent proposal [15] suggests that classical driving fields can transfer quantum fluctuations between two suspended membranes in an optomechanical cavity system.

Given these results, one may wonder whether it is possible for spatially separated mesoscopic or macroscopic bodies to interact at a quantum level by means of the vacuum fluctuations of the electromagnetic field. It is known that, owing to quantum fluctuations, the electromagnetic vacuum is able, in principle, to affect the motion of objects through it, like a complex fluid [16]. For example, it can induce dissipation and decoherence effects on the motion of moving objects [17–19]. By using linear dispersion theory, it has also been shown that vacuum fluctuations can induce motional forces exerted upon one mirror when the other one moves [20]. Here, we show that two spatially separated moveable mirrors, constituting a cavity-optomechanical system, can exchange energy coherently and reversibly, by exchanging virtual photon pairs. The effects described here can be experimentally demonstrated with circuit-optomechanical systems, using ultra-high-frequency mechanical microresonators or nanoresonators in the GHz spectral range [21,22]. Coupling such a mechanical oscillator to a superconducting qubit, quantum control over a macroscopic mechanical system has been demonstrated [21]. Our results show that the electromagnetic quantum vacuum is able to transfer mechanical energy somewhat like an ordinary fluid. It would be as if the vibration of a string (mechanical

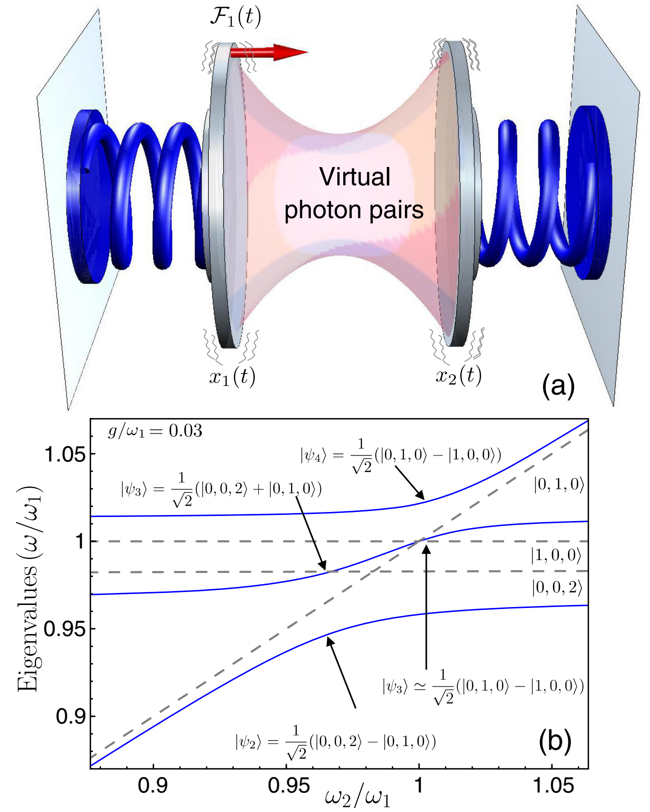

FIG. 1. (a) Schematic of an optomechanical system constituted by two vibrating mirrors. If one of the two vibrating mirrors is excited by an external drive $\mathcal{F}_1(t)$, its excitation can be transferred coherently and reversibly to the other mirror. The interaction is mediated by the exchange of virtual photon pairs. (b) Relevant energy levels of the system Hamiltonian $\hat{H}_s$ as a function of the ratio between the mechanical frequency of mirror 2 and that of mirror 1. An optomechanical coupling $g/\omega_1 = 0.03$ has been used; the cavity-mode resonance frequency is $\omega_c = 0.495\omega_1$. The lowest-energy anticrossing corresponds to the resonance condition for the DCE [26]. The higher energy one is the signature of the mirror-mirror interaction mediated by the virtual DCE photons.

oscillator 1) could be transferred to the membrane of a microphone (mechanical oscillator 2) in the absence of air (or any excited medium filling the gap).

