Entangling two oscillators with arbitrary asymmetric initial states

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We present a Hamiltonian which can be used to convert any asymmetric state $|\varphi\rangle_a|\phi\rangle_b$ of two oscillators a and b into an entangled state via a single-step operation. Furthermore, with this Hamiltonian and only local operations, two oscillators, initially in any asymmetric initial states, can be entangled with a third oscillator. The prepared entangled states can be engineered with an arbitrary degree of entanglement. A discussion of the realization of this Hamiltonian is given. Numerical simulations show that, with current circuit QED technology, it is feasible to generate high-fidelity entangled states of two microwave optical fields, such as entangled coherent states, entangled squeezed states, entangled coherent-squeezed states, and entangled cat states. Our finding opens a avenue for creating not only wavelike or particlelike entanglement but also wavelike and particlelike hybrid entanglement.

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I. INTRODUCTION

Entangled states of light are a fundamental resource for many quantum information tasks [1–8]. In the regime of discrete variables, entanglement of up to eight photons has been experimentally demonstrated via linear optical devices [9,10]. In the regime of continuous variables, Einstein-Podolsky-Rosen (EPR) states of light have been experimentally generated from two independent squeezed fields [11,12], two independent coherent fields [13], or a single squeezed light source [14]; two- or three-color entangled states of light have been experimentally prepared by means of nondegenerate optical parametric oscillators [15–17]. Recently, hybrid entanglement between particlelike and wavelike optical qubits or between quantum and classical states of light [18,19] has also been demonstrated in experiments, which has drawn increasing attention because hybrid entanglement of light is a key resource in establishing hybrid quantum networks and connecting quantum processors with different encoding qubits. Moreover, a large number of theoretical proposals have been presented for generating *particular types* of entangled states of light or photons in various physical systems [20–33].

In this paper, we propose a Hamiltonian which can be used to convert any asymmetric state $|\varphi\rangle_a|\phi\rangle_b$ of two oscillators a and b into an entangled state $\alpha|\varphi\rangle_a|\phi\rangle_b \pm \beta|\phi\rangle_a|\varphi\rangle_b$. Here, the term asymmetric state refers to the product state $|\varphi\rangle_a|\phi\rangle_b$, with $|\varphi\rangle \neq |\phi\rangle$. The procedure consists of *a single unitary operation* and a posterior measurement on the states of the qudit coupler that is used to couple the oscillators. Furthermore, by combining this Hamiltonian with additional local operations, two oscillators a and b initially in any asymmetric state $|\varphi\rangle_a|\phi\rangle_b$ and a third oscillator in the vacuum state $|0\rangle_c$ can be converted to a tripartite entangled state $\alpha|\varphi\rangle_a|\phi\rangle_b|0\rangle_c + \beta|\phi\rangle_a|\varphi\rangle_b|1\rangle_c$ with no measurement required. Hereafter, we call them the bipartite and tripartite protocols, respectively. In both cases, the degree of entanglement, determined by the two coefficients α and β, is adjustable by controlling the initial state of the qudit coupler. More importantly, the light fields involved can be wavelike entangled states, particlelike entangled states, or wavelike and particlelike hybrid entangled states, depending on whether the states $|\varphi\rangle$ and $|\phi\rangle$ are both wavelike states (e.g., coherent states, squeezed states, and cat states), both particlelike states (e.g., Fock states), or one wavelike state and one particlelike state (e.g., coherent states and Fock states).

Independent of the nature of the two nonidentical states $|\varphi\rangle$ and $|\phi\rangle$, the bipartite protocol requires postselection by measurement while the tripartite protocol does not. So they are not the "same". The protocol can be applied to optical cavities, microwave resonators, nanomechanical oscillators, and even hybrids of these systems. Mechanical oscillators preserve their ability to interact with almost anything and can be utilized for preparing nonclassical states of light [34–36] or matter [37]. In recent years, optical and microwave cavities and resonators as well as mechanical oscillators have played crucial roles in quantum information processing and manipulating light or microwave photons. In fact, hybrid quantum systems have become one of the most exciting areas of quantum science and technology [38].

