Supplemental material for “Cavity-free optical isolators and circulators using a chiral cross-Kerr nonlinearity”

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The main goal of this supplementary material is to present the derivation of the unidirectional wave equations for the probe field and the chiral cross-Kerr nonlinearity. Also, it discusses the effect of the phase shifter in the lower channel of the Mach-Zehnder interferometer.

1 Unidirectional wave equations

In this section, we derive the unidirectional wave equations for the forward and backward traveling probe fields in1D space, shown in Fig. S1. The right-moving (blue) pulse travels in the positive z direction, while the left-moving (red) pulse propagates in the negative z direction. We also refer to the right-moving (left-moving) mode as the forward-moving (backward-moving) modes. The medium has length L.

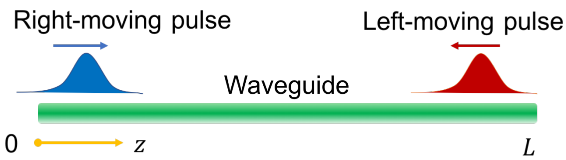

Figure S1: Fields propagating in a 1D space. The right-moving (left-moving) pulse, shown in blue (red), propagates in the positive (negative) z direction. The waveguide length is L.

We start our derivation of unidirectional wave equations from the general wave equation for the electric field $\mathbf{E}$ in a nonlinear medium, which can be written as

$$\nabla \times (\nabla \times \tilde{\mathbf{E}}) - \frac{n_s^2}{c^2}\frac{\partial^2 \tilde{\mathbf{E}}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \tilde{\mathbf{P}}^{\mathrm{NL}}}{\partial t^2} , \tag{S1}$$

where c is the speed of light in vacuum, and ε_0 is the vacuum permittivity. Here, n_s is the usual linear refractive index of the waveguide without atoms and $n_s \approx 1$. Also, $\mathbf{P}^{\mathrm{NL}}$ is the macroscopic nonlinear polarization of the medium. In our case, the linear susceptibility is negligible because the atomic states related to the probe are depopulated. Therefore, $\mathbf{P}^{\mathrm{NL}}$ represents the cross-Kerr nonlinearity controlled by another laser beam, and is linear in terms of $\mathbf{E}$ but proportional to the intensity of that beam. This nonlinearity is attributed to the polarizability of the atoms.

For a right-moving and monochromatic planar wave in a 1D waveguide, we can write the electric field $\tilde{\mathbf{E}}$ and the polarization $\tilde{\mathbf{P}}^{\mathrm{NL}}$ as the summation of the positive- and negative-frequency components:

$$\begin{aligned} \tilde{\mathbf{E}} &= \tilde{\mathbf{E}}^{+} + \tilde{\mathbf{E}}^{-} , \\ \tilde{\mathbf{P}}^{\mathrm{NL}} &= \tilde{\mathbf{P}}^{\mathrm{NL},+} + \tilde{\mathbf{P}}^{\mathrm{NL},-} , \end{aligned} \tag{S2}$$

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with

$$\tilde{\mathbf{E}}^{+} = \mathbf{E}e^{i(\omega t-kz)} \ , \tag{S3a}$$

$$\tilde{\mathbf{E}}^{-} = \mathbf{E}e^{-i(\omega t-kz)} \ , \tag{S3b}$$

$$\tilde{\mathbf{P}}^{\mathrm{NL},+} = \mathbf{P}^{\mathrm{NL}}e^{i(\omega t-kz)} \ , \tag{S3c}$$

$$\tilde{\mathbf{P}}^{\mathrm{NL},-} = \mathbf{P}^{\mathrm{NL}}e^{-i(\omega t-kz)} \ , \tag{S3d}$$

where $\mathbf{E}$ and $\mathbf{P}^{\mathrm{NL}}$ are the slowly-varying envelopes of the electric field and the polarization, respectively. k is the wave vector of the field. Then we have, for the negative-frequency part,

$$\frac{\partial^2\tilde{\mathbf{E}}^{+}}{\partial z^2} = \left(\frac{\partial^2\mathbf{E}}{\partial z^2} + 2ik\frac{\partial\mathbf{E}}{\partial z} - k^2\mathbf{E}\right)e^{-i(\omega t-kz)} \ , \tag{S4a}$$