We consider a system constituted by two vibrating mirrors interacting via radiation pressure [see Fig. 1(a)]. Very recently, entanglement between two mechanical oscillators has been demonstrated in a similar system, where, however, the two entangled mechanical oscillators have much lower resonance frequencies and the system is optically pumped [23]. This system can be described by a Hamiltonian that is a direct generalization to two mirrors of the Law Hamiltonian, describing the coupled mirror-field system [24–28]. It provides a unified description of cavity-optomechanics experiments [29] and of the dynamical Casimir effect (DCE) [30–34] in a cavity with a vibrating mirror [26]. It has been shown [32–38] that the photon pairs generated by the DCE can be used to produce entanglement. However, in the present case, the interaction and the entanglement between two mechanical oscillators is determined by virtual photon pairs. Both the cavity field and the position of the mirror are treated as dynamical variables and a canonical quantization procedure is adopted [24]. By considering only one mechanical mode for each mirror, with resonance frequency ω_i $(i = 1,\ 2)$ and bosonic operators $\hat{b}_i$ and $\hat{b}_i^\dagger$, the displacement operators can be expressed as $\hat{x}_i = X_{\text{zpf}}^{(i)}(\hat{b}_i^\dagger + \hat{b}_i)$, where $X_{\text{zpf}}^{(i)}$ is the zero-point-fluctuation amplitude of the ith mirror. The mirrors form a single-mode optical resonator with frequency ω_c and bosonic photon operators $\hat{a}$ and $\hat{a}^\dagger$. The system Hamiltonian can be written as $\hat{H}_s = \hat{H}_0 + \hat{H}_\text{I}$, where $(\hbar = 1)$ $\hat{H}_0 = \omega_c \hat{a}^\dagger \hat{a} + \sum_i \omega_i \hat{b}_i^\dagger \hat{b}_i$ is the unperturbed Hamiltonian. The mirror-field interaction Hamiltonian can be written as $\hat{H}_\text{I} = \hat{V}_\text{om} + \hat{V}_\text{DCE}$, where $\hat{V}_\text{om} = \hat{a}^\dagger \hat{a} \sum_i g_i (\hat{b}_i + \hat{b}_i^\dagger)$ is the standard optomechanical interaction conserving the number of photons, $\hat{V}_\text{DCE} = (1/2)(\hat{a}^2 + \hat{a}^{\dagger 2}) \sum_i g_i (\hat{b}_i + \hat{b}_i^\dagger)$ describes the creation and annihilation of photon pairs, and g_i is the optomechanical coupling rate for mirror i. The linear dependence of the interaction Hamiltonian on the mirror operators is a consequence of the usual small-displacement assumption [24]. This Hamiltonian can be directly generalized to include additional cavity modes. However, in most circuit-optomechanics experiments, the electromagnetic resonator is provided by a superconducting LC circuit, which only supports a *single* mode.

When describing most of the optomechanics experiments to date [29], $\hat{V}_\text{DCE}$ is neglected. This is a very good approximation when $\omega_i \ll \omega_c$ (which is the most common experimental situation). However, when ω_i are of the order of ω_c, $\hat{V}_\text{DCE}$ cannot be neglected. We are interested in studying this regime, which can be achieved using microwave resonators and ultra-high-frequency mechanical microresonators or nanoresonators [21,22]. The Hamiltonian $\hat{H}_s$ describes the interaction between two vibrating mirrors and the radiation pressure of a cavity field. However, the same radiation-pressure-type coupling is obtained for microwave optomechanical circuits (see, e.g., Ref. [39]).

In order to properly describe the system dynamics, including external driving and dissipation, the coupling to external degrees of freedom needs to be considered. A coherent external drive of the vibrating mirror i can be described by including the time-dependent Hamiltonian

$$\hat{V}_i(t) = \mathcal{F}_i(t)(\hat{b}_i + \hat{b}_i^\dagger), \qquad (1)$$

where $\mathcal{F}_i(t)$ is equal to the external force applied to the mirror times the mechanical zero-point-fluctuation amplitude. Dissipation and decoherence effects are taken into account by adopting a master-equation approach. For strongly coupled hybrid quantum systems, the description offered by the standard quantum-optical master equation breaks down [40,41]. Following Refs. [41–43], we express the system-bath interaction Hamiltonian in the basis formed by the energy eigenstates of $\hat{H}_s$ [26].