As shown below, the entanglement generation operates essentially via the quantum state swapping conditioned on the state of the coupler. Namely, when the coupler is in the state $|g'\rangle$, the two-oscillator initial state $|\varphi\rangle_a|\phi\rangle_b$ remains unchanged; however, when the coupler is in the state $|g\rangle$, the two-oscillator initial state $|\varphi\rangle_a|\phi\rangle_b$ changes to $|\phi\rangle_a|\varphi\rangle_b$ via the state swapping $|\varphi\rangle \leftrightarrow |\phi\rangle$. Hence, the physical mechanism used for the entanglement creation here is quite different from those based on state synthesis algorithms [39–43].

The previous protocols for entangling two oscillators in high-dimension Hilbert space are based on complex state synthesis algorithms that require a sequence of unitary operations [39–43]. In stark contrast, the present proposal requires only a single unitary operation, significantly

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simplifying the experimental implementation and reducing the operation time and thus the negative effect of decoherence on fidelity. According to [35–39], the number of operations, required by state-synthesis algorithms for preparing the target states $|\Psi\rangle_{\rm target} = \sum_{m,n} C_{mn}|m,n\rangle$, increases drastically with the dimensionality of the subspace of the Fock-state space in which the target states are embedded [39–43].

Interestingly, it is also noted that based on the proposed Hamiltonian, a SWAP gate of two *discrete-variable* qubits or two *continuous-variable* qubits can be realized in a single operation, including the two-qubit SWAP gate with *cat-state* encoding qubits which have attracted increasing attention recently [44].

II. HAMILTONIAN AND INTUITION

Two oscillators a and b are coupled to a coupler with an energy level $|g\rangle$. In the interaction picture, the Hamiltonian considered here is given by (assuming $\hbar = 1$)

$$H = \omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})|g\rangle\langle g| + \lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)|g\rangle\langle g|, \quad (1)$$

where a (b) is the photon annihilation operator of oscillator a (b), $|\omega|$ (with ω being either positive or negative) is the frequency or frequency shift of both oscillators, and $|\lambda|$ (with λ being either positive or negative) is the coupling strength between the two oscillators. The second term, $\lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)|g\rangle\langle g|$, represents the interaction between the two oscillators when the coupler is in the state $|g\rangle$. After some interaction time, this term results in the exchange of the states of the two oscillators when the coupler is in the state $|g\rangle$. However, the two-oscillator state exchange is imperfect without including the first term, $\omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})|g\rangle\langle g|$, because the state exchange resulting from the second term, $\lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)|g\rangle\langle g|$, comes with inevitable photon-number-dependent phase errors. For instance, the state $|\varphi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$ of oscillator a (with $|n\rangle$ being the n-photon Fock state) is transferred onto oscillator b initially in a vacuum state by an error state $|\varphi\rangle_{\rm er} = \sum_{n=0}^{\infty} c_n e^{i\phi_n}|n\rangle$ (see the discussion below).

Note that Eq. (1) is different from the well-known Hamiltonian $\widetilde{H} = \omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}) + \lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)$ describing two single-mode interacting oscillators. This is because each term in Eq. (1) contains a coupler operator $|g\rangle\langle g|$, which is, however, not involved in $\widetilde{H}$.

III. ENTANGLING OSCILLATORS

Suppose that oscillator a is in an arbitrary pure state $|\varphi\rangle_a$ and oscillator b is in another arbitrary pure state $|\phi\rangle_b$. Assume that a coupler is in a superposition state $\alpha|g'\rangle + \beta|g\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. Here, $|g'\rangle$ is an excited state of the coupler. Under the Hamiltonian in Eq. (1), the initial state of the system $|\varphi\rangle_a|\phi\rangle_b(\alpha|g'\rangle + \beta|g\rangle)$ evolves into

$$\begin{aligned} &e^{-iHt}|\varphi\rangle_a|\phi\rangle_b(\alpha|g'\rangle + \beta|g\rangle) \\ &\quad = \alpha|\varphi\rangle_a|\phi\rangle_b|g'\rangle + \beta(e^{-iH_e t}|\varphi\rangle_a|\phi\rangle_b)\otimes|g\rangle, \end{aligned} \quad (2)$$