$$\frac{\partial^2\tilde{\mathbf{E}}^{+}}{\partial t^2} = \left(\frac{\partial^2\mathbf{E}}{\partial t^2} - 2i\omega\frac{\partial\mathbf{E}}{\partial t} - \omega^2\mathbf{E}\right)e^{-i(\omega t-kz)} \ , \tag{S4b}$$

$$\frac{\partial^2\tilde{\mathbf{P}}^{\mathrm{NL},+}}{\partial t^2} = \left(\frac{\partial^2\mathbf{P}^{\mathrm{NL}}}{\partial t^2} - 2i\omega\frac{\partial\mathbf{P}^{\mathrm{NL}}}{\partial t} - \omega^2\mathbf{P}^{\mathrm{NL}}\right)e^{-i(\omega t-kz)} \ . \tag{S4c}$$

To derive the unidirectional wave equation in the 1D waveguide, we only need to consider either the positive- or the negative-frequency components of the field and the polarization. Substituting Eq. S4 into Eq. S1 and applying the conditions

$$\frac{\partial^2\mathbf{E}}{\partial z^2} \ll 2k\frac{\partial\mathbf{E}}{\partial z} \ , \tag{S5a}$$

$$\frac{\partial^2\mathbf{E}}{\partial t^2} \ll 2\omega\frac{\partial\mathbf{E}}{\partial t} \ , \tag{S5b}$$

$$\frac{\partial^2\mathbf{P}^{\mathrm{NL}}}{\partial t^2} \ll \omega^2\mathbf{P}^{\mathrm{NL}} \ , \tag{S5c}$$

$$2\omega\frac{\partial\mathbf{P}^{\mathrm{NL}}}{\partial t} \ll \omega^2\mathbf{P}^{\mathrm{NL}} \ , \tag{S5d}$$

for a long pulse, we approximately have

$$\frac{\partial\mathbf{E}}{\partial z} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = i\frac{\omega}{2\varepsilon_0 c}\mathbf{P}^{\mathrm{NL}} \ , \tag{S6}$$

for the right-moving field. Similarly, the unidirectional wave equation of the left-moving field has the form

$$\frac{\partial\mathbf{E}}{\partial z} - \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = -i\frac{\omega}{2\varepsilon_0 c}\mathbf{P}^{\mathrm{NL}} \ . \tag{S7}$$

Note that the backward-moving field enters the medium from the right side at $z = L$ and propagates towards $z = 0$. For simplicity, we replace z with $(L - z')$. In doing so, we transform the backward-moving part to propagate in the positive z' direction. Thus, Eq. S7 becomes

$$\frac{\partial\mathbf{E}}{\partial z'} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = i\frac{\omega}{2\varepsilon_0 c}\mathbf{P}^{\mathrm{NL}} \ . \tag{S8}$$

In 1D space we can treat the vectors $\mathbf{E}$ and $\mathbf{P}^{\mathrm{NL}}$ as scalar quantities and replace them with E and P, respectively.

Without loss of generality, we assume that the field E drives a transition between two atomic states, namely, the excited state $|e\rangle$ and the ground state $|g\rangle$. This field is subject to modification due to the atomic polarizability ρ_{eg}. For an atomic medium, we have $P = N_a\langle\hat{\mu}\rangle\rho_{\mathrm{eg}}$ [1, 2], where $\hat{\mu} = -e\mathbf{r}$ is the electric dipole moment operator associated with the transition, and $-e$ is the charge of the electron. N_a is the number density of atoms. The matrix representation of the dipole moment operator is $\hat{\mu} = \mu_{\mathrm{ge}}|g\rangle\langle e| + \mu_{\mathrm{eg}}|e\rangle\langle g|$, where $\mu_{ij} = \langle i|\hat{mu}|j\rangle$ $(i, j = e, g)$. We use the notations $d = |\mu_{\mathrm{eg}}|$ for the average of the electric dipole moment. Thus, $P = -N_a d\rho_{\mathrm{eg}}$ for the negative-frequency part. The Rabi frequency of the field is defined as $\Omega = dE/\hbar$. Thus, in terms of the Rabi frequency, the unidirectionally propagating wave equations become

$$\frac{\partial\Omega_f}{\partial z} + \frac{1}{c}\frac{\partial\Omega_f}{\partial t} = -iN_a\frac{\omega d^2}{2\varepsilon_0\hbar c}\rho_{\mathrm{eg}} \ , \tag{S9a}$$

$$\frac{\partial\Omega_b}{\partial z'} + \frac{1}{c}\frac{\partial\Omega_b}{\partial t} = -iN_a\frac{\omega d^2}{2\varepsilon_0\hbar c}\rho_{\mathrm{eg}} \ . \tag{S9b}$$