We begin our analysis by numerically diagonalizing the Hamiltonian $\hat{H}_s$ in a truncated finite-dimensional Hilbert space. The truncation is realized by only including eight Fock states for each of the three harmonic oscillators. The blue solid curves in Fig. 1(b) describe the eigenvalue differences $E_j - E_0$ (E_0 is the ground-state energy) of the total Hamiltonian $\hat{H}_s$ (including $\hat{V}_{\rm DCE}$) as a function of ω_2/ω_1. For the optomechanical couplings, we use $g_1 = g_2 = g = 0.03\omega_1$. Such a coupling strength is quite high, but nevertheless below the onset of the so-called ultrastrong optomechanical coupling regime [41,44–46]. The cavity-mode resonance frequency is fixed at $\omega_c = 0.495\omega_1$. This value is chosen close to the resonance condition for the DCE [26] in order to increase the effective coupling between the mirrors. For comparison, we also show in Fig. 1(b) (dashed gray lines) the lowest-energy levels $E_{n,k_1,k_2} = \omega_c n - \sum_i g_i^2 n^2/\omega_i + \sum_i \omega_i k_i$ of the standard optomechanics Hamiltonian $\hat{H}_0 + \hat{V}_{\rm om}$. This Hamiltonian has the eigenstates $|k_1, k_2, n\rangle \equiv D_1(n\beta_1)|k\rangle_1 \otimes D_2(n\beta_2)|k\rangle_2 \otimes |n\rangle_c$, where $|n\rangle_c$ are the cavity Fock states and $|k\rangle_i$ are the bare mechanical states for the ith mirror.

The bare mechanical states $|k\rangle_i$ are displaced by the optomechanical interaction, $\hat{D}_i(n\beta_i) = \exp[n\beta_i(\hat{b}_i^\dagger - \hat{b}_i)]$, with $\beta_i = g_i/\omega_i$ (see Sec. I of Supplemental Material [47]). The main differences between the blue solid and the gray dashed curves are the appearance of small energy shifts, and of level anticrossings in the region $\omega_2/\omega_1 \sim 1$. We indicate by $|\psi_n\rangle$ $(n = 0, 1, 2\ldots)$ the eigenvectors of $\hat{H}_s$ and by E_n the corresponding eigenvalues, choosing the labeling of the states such that $E_j > E_k$ for $j > k$. The lowest-energy anticrossing corresponds to the resonance condition for the DCE [26]. The higher-energy splitting in Fig. 1(b) originates from the coherent coupling of the zero-photon states $|1,0,0\rangle$ and $|0,1,0\rangle$. At the minimum energy splitting $2\lambda_{10}^{01} \simeq 2.11 \times 10^{-2}\omega_1$, the resulting states are well approximated by $|\psi_{3,4}\rangle \simeq (1/\sqrt{2})(|1,0,0\rangle \pm |0,1,0\rangle)$. As we will show explicitly below by using perturbation theory, this mirror-mirror interaction is a result of virtual exchange of cavity photon pairs. When the mirrors have the same resonance frequency, an excitation in one mirror can be transferred to the other by virtually becoming a photon pair in the cavity, thanks to the DCE. The resulting minimum energy splitting provides a measure of the effective coupling strength between the two mirrors. At higher energy for $\omega_2 \simeq \omega_1$ a ladder of increasing level splittings, involving higher number phonon states, is present (see Sec. III in [47]).

The origin of the higher-energy avoided-level crossing shown in Fig. 1(b) can be understood by deriving an effective Hamiltonian, using second-order perturbation theory or, equivalently, the James' method [52,53] (see Sec. II in [47]). The resulting effective Hamiltonian, describing the coherent coupling of states $|1,0,0\rangle$ and $|0,1,0\rangle$, is

$$\hat{H}_{\rm eff} = \Omega_1|1,0,0\rangle\langle 1,0,0| + \Omega_2|0,1,0\rangle\langle 0,1,0| + (\lambda_{10}^{01}|1,0,0\rangle\langle 0,1,0| + \text{H.c.}), \tag{2}$$

where $\Omega_1 = \omega_1 + \Delta_{10}$ and $\Omega_2 = \omega_2 + \Delta_{01}$ denote the Lamb-shifted levels [47]. The effective coupling strength is

$$\lambda_{10}^{01} = \sum_{k,q} \frac{\langle 0,1,0|\hat{V}_{\rm DCE}|k,q,2\rangle\langle k,q,2|\hat{V}_{\rm DCE}|1,0,0\rangle}{E_{0,1,0} - E_{k,q,2}}. \tag{3}$$

Equations (2) and (3) clearly show that the one-phonon state of mirror 1 can be transferred to mirror 2 through a virtual transition via the two-photon intermediate states $|k,q,2\rangle$. We notice that the largest contribution is provided by the zero-phonon intermediate state ($k = q = 0$). This perturbative calculation gives rise to an effective coupling strength λ and energy shifts Δ in good agreement with the numerical calculation shown in Fig. 1(b) (see Sec. II of [47]). Analogous effective Hamiltonians can be derived for the avoided-level crossings at higher energy (see Sec. II of [47]).