where we have used $\langle g|g'\rangle = 0$. Here, $H_e = H_0 + H_I$, with $H_0 = \omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})$ and $H_I = \lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)$. H_e describes the dynamics of the oscillators, which arises from Eq. (1) when the coupler is in the state $|g\rangle$. Because of $[H_0, H_I] = 0$, the oscillator state $e^{-iH_e t}|\varphi\rangle_a|\phi\rangle_b$ of Eq. (2) can be written as

$$e^{-iH_e t}|\varphi\rangle_a|\phi\rangle_b = U_2U_1|\varphi\rangle_a|\phi\rangle_b, \quad (3)$$

with $U_1 = e^{-iH_I t}$ and $U_2 = e^{-iH_0 t}$.

U_1 leads to the transformations $U_1\hat{a}^\dagger U_1^+ = \cos(\lambda t)\hat{a}^\dagger - i\sin(\lambda t)\hat{b}^\dagger$ and $U_1\hat{b}^\dagger U_1^+ = \cos(\lambda t)\hat{b}^\dagger - i\sin(\lambda t)\hat{a}^\dagger$. For $|\lambda|t = (2m + 1/2)\pi$ (m is an integer), one has $U_1(\hat{a}^\dagger)^n U_1^+ = (\mp i\hat{b}^\dagger)^n$ and $U_1(\hat{b}^\dagger)^n U_1^+ = (\mp i\hat{a}^\dagger)^n$, which will be applied in the derivation of Eq. (5) below. Here and below, a minus sign (−) corresponds to $\lambda > 0$, while a plus (+) corresponds to $\lambda < 0$. The arbitrary pure states $|\varphi\rangle_a$ and $|\phi\rangle_b$ can be expressed as

$$|\varphi\rangle_a = \sum_{n=0}^{\infty} c_n|n\rangle_a, \quad |\phi\rangle_b = \sum_{m=0}^{\infty} d_m|m\rangle_b, \quad (4)$$

where c_n and d_m are normalized coefficients and $|n\rangle_a = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle_a$ [$|m\rangle_b = \frac{(\hat{b}^\dagger)^m}{\sqrt{m!}}|0\rangle_b$] represents the $n-$photon (m-photon) Fock state of oscillator a (b).

By performing a unitary transformation U_1, after $t = \pi/(2|\lambda|)$, the state $|\varphi\rangle_a|\phi\rangle_b$ evolves into

$$\begin{aligned} &U_1|\varphi\rangle_a|\phi\rangle_b \\ &= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{c_n d_m}{\sqrt{n!m!}}[U_1(\hat{a}^\dagger)^n U_1^+][U_1(\hat{b}^\dagger)^m U_1^+]U_1|0\rangle_a|0\rangle_b \\ &= \sum_{n=0}^{\infty} c_n(\mp i)^n\frac{(\hat{b}^\dagger)^n}{\sqrt{n!}}|0\rangle_a \sum_{m=0}^{\infty} d_m(\mp i)^m\frac{(\hat{a}^\dagger)^m}{\sqrt{m!}}|0\rangle_b \\ &= \sum_{n=0}^{\infty} c_n e^{\mp in\pi/2}|n\rangle_b \otimes \sum_{m=0}^{\infty} d_m e^{\mp im\pi/2}|m\rangle_a, \end{aligned} \quad (5)$$

where the positions of $|0\rangle_a$ and $|0\rangle_b$ in the third line are exchanged in the last line and $U_1|0\rangle_a|0\rangle_b = |0\rangle_a|0\rangle_b$ is applied. The first (second) part of the product in the last line represents the state of oscillator b (a). Comparing the last line with the original states $|\varphi\rangle_a$ and $|\phi\rangle_b$ given in Eq. (4), one can see that the two oscillators exchange their states while accumulating photon-number-dependent phase errors $e^{\mp in\pi/2}$ and $e^{\mp im\pi/2}$, respectively.