In the above unidirectional wave equation, we neglect the backscattering between two oppositely propagating probe modes. This is reasonable because: (i) The atoms are mostly trapped in their ground states and therefore the backscattering due to atomic radiance is negigible [3, 4]. (ii) The forward- and backward-moving probe photons have opposite momenta. Their coherent coupling, caused by the perturbance of the medium or waveguide structure, is greatly suppressed [5, 6].

2 Chiral cross-Kerr nonlinearity

Next we derive the chiral XKerr nonlinearity for Eq. S9, i.e. Eqs. (1-2). The key point is to find the atomic polarization ρ_{eg} for our configuration.

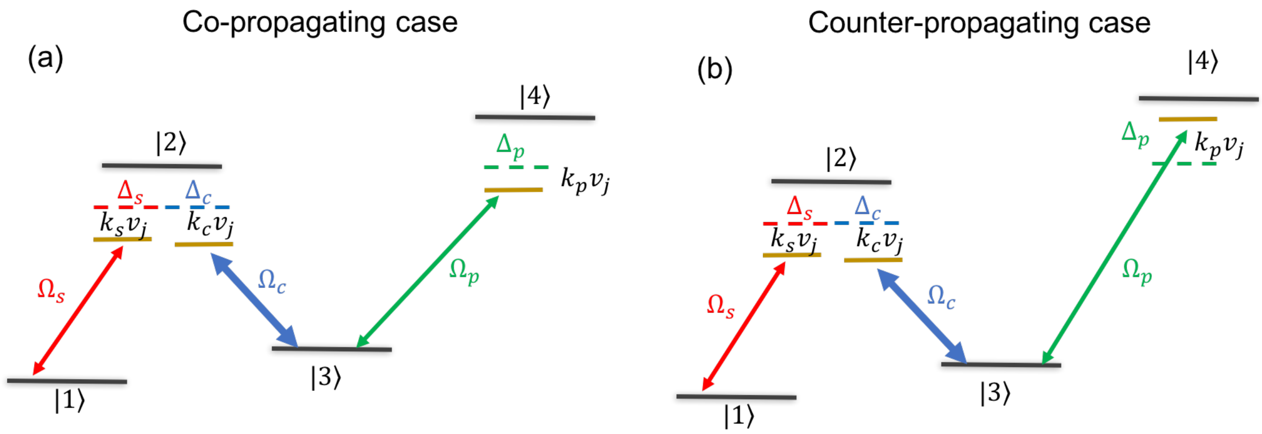

Figure S2: (color online) Level diagrams for (a) the co-propagating case and (b) the counter-propagating case. The N-type atom is driven by the coupling field Ω_c, the switching field Ω_s and the probe field Ω_p, with detunings Δ_c, Δ_s and Δ_p, respectively. The wave vector of these fields are k_c, k_s and k_p. The thermal motion of the jth atom with velocity v_j causes the microscopic Doppler shifts $k_c v_j$, $k_s v_j$, and $k_p v_j$, seen by the corresponding fields Ω_c, Ω_s and Ω_p, respectively. The shifts are shown by the yellow lines. The coupling and the switching modes are arranged to be left-moving. The left-moving (right-moving) probe light sees the same (opposite) Doppler shift to the control and signal photons in the co-propagating (counter-propagating) case.