If the optomechanical couplings g_i are strong enough to ensure that the DCE-induced effective coupling (3) becomes larger than the relevant decoherence rates in the system, the transfer of one-phonon excitations between the two mirrors can be deterministic and reversible. Neglecting decoherence (calculations including losses can be found in Secs. V and VI of [47]), if the system is initially prepared in the state $|1,0,0\rangle$, it will evolve as

$$|\psi(t)\rangle = \cos(\lambda_{10}^{01}t)|1,0,0\rangle - i\sin(\lambda_{10}^{01}t)|0,1,0\rangle. \tag{4}$$

After a time $t = \pi/(2\lambda_{10}^{01})$, the excitation will be completely transferred to mirror 1. After a time $t = \pi/(4\lambda_{10}^{01})$, the two mirrors will be in a maximally entangled motional state.

We now investigate the system dynamics starting from a low-temperature thermal state and introducing the excitation of mirror 1 by a single-tone continuous-wave mechanical drive $\mathcal{F}_1(t) = \mathcal{A}\cos(\omega_d t)$. We numerically solve the master equation for hybrid quantum systems in a truncated Hilbert space [54]. Figure 2 shows the time evolution of the mean phonon numbers of the two mirrors $\langle \hat{B}_i^\dagger \hat{B}_i\rangle$ and the intracavity mean photon number $\langle \hat{A}^\dagger \hat{A}\rangle$. Here, $\hat{A}$, $\hat{B}_i$ are the *physical* photon and phonon operators. Such operators $\hat{O} = \hat{A}, \hat{B}_i$ can be defined in terms of their bare counterparts $\hat{o} = \hat{a}$, $\hat{b}_i$ as [55–58] $\hat{O} = \sum_{E_n > E_m} \langle\psi_m|(\hat{o} + \hat{o}^\dagger)|\psi_n\rangle|\psi_m\rangle\langle\psi_n|$. We

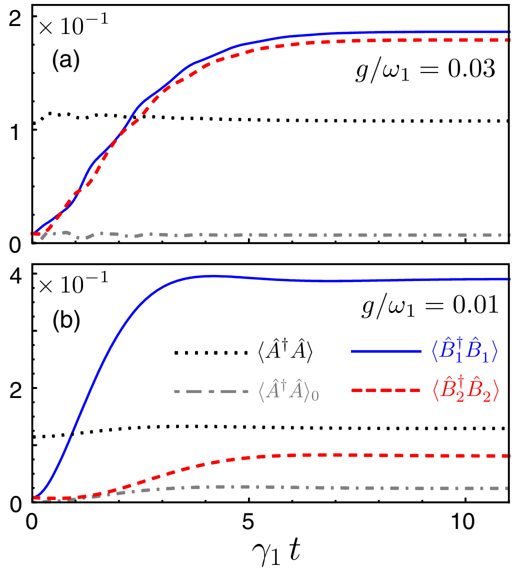

FIG. 2. System dynamics for $\omega_c \simeq 1.5\omega_1$ under continuous-wave drive of mirror 1. The blue solid and red dashed curves describe the mean phonon numbers $\langle \hat{B}_1^\dagger \hat{B}_1 \rangle$ and $\langle \hat{B}_2^\dagger \hat{B}_2 \rangle$, respectively, while the black dotted curve describes the mean intracavity photon number $\langle \hat{A}^\dagger \hat{A} \rangle$ and the gray dash-dotted curve shows the same photon number $\langle \hat{A}^\dagger \hat{A} \rangle_0$, calculated assuming zero temperature.