By performing a unitary transformation U_2 with $t = \pi/(2|\lambda|)$ and setting $\mp\pi/2 - \omega t = 2k\pi$ (k is an integer), state (5) becomes

$$\begin{aligned} &U_2(U_1|\varphi\rangle_a|\phi\rangle_b) \\ &\quad = \sum_{n=0}^{\infty} c_n e^{in(\mp\pi/2-\omega t)}|n\rangle_b \otimes \sum_{m=0}^{\infty} d_m e^{im(\mp\pi/2-\omega t)}|m\rangle_a \\ &\quad = \sum_{n=0}^{\infty} c_n|n\rangle_b \otimes \sum_{m=0}^{\infty} d_m|m\rangle_a = |\varphi\rangle_b|\phi\rangle_a, \end{aligned} \quad (6)$$

where $|\varphi\rangle_b$ ($|\phi\rangle_a$) takes the same form the state $|\varphi\rangle_a$ ($|\phi\rangle_b$) with the subscript a (b) replaced by b (a). Combining Eqs. (3) and (6), one finds that state (2) would be

$$\alpha|\varphi\rangle_a|\phi\rangle_b|g'\rangle + \beta|\phi\rangle_a|\varphi\rangle_b|g\rangle. \quad (7)$$

Now apply a classical pulse to the coupler, resulting in $|g'\rangle \to (|g\rangle + |g'\rangle)/\sqrt{2}$ and $|g\rangle \to (|g\rangle - |g'\rangle)/\sqrt{2}$. Thus, state (7)

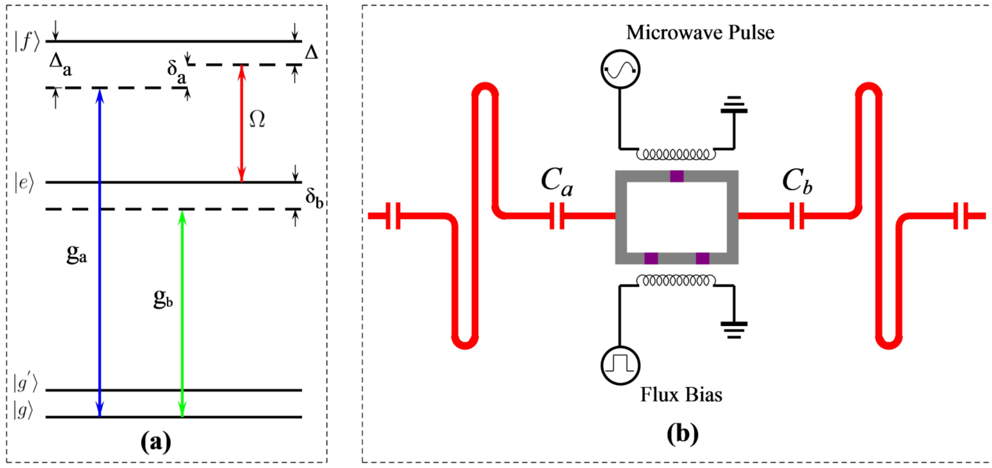

FIG. 1. (a) Illustration of the coupler interacting with two oscillators and a classical pulse. Here, $\delta_a = \omega_p + \omega_{eg} - \omega_a = \delta_b$, which can be readily met by adjusting the pulse frequency ω_p. (b) Setup of two cavities coupled to a flux device via a capacitor, C_a or C_b.

becomes

$$\frac{1}{\sqrt{2}}(|\psi^+\rangle \otimes |g\rangle + |\psi^-\rangle \otimes |g'\rangle), \tag{8}$$

with

$$|\psi^\pm\rangle = \alpha|\varphi\rangle_a|\phi\rangle_b \pm \beta|\phi\rangle_a|\varphi\rangle_b. \tag{9}$$

Equation (8) shows that when the coupler is measured in the state $|g\rangle$ ($|g'\rangle$), the two oscillators are prepared in an entangled state $|\psi\rangle^+$ ($|\psi^-\rangle$), for which the degree of entanglement can be adjusted by varying α and β during the preparation of the initial state of the coupler.

It is straightforward to show that state (7) can be transformed to a three-oscillator entangled state

$$\alpha|\varphi\rangle_a|\phi\rangle_b|1\rangle_c + \beta|\phi\rangle_a|\varphi\rangle_b|0\rangle_c \tag{10}$$

by performing local operations on the coupler and a third oscillator c initially in the vacuum state. For instance, this transformation from state (7) to state (10) can be achieved by tuning the frequency of oscillator c on resonance with the $|g\rangle \leftrightarrow |g'\rangle$ transition or vice versa to have a single photon emitted into oscillator c when the coupler is in the excited state $|g'\rangle$.