We consider the configuration shown in Fig. S2. An ensemble of N-type atoms with states $|j\rangle$ ($j \in \{1,2,3,4\}$), embedded in a 1D waveguide, is driven by the coupling field Ω_c with a wavevector k_c, the switching field Ω_s with a wavevector k_s, respectively. We now consider that these two fields are backward-moving (left-moving). The probe field Ω_p has a wave vector k_p. In the absence of thermal motion, the field Ω_c (Ω_s, Ω_p) is detuned from the transition $|2\rangle \leftrightarrow |3\rangle$ ($|1\rangle \leftrightarrow |2\rangle$, $|3\rangle \leftrightarrow |4\rangle$) by a value Δ_c (Δ_s, Δ_p). In the rotating frame, the Hamiltonian describing the field-atom interaction takes the form

$$H_1 = -\Delta_s\sigma_{11} - \Delta_c\sigma_{33} + (\Delta_p - \Delta_c)\sigma_{44} + (\Omega_s\sigma_{21} + \Omega_s^*\sigma_{12}) + (\Omega_s\sigma_{21} + \Omega_s^*\sigma_{12}) + (\Omega_p\sigma_{43} + \Omega_p^*\sigma_{34}) \,, \tag{S10}$$

where $\sigma_{ij} = |i\rangle\langle j|$. For simplicity, here we neglect the superscript for the jth atom. The excited state $|i\rangle$ ($i = 2, 4$) decays to the ground state $|j\rangle$ ($j = 1, 3$) at a rate γ_{ij}. The state $|3\rangle$ slowly decays to $|1\rangle$ at a rate Γ_{31} and is also pumped from $|1\rangle$ at a rate of Γ_{13} due to thermal excitations. The decay and dephasing of the atoms can be described by the Lindblad operator:

$$\mathscr{L}[\gamma, A]\rho = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A \,, \tag{S11}$$

where $\gamma = \{\gamma_{21}, \gamma_{23}, \gamma_{43}, \gamma_{31}, \gamma_{13}\}$ and $A = \{\sigma_{12}, \sigma_{32}, \sigma_{34}, \sigma_{13}, \sigma_{31}\}$. For atoms, we can choose $\gamma_{13} = \gamma_{31} = \Gamma_3$. The polarization, ρ_{eg}, in Eq. S9 is ρ_{43} in our case, i.e. $\rho_{\mathrm{eg}} = \rho_{43}$.

Our scheme makes use of the Doppler shifts to create a chiral cross-Kerr nonlinearity for the weak probe laser beam at room temperature. The above Hamiltonian ignores the Doppler shifts of atomic transitions due to the random thermal motion of atoms. In the presence of thermal motion, a laser beam with wavevector k moving towards (away) an atom moving with velocity v "sees" the atomic frequency upshifted (downshifted) by an amount kv. This Doppler shift is considerable at room temperature. Next, we take into account the effect of thermal motion in our model and calculate the cross-Kerr nonlinearity. In the presence of the Doppler shift, we need to replace the detunings Δ_c, Δ_s and Δ_p with $\Delta_c + k_c v$, $\Delta_s + k_s v$, and $\Delta_p + k_p v$, respectively. To achieve a large nonlinearity, we simply choose $\Delta_c = \Delta_s = \delta$ and arrange that the coupling and switching laser beams co-propagate in the negative z direction. The atom is on two-photon resonance with these two fields. The probe field can propagate in two opposite (forward and backward) directions in the atomic medium.

Below we will see that the thermal-motion-induced Doppler shift, combining with the unidirectionally propagating coupling and switching fields, can induce a chiral cross-Kerr (XKerr) nonlinearity in atoms for the probe mode. As a result, the cross phase modulation and absorption of the probe field due the existence of the switching field becomes locked to their relative propagation directions. The equation of motion for the atomic density matrix takes the form