consider the system initially in a thermal state with a normalized thermal energy $k_B T/\omega_1 = 0.208$, corresponding to a temperature $T = 60$ mK for $\omega_1/2\pi = 6$ GHz. During its time evolution, the system interacts with thermal reservoirs all with the same temperature T. We use $\gamma_1 = \gamma_2 = \gamma = \omega_1/260$ and $\kappa = \gamma$ for the mechanical and photonic loss rates. We consider a weak ($\mathcal{A}/\gamma = 0.95$) resonant excitation of mirror 1 ($\omega_d = \omega_1$). We present results for two normalized coupling strengths ($g/\omega_1 = 0.01, 0.03$), and set $\omega_2 = \omega_1$. The results shown in Fig. 2(a) demonstrate that the excitation transfer mechanism via virtual DCE photon pairs, proposed here, works very well for $g/\omega_1 = 0.03$. In steady state, mirror 2 reaches almost the same excitation intensity as the driven mirror 1. The photon population differs only slightly from the thermal one at $t = 0$, showing that a negligible amount of DCE photon pairs are generated. We also observe that the influence of temperature on the mechanical expectation values is almost negligible (see Supplemental Material [47]). On the contrary, the cavity mode at lower frequency is much more affected by the temperature. We observe that for $g/\omega_1 = 0.01$, although the transfer is reduced, the effect is still measurable. The mean photon number obtained at $T = 0$ is also shown for comparison (dash-dotted curves) in both the panels. The mirror-mirror excitation transfer at $g/\omega_1 = 0.01$ can be significantly improved [47] by taking advantage of the DCE resonance condition $\omega_c = 2\omega_1$. However, in this case, a significant amount of real photon pairs are generated. This configuration can be used to probe the DCE effect in the presence of thermal photons.

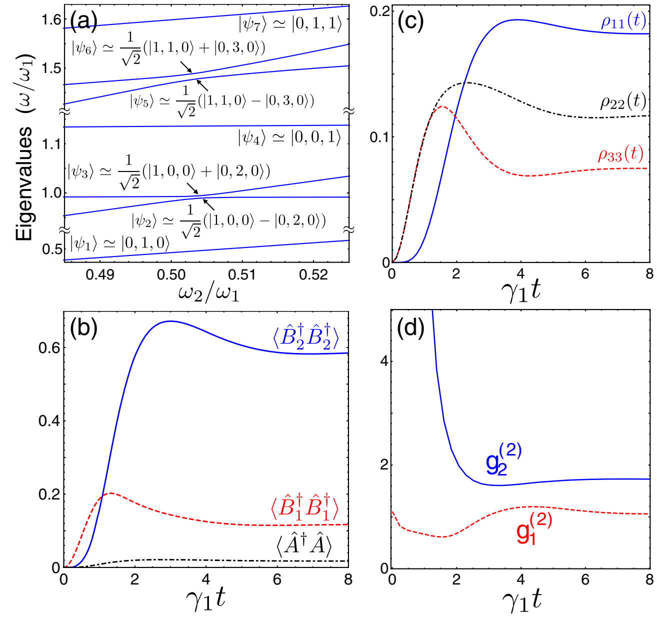

FIG. 3. Mechanical parametric down-conversion. (a) Lowest-energy levels of the system Hamiltonian as a function of the ratio between the mechanical frequency of mirror 2 and that of mirror 1. An optomechanical coupling $g/\omega_1 = 0.12$ has been used and the cavity-mode resonance frequency is $\omega_c = 1.2\omega_1$. Two avoided-level crossings are clearly visible. The one at lower energy corresponds to the resonant coupling of the one-phonon state of mirror 1 with the two-phonon state of mirror 2, whose resonance frequency is half that of mirror 1. The higher-energy anticrossing corresponds to the resonant coupling of the states $|1,1,0\rangle$ and $|0,3,0\rangle$. (b) Time evolution of the mean phonon and photon numbers. (c) Time evolution of the population of the first three energy states. (d) Equal-time phonon-phonon normalized correlation functions $g_i^{(2)}(t,t)$ for the two mirrors.