IV. HAMILTONIAN CONSTRUCTION

The four levels of the coupler are denoted as $|g\rangle$, $|g'\rangle$, $|e\rangle$, and $|f\rangle$ [Fig. 1(a)]. The level $|g'\rangle$ can remain unaffected, for example, by having the transition between $|g'\rangle$ and any other level highly detuned from the frequencies of the two oscillators and the classical pulse. Oscillator a (b) is coupled to the $|g\rangle \leftrightarrow |f\rangle$ ($|g\rangle \leftrightarrow |e\rangle$) transition with coupling strength g_a (g_b) and detuning $\Delta_a = \omega_{fg} - \omega_a$ ($\delta_b = \omega_{eg} - \omega_b$) [Fig. 1(a)]. Here, ω_{fg} (ω_{eg}) is the $|g\rangle \leftrightarrow |f\rangle$ ($|g\rangle \leftrightarrow |e\rangle$) transition frequency, and ω_a (ω_b) is the frequency of oscillator a (b). A classical pulse of frequency ω_p is coupled to the $|e\rangle \leftrightarrow |f\rangle$ transition with detunings $\Delta = \omega_{fe} - \omega_p$ [Fig. 1(a)]. In the interaction picture under the free Hamiltonian $H_{\text{field}} + H_{\text{atom}}$, with $H_{\text{field}} = \omega_a\hat{a}^\dagger\hat{a} + \omega_b\hat{b}^\dagger\hat{b}$, the Hamiltonian is given by

$$\begin{aligned} H = {} & (g_a e^{i\Delta_a t}\hat{a}\sigma_{fg}^+ + g_b e^{i\delta_b t}\hat{b}\sigma_{eg}^+ + \text{H.c.}) \\ & + (\Omega e^{i\Delta t}\sigma_{fe}^+ + \text{H.c.}), \end{aligned} \tag{11}$$

where $\sigma_{fg}^+ = |f\rangle\langle g|$, $\sigma_{fe}^+ = |f\rangle\langle e|$, Ω is the Rabi frequency of the classical pulse, and $\hat{a}$ ($\hat{b}$) is the photon annihilation operator of oscillator a (b).

Under large-detuning conditions and when the levels $|e\rangle$ and $|f\rangle$ are not occupied, the Hamiltonian of Eq. (11) can be expressed as the following effective Hamiltonian (see the Appendix):

$$\begin{aligned} H_{\text{eff}} = {} & -\left(g_a^2/\Delta_a + \tilde{g}_a^2/\delta\right)\hat{a}^\dagger\hat{a}|g\rangle\langle g| - g_b^2/\delta\hat{b}^\dagger\hat{b}|g\rangle\langle g| \\ & + \lambda(\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b})|g\rangle\langle g|, \end{aligned} \tag{12}$$

where $\tilde{g}_a = g_a\Omega(\Delta_a^{-1} + \Delta^{-1})/2$, $\delta_a = \Delta_a - \Delta$, and $\lambda = \tilde{g}_a g_b/\delta > 0$. In Eq. (12), we have set $\delta_a = \delta_b \equiv \delta > 0$, i.e., $\omega_p = \omega_a - \omega_b$, which can be readily achieved by adjusting the pulse frequency ω_p. By setting

$$\frac{g_a^2}{\Delta_a} + \frac{g_a^2\Omega^2}{4\delta}\left(\Delta_a^{-1} + \Delta^{-1}\right)^2 = \frac{g_b^2}{\delta} = -\omega \tag{13}$$

(e.g., by adjusting the pulse Rabi frequency Ω), one sees that Eq. (12) takes the same form as the Hamiltonian (1). Based on Eq. (13) and setting $\mp\pi/2 - \omega t = 2k\pi$, we can obtain the following relationship between the various parameters:

$$\begin{aligned} g_b &= \frac{|4k \pm 1|}{2\sqrt{2k(2k \pm 1)\Delta_a/\delta}} g_a, \\ \Omega &= \frac{\Delta\Delta_a}{\Delta + \Delta_a}\sqrt{\delta/[2k(2k \pm 1)\Delta_a]}, \end{aligned} \tag{14}$$

which shows that the pulse Rabi frequency Ω is independent of the coupling strengths g_a and g_b.