$$\dot{\rho}_{11} = 2\gamma_{21}\rho_{22} + 2\gamma_{31}\rho_{33} - 2\gamma_{13}\rho_{11} + i(\Omega_s\rho_{12} - \Omega_s^*\rho_{21}) \tag{S12a}$$
$$\dot{\rho}_{22} = -2(\gamma_{21} + \gamma_{23})\rho_{22} - i(\Omega_s\rho_{12} - \Omega_s^*\rho_{21}) - i(\Omega_c\rho_{32} - \Omega_c^*\rho_{23}) \tag{S12b}$$
$$\dot{\rho}_{33} = 2\gamma_{43}\rho_{44} + 2\gamma_{13}\rho_{11} - 2\gamma_{31}\rho_{33} + 2\gamma_{23}\rho_{22} + i(\Omega_c\rho_{32} - \Omega_c^*\rho_{23}) + i(\Omega_p\rho_{34} - \Omega_p^*\rho_{43}) \tag{S12c}$$
$$\dot{\rho}_{44} = -2\gamma_{43}\rho_{44} - i(\Omega_p\rho_{34} - \Omega_p^*\rho_{43}) \tag{S12d}$$
$$\dot{\rho}_{21} = -i(\delta + k_s v)\rho_{21} - (\gamma_{21} + \gamma_{23} + \gamma_{13})\rho_{21} + i\Omega_s(\rho_{22} - \rho_{11}) - i\Omega_c\rho_{31}\ , \tag{S12e}$$
$$\dot{\rho}_{31} = -i(k_s v - k_c v)\rho_{31} - (\gamma_{31} + \gamma_{13})\rho_{31} + i\Omega_s\rho_{32} - i\Omega_c^*\rho_{21} - i\Omega_p^*\rho_{41}\ , \tag{S12f}$$
$$\dot{\rho}_{43} = -i(\Delta_p + k_p v + k_c v)\rho_{43} + i\Omega_c\rho_{42} + i\Omega_p(\rho_{44} - \rho_{33}) - (\gamma_{43} + \gamma_{31})\rho_{43}\ . \tag{S12g}$$

We solve Eq. S12 using the perturbation method [1, 7–9]. The density matrix elements can be expanded as $\rho_{nm} = \rho_{nm}^{(0)} + \rho_{nm}^{(1)} + \rho_{nm}^{(2)} + \rho_{nm}^{(3)} + \cdots$. In our case, we have $\Omega_c \gg \Omega_s$ that, to a good approximation, all the populations can be assumed in the ground state $|1\rangle$ to zeroth order, i.e. $\rho_{11}^{(0)} = 1, \rho_{22}^{(0)} = \rho_{33}^{(0)} = \rho_{44}^{(0)} = 0$. Therefore, we also have $\rho_{32}^{(0)} = \rho_{41}^{(0)} = \rho_{43}^{(0)} = 0$. We arrange that the probe field is much weaker than the control and signal fields, i.e. $\Omega_p \ll \Omega_s, \Omega_c$. Under the weak-probe approximation, the off-diagonal terms with $\rho_{nm}\Omega_p$ $(n \neq m)$ can be neglected when solving for ρ_{nm}. Solving Eq. S12, we obtain the off-diagonal density-matrix elements, to first order, to be

$$\rho_{21}^{(1)} = \frac{-i\Omega_s}{F_1}\ , \tag{S13a}$$
$$\rho_{31}^{(1)} = \frac{-i\Omega_c^*\rho_{21}^{(1)}}{i(k_s v - k_c v) + (\gamma_{31} + \gamma_{13})}\ , \tag{S13b}$$
$$\rho_{32}^{(1)} = \rho_{43}^{(1)} = 0\ , \tag{S13c}$$

where

$$F = i(\delta + k_s v) + (\gamma_{21} + \gamma_{23} + \gamma_{13}) + |\Omega_c|^2/[i(k_s v - k_c v) + (\gamma_{31} + \gamma_{13})]\ . \tag{S14}$$

Since the total population in a closed atomic system is conserved, i.e. $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$, it is straightforward to find the second-order diagonal density-matrix elements satisfying the relationship

$$\rho_{11}^{(2)} + \rho_{22}^{(2)} + \rho_{33}^{(2)} + \rho_{44}^{(2)} = 0\ . \tag{S15}$$

Substituting Eqs. S13 and S15 to Eqs. S12(a-d), the second-order diagonal density-matrix elements can be obtained

$$\rho_{33}^{(2)} = \frac{\gamma_{23} - \Gamma_3}{4\Gamma_3(\gamma_{21} + \gamma_{23})}|\Omega_s|^2\left(\frac{1}{F} + \frac{1}{F^*}\right) \tag{S16a}$$
$$\approx \frac{\gamma_{23}}{4\Gamma_3(\gamma_{21} + \gamma_{23})}|\Omega_s|^2\left(\frac{1}{F} + \frac{1}{F^*}\right)$$
$$\rho_{44}^{(2)} = 0\ . \tag{S16b}$$

Again, substituting Eq. S16 into Eq. S12(g) and solving it, we obtain the third-order atomic polarization for the probe field

$$\rho_{43}^{(3)} = \frac{-i\Omega_p|\Omega_s|^2}{4\Gamma_3(\gamma_{21} + \gamma_{23})}\frac{\gamma_{23}}{i(\Delta_p + k_p v + k_c v) + \gamma_{43}}\left(\frac{1}{F} + \frac{1}{F^*}\right)\ . \tag{S17}$$

Because $\rho_{43}^{(0)} \approx \rho_{43}^{(1)} \approx \rho_{43}^{(2)} \approx 0$, we have $\rho_{43} \approx \rho_{43}^{(3)}$ for the total atomic polarization summed over from the first order to the third.