In order to put forward the potentialities and the flexibility of this vacuum-field-mediated interaction between mechanical oscillators, we now show that this system also can operate as a mechanical parametric down-converter. For mechanical frequencies such that $\omega_1 \simeq 2\omega_2$, a ladder of avoided-level crossings manifests. Two of them are shown in Fig. 3(a). Also in this case, the avoided-level crossings originate from the exchange of virtual photon pairs, as can be understood by using second-order perturbation theory. For example, the dominant path for the lowest-energy level anticrossing goes through the intermediate state $|0,0,2\rangle$: $|1,0,0\rangle \leftrightarrow |0,0,2\rangle \leftrightarrow |0,2,0\rangle$ [47]. We note that these avoided-level crossings, in contrast to those shown in Fig. 1(b), do not conserve the excitation number. Analogous coherent coupling effects can be observed in the ultrastrong-coupling regime of cavity QED [9,11,43,59,60]. Using $\omega_c = 1.2\omega_1$ and $g/\omega_1 = 0.12$, we obtain a minimum energy splitting $\lambda_{10}^{02}/\omega_1 \simeq 4 \times 10^{-3}$. We fix the resonance frequency of mirror 2 at the value providing the minimum level splitting, and calculate the system dynamics considering a weak resonant excitation of mirror 1, $\mathcal{F}_1(t) = \mathcal{A}\cos(\omega_d t)$, with $\omega_d = (E_3 + E_2 - 2E_0)/2$, and $\mathcal{A}/\gamma = 0.7$. We also used $\gamma = 2 \times 10^{-3}\omega_1$ and $\kappa = \gamma/2$. The results shown in Fig. 3(b) demonstrate a very efficient excitation transfer between the two mechanical oscillators of different frequency. We also observe that the transfer occurs even in the presence of a very weak excitation of mirror 1 (peak mean phonon number of mirror 1: $\langle \hat{B}_1^\dagger \hat{B}_1 \rangle \simeq 0.2$). It may appear surprising that the steady-state mean phonon number of mirror 2 is significantly *larger* than that of mirror 1, even though it receives all the energy from the latter. This phenomenon can be partly understood by observing that a phonon of mirror 1 converts into two phonons (each at half energy) of mirror 2. In addition, once the system decays to the state $|\psi_1\rangle \simeq |0,1,0\rangle$, the remaining excitation in mirror 2 will not be exchanged back and forth with mirror 1, since the corresponding energy level is not resonantly coupled to other energy levels [see Fig. 3(a)]. Figure 3(c) displays the populations of the three lowest-energy levels, which are the levels that are most populated at this input power. This panel confirms that $|\psi_1\rangle$ has the higher population in steady state.

We also calculated the equal-time phonon-phonon normalized correlation functions

$$g_i^{(2)}(t,t) = \frac{\langle \hat{B}_i^\dagger(t)\hat{B}_i^\dagger(t)\hat{B}_i(t)\hat{B}_i(t)\rangle}{\langle \hat{B}_i^\dagger(t)\hat{B}_i(t)\rangle^2}. \quad (5)$$

The high value at early times obtained for mirror 2 [see Fig. 3(d)] confirms the *simultaneous* excitation of phonon pairs.

In conclusion, we demonstrated that mechanical quantum excitations can be coherently transferred among spatially separated mechanical oscillators, through a dissipationless quantum bus, due to the exchange of virtual photon pairs. The experimental demonstration of these processes would show that the electromagnetic quantum vacuum is able to transfer mechanical energy somewhat like an ordinary fluid [16]. The results presented here open up exciting possibilities of applying ideas from fluid dynamics in the study of the electromagnetic quantum vacuum. Furthermore, these results show that the DCE in high-frequency optomechanical systems can be a versatile and powerful new resource for the development of quantum-optomechanical technologies. If, in the future, it will be possible to control the interaction time (as currently realized in superconducting artificial atoms), e.g., changing rapidly the resonance frequencies of mechanical oscillators (see Sec. VI of [47]), the interaction scheme proposed here would represent an attractive architecture for quantum information processing with optomechanical systems [61]. The best platform to experimentally demonstrate these results is circuit optomechanics using ultra-high-frequency (ω_1 at 5–6 GHz) mechanical oscillators. Their quantum interaction with superconducting artificial atoms has been experimentally demonstrated [21,22]. Considering instead their interaction with a superconducting microwave resonator should allow the observation of the effects predicted here. Specifically, combining circuit-optomechanics schemes able to increase the coupling [39,62] with already demonstrated ultra-high-frequency mechanical resonators [21,22] represents a very promising setup for entangling spatially separated vibrations via virtual photon pairs (see Sec. VII of [47]).

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