Note that the four-level structure in Fig. 1(a) is widely available in natural or artificial atoms such as quantum dots, nitrogen-vacancy centers, and various superconducting

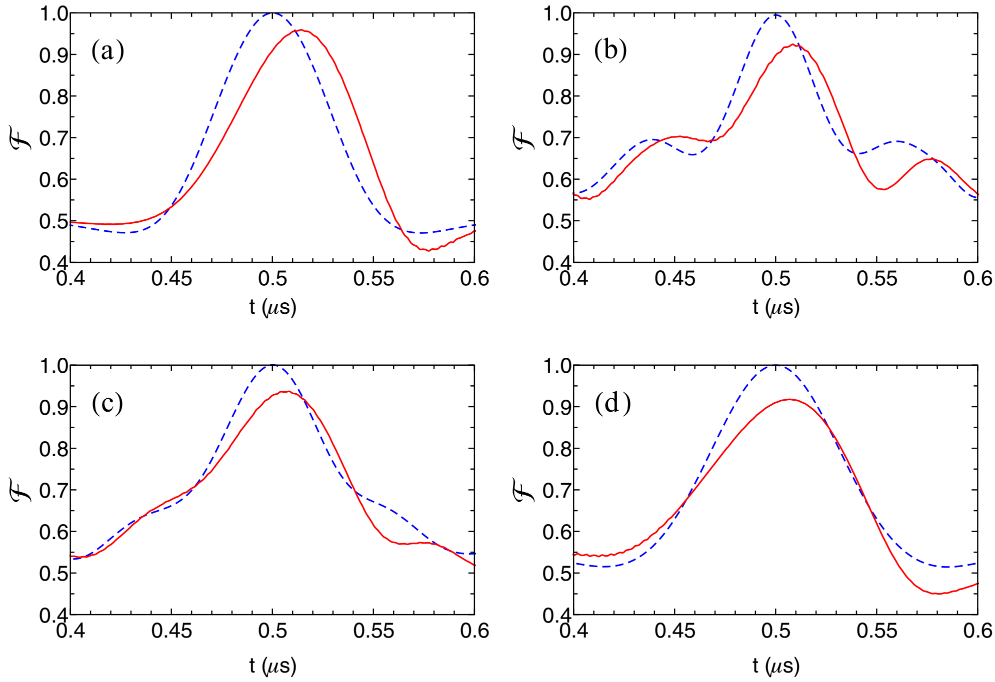

FIG. 2. Fidelities versus the operation time t for (a) entangled coherent states, (b) entangled squeezed states, (c) entangled coherent-squeezed states, and (d) entangled cat states. Dashed blue curves were based on the effective Hamiltonian (12) without considering decoherence, while red curves were based on the master equation (A1) by taking decoherence into consideration.

devices [45]. Thus, the Hamiltonian (1) can be realized with a variety of physical systems. As shown above, the Hamiltonian (12), i.e., Eq. (1), was constructed based on the Raman transition induced by the field-pulse cooperation. Note that it is possible to construct the proposed Hamiltonian (1) based on other physical mechanisms.

V. CIRCUIT-QED IMPLEMENTATION

Circuit QED with resonators and superconducting qubits is one of the most promising candidates for quantum information processing (for reviews, see [46–49]). We now consider a setup consisting of two microwave resonators coupled via a superconducting artificial atom [Fig. 1(b)]. Each resonator here is a one-dimensional transmission-line resonator (TLR). The four levels of the coupler are illustrated in Fig. 1(a). The pulse- or resonator-induced unwanted transitions between irrelevant levels are assumed to be negligibly small. This can be achieved by a prior design of the coupler with a strong anharmonicity (e.g., a superconducting flux device). Alternatively, this condition can be satisfied by adjusting the coupler level spacings or the resonator frequencies. In practice, level spacings of superconducting devices can be rapidly adjusted within a few nanoseconds (e.g., see [50] and references therein), and to a lesser extent, frequencies of the resonators can be quickly tuned in 1–3 ns [51,52]. When the interresonator cross talk is taken into account, the Hamiltonian (11) becomes $H' = H + \varepsilon$, where ε describes the unwanted interresonator cross talk, given by $\varepsilon = g_{ab}e^{i\Delta_{ab}t}\hat{a}^{\dagger}\hat{b} + \text{H.c.}$, with the two-resonator coupling strength g_{ab} and the resonator frequency detuning $\Delta_{ab} = \omega_a - \omega_b$. Here, ω_a (ω_b) is the frequency of resonator a (b).