The decay rate γ_{43} is related to the atomic transition frequency ω_{43} via the relation $\gamma_{43} = d^2\omega_{43}^3/(3\pi\varepsilon_0\hbar c^3)$ [1, 10]. The carrier frequency of the probe laser beam is much larger than its detuning to the atom, so we can take the approximation $\omega_p \approx \omega_{43}$ in the calculation of the cross-Kerr nonlinearity. We assume that $|k_c| = |k_s| = |k_p|$. In our configuration, the forward-moving (backward-moving) probe field counter-propagates (co-propagates) with the coupling and switching fields. Thus we have $k_p v = -k_c v$ for the forward-moving probe, and $k_p v = k_c v$ for the backward-moving probe, respectively. The number of atoms is typically large enough that we can convert the sum of atoms into an integral over the velocity distribution in the 1D space. Substituting Eq. S17 into the unidirectional wave equation Eq. S9, we get

$$\frac{\partial\Omega_f(z,t)}{\partial z} + \frac{1}{c}\frac{\partial\Omega_f(z,t)}{\partial t} = -\chi_f|\Omega_s|^2\Omega_p^f(z,t)\ , \tag{S18a}$$
$$\frac{\partial\Omega_b(z',t)}{\partial z'} + \frac{1}{c}\frac{\partial\Omega_b(z',t)}{\partial t} = -\chi_b|\Omega_s|^2\Omega_p^b(z',t)\ , \tag{S18b}$$

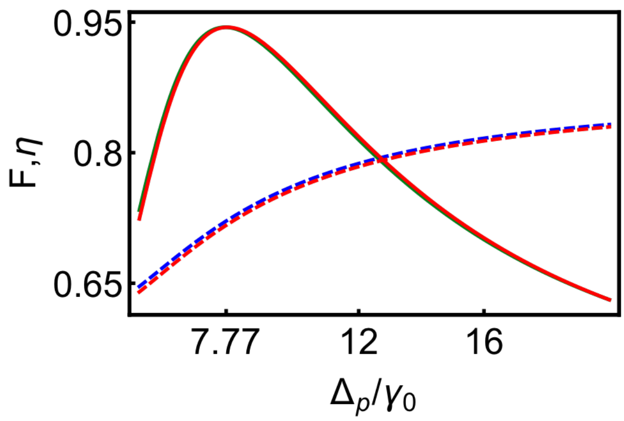

Figure S3: (color online) Fidelity (solid curves) and average insertion loss (dashed curves) of a circulator. Red curves are for $\vartheta = 0$. Blue and green curves for $\vartheta = 0.01\pi$. Other parameters are $N_a = 5 \times 10^{12}$ cm^{-3}, $\Gamma_3 = 0.1\gamma_0$, $\Omega_c = 20\gamma_0$, $\Omega_s = 4\gamma_0, \Delta_p = 7.77\gamma_0$, and $L = 3.33$ mm.

where the effective cross-Kerr nonlinearity is given by

$$\chi_f = X_0 \int \frac{\gamma_{23}}{(i\Delta_p + \gamma_{43})} \left(\frac{1}{\zeta} + \frac{1}{\zeta^*} \right) N(v) dv \,, \tag{S19}$$

for the forward-moving probe, and

$$\chi_b = X_0 \int \frac{\gamma_{23}}{[i(\Delta_p + 2kv) + \gamma_{43}]} \left(\frac{1}{\zeta} + \frac{1}{\zeta^*} \right) N(v) dv \,, \tag{S20}$$

for the backward-moving probe, and $X_0 = 3\pi c^2 \gamma_{43}/8\omega_p^2 \Gamma_3(\gamma_{21} + \gamma_{23})$, $\zeta = i(\delta + kv) + (\gamma_{21} + \gamma_{23} + \Gamma_3) + |\Omega_c|^2/2\Gamma_3$. The velocity distribution is conventionally taken to be Maxwellian, i.e. $N(v) = N_a \exp(-v^2/u^2)/\sqrt{\pi} u$, where u is the room-mean-square atomic velocity, and $ku \approx 2\pi \times 300$ MHz for Rb atoms at room temperature [11].