The fidelity of the operation is given by $\mathcal{F} = \sqrt{\langle\psi_{id}|\rho|\psi_{id}\rangle}$, where $|\psi_{id}\rangle$ is the ideal state given in Eq. (7), while ρ is the final density operator of the whole system after the operation is performed in a realistic system. As an example, we consider $\alpha = \beta = 1/\sqrt{2}$.

By solving the master equation and choosing the system parameters appropriately (see the Appendix), the simulated fidelity $\mathcal{F}$ versus the operation time t is shown in Fig. 2 for $\eta = \Delta_a/g_a = 25$, $k = 1$, and $\alpha = \xi = 1$, where $|\pm\xi\rangle$ are squeezed vacuum states. One can see that for $t \sim 0.5\,\mu$s, a high fidelity can be obtained: (i) $\mathcal{F} \simeq 0.959$ for the entangled coherent states $\frac{1}{\sqrt{2}}(|\alpha\rangle_a|-\alpha\rangle_b \pm |-\alpha\rangle_a|\alpha\rangle_b)$ [Fig. 2(a)], (ii) $\mathcal{F} \simeq 0.912$ for the entangled squeezed states $\frac{1}{\sqrt{2}}(|\xi\rangle_a|-\xi\rangle_b \pm |-\xi\rangle_a|\xi\rangle_b)$ [Fig. 2(b)], (iii) $\mathcal{F} \simeq 0.929$ for the entangled coherent-squeezed states $\frac{1}{\sqrt{2}}(|\alpha\rangle_a|\xi\rangle_b \pm |\xi\rangle_a|\alpha\rangle_b)$ [Fig. 2(c)], and (iv) $\mathcal{F} \simeq 0.918$ for the entangled cat states $\frac{1}{\sqrt{2}}(|\text{cat}\rangle_a|\overline{\text{cat}}\rangle_b \pm |\overline{\text{cat}}\rangle_a|\text{cat}\rangle_b)$.

For $\eta = 25$, we have $g_a/2\pi \sim 60$ MHz, $g_b/2\pi \sim 25$ MHz, and $\Omega/2\pi \sim 114$ MHz, which are available in experiments [53,54]. The frequency of a circuit resonator is typically a few gigahertz. For the sake of concreteness, consider $\omega_a/(2\pi) \sim 7.5$ GHz and $\omega_b/(2\pi) \sim 4.5$ GHz. For the values of κ_a^{-1} and κ_b^{-1} used in the numerical simulation, the required quality factors for the two resonators are $Q_a \sim 9.4 \times 10^5$ and $Q_b \sim 5.6 \times 10^5$, which are readily available in experiments [55,56]. The analysis here demonstrates that by applying the proposed protocol, the high-fidelity generation of entanglement between asymmetric states of two oscillators is feasible with current circuit QED technology. Finally, we remark that the fidelity obtained above was calculated without considering the initial-state preparation and measurement errors, which, however, could be negligible due to progress in accurate preparation and measurement of the states of superconducting artificial atoms [57].

Finally, it is noted that based on the Hamiltonian (1), when the coupler is in the state $|g\rangle$, a SWAP gate of two *discrete-variable* qubits or two *continuous-variable* qubits, defined by $|\varphi\rangle_a|\varphi\rangle_b \to |\varphi\rangle_a|\varphi\rangle_b$, $|\varphi\rangle_a|\phi\rangle_b \to |\phi\rangle_a|\varphi\rangle_b$, $|\phi\rangle_a|\varphi\rangle_b \to |\varphi\rangle_a|\phi\rangle_b$, and $|\phi\rangle_a|\phi\rangle_b \to |\phi\rangle_a|\phi\rangle_b$, can be realized via a single operation. Here, a qubit is encoded by the two states $|\varphi\rangle$ and $|\phi\rangle$ of each oscillator. For $|\varphi\rangle = |\text{cat}\rangle$ and $|\phi\rangle = |\overline{\text{cat}}\rangle$, the two-qubit SWAP gate is implemented with *cat-state* encoding qubits [44].

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APPENDIX

1. Derivation of an effective Hamiltonian

Let us start with the original Hamiltonian given in Eq. (11), i.e.,

$$\begin{aligned} H &= g_a(\hat{a}\sigma_{fg}^{+}e^{i\Delta_a t} + \text{H.c.}) + g_b(\hat{b}\sigma_{eg}^{+}e^{i\delta_b t} + \text{H.c.}) \\ &\quad + \Omega(e^{i\Delta t}\sigma_{fe}^{+} + \text{H.c.}), \end{aligned} \tag{A1}$$

where $\sigma_{eg}^{+} = |e\rangle\langle g|$ and $\sigma_{fg}^{+} = |f\rangle\langle g|$, Ω is the Rabi frequency of the pulse, and $\hat{a}$ ($\hat{b}$) is the photon annihilation operator for quantum oscillator a (b).

Under the large-detuning conditions $\Delta_a \gg g_a$ and $\Omega \gg \Delta$, there is no energy exchange between oscillator a and the coupler or between the pulse and the coupler [Fig. 1(a)]. In addition, under the conditions $\Delta_a - \delta_b \gg g_a g_b(\Delta_a^{-1} + \delta_b^{-1})/2$ and $\Delta - \delta_b \gg \Omega g_b(\Delta^{-1} + \delta_b^{-1})/2$, there is no interaction between oscillator b and either oscillator a or the pulse [Fig. 1(a)]. In this case, the effective Hamiltonian can be expressed as [58]

$$\begin{aligned} H_{\text{eff}} &= \frac{g_a^2}{\Delta_a}[|f\rangle\langle f| + \hat{a}^{\dagger}\hat{a}(|f\rangle\langle f| - |g\rangle\langle g|)] \\ &\quad + \frac{\Omega^2}{\Delta}(|f\rangle\langle f| - |e\rangle\langle e|) - \tilde{g}_a(\hat{a}\sigma_{eg}^{+}e^{i\delta_a t} + \text{H.c.}) \\ &\quad + g_b(\hat{b}\sigma_{eg}^{+}e^{i\delta_b t} + \text{H.c.}), \end{aligned} \tag{A2}$$

where $\tilde{g}_a = g_a\Omega(\Delta_a^{-1} + \Delta^{-1})/2$ and $\delta_a = \Delta_a - \Delta$. Under the large-detuning conditions $\delta_a \gg \{\tilde{g}_a, g_a^2/\Delta_a, \Omega^2/\Delta\}$ and $\delta_b \gg \{g_b, g_a^2/\Delta_a, \Omega^2/\Delta\}$, the effective Hamiltonian H_{eff} becomes [58]

$$\begin{aligned} H_{\text{eff}} &= \frac{\tilde{g}_a^2}{\delta}[|e\rangle\langle e| + \hat{a}^{\dagger}\hat{a}(|e\rangle\langle e| - |g\rangle\langle g|)] \\ &\quad + \frac{g_b^2}{\delta_b}[|e\rangle\langle e| + \hat{b}^{\dagger}\hat{b}(|e\rangle\langle e| - |g\rangle\langle g|)] \\ &\quad + \frac{g_a^2}{\Delta_a}[|f\rangle\langle f| + \hat{a}^{\dagger}\hat{a}(|f\rangle\langle f| - |g\rangle\langle g|)] \\ &\quad + \frac{\Omega^2}{\Delta}(|f\rangle\langle f| - |e\rangle\langle e|) - \frac{\tilde{g}_a g_b}{2}\left(\frac{1}{\delta_a} + \frac{1}{\delta_b}\right) \\ &\quad \times [(\hat{a}\hat{b}^{\dagger}|e\rangle\langle e| - \hat{a}^{\dagger}\hat{b}|g\rangle\