We are interested in the atomic response to a long pulse such that $\frac{1}{c}\frac{\partial \Omega_f}{\partial t} \approx 0$ and $\frac{1}{c}\frac{\partial \Omega_b}{\partial t} \approx 0$. After passing through the medium with a length L, the probe fields become

$$\Omega_p^f(L) = \xi_f e^{i\phi_f} \Omega_p^f(0) \,, \tag{S21}$$

in the 0–z coordinate system, and

$$\Omega_p^f(L) = \xi_f e^{i\phi_f} \Omega_p^f(0) \,, \tag{S22}$$

in the 0–z' coordinate system. Also, $\xi_j = \exp(-\mathrm{Re}[\chi_j]|\Omega_s|^2 L)$ and $\phi_j = -\mathrm{Im}[\chi_j]|\Omega_s|^2 L$, with $j = f, b$, are the corresponding transmission amplitude and phase shift, respectively.

Obviously, the XKerr nonlinearity induced in atoms is crucially dependent on the probe propagation with respect to the coupling and the switching lasers. It can be very different, i.e. chiral, in a specially engineered medium.

3 Zero phase shift in the lower path

To achieve the optimal performance of the optical circulator, we apply a small phase shift, $\vartheta = 0.01\pi$, to compensate the phase modulation on the backward-moving photon in the upper path of the Mach-Zehnder interferometer. However, a simpler version without this phase shifter ($\vartheta = 0$) only very slightly reduces the performance of the device, see Fig. S3. In the main paper, we present a general scheme for the optical circulator. Because the phase modulation on the backward laser beam is very small, one can remove the phase shifter in the lower path to simplify the setup while maintaining a high performance.

References

[1] Robert W. Boyd, *Nonlinear Optics*, Elsevier, 3rd edition, 2008.

[2] Keyu Xia, Shangqing Gong, Chengpu Liu, Xiaohong Song, and Yueping Niu, Near dipole-dipole effects on the propagation of few-cycle pulse in a dense two-level medium, *Opt. Express*, **13**, 5913–5924 (2005).

[3] Michael Fleischhauer, Atac Imamoglu, and Jonathan P. Marangos, Electromagnetically induced transparency: Optics in coherent media, *Rev. Mod. Phys.*, **77**, 633–673 (2005).

[4] Z.-Y. Liu, Y.-H. Chen, Y.-C. Chen, H.-Y. Lo, P.-J. Tsai, I. A. Yu, Y.-C. Chen, and Y.-F. Chen, Large cross-phase modulation at the few-photon level, *Phys. Rev. Lett.*, **117**, 203601 (2016).

[5] Keyu Xia, M. Alamri, and M. Suhail Zubairy, Ultrabroadband nonreciprocal transverse energy flow of light in linear passive photonic circuits, *Opt. Express*, **21**, 25619–25631 (2013).

[6] Katsunari Okamoto, *Fundamental of Optical Waveguides*, Elsevier, 2nd edition, 2006.

[7] Yong qing Li and Min Xiao, Electromagnetically induced transparency in a three-level Λ-type system in rubidium atoms, *Phys. Rev. A*, **51**, R2703 (1995).

[8] Jiteng Sheng, Xihua Yang, Haibin Wu, and Min Xiao, Modified self-Kerr-nonlinearity in a four-level N-type atomic system, *Phys. Rev. A*, **84**, 053820 (2011).

[9] Hai Wang, D. J. Goorskey, and Min Xiao, Atomic coherence induced Kerr nonlinearity enhancement in Rb vapour, *J. Mod. Opt.*, **49**, 335–347 (2002).

[10] Marlan O. Scully and M. Suhail Zubairy, *Quantum Optics*, Cambridge University Press, 1997.

[11] Haibin Wu, J. Gea-Banacloche, and Min Xiao, Observation of intracavity electromagnetically induced transparency and polariton resonances in a Doppler-broadended medium, *Phys. Rev. Lett.*, **100**, 173602 (2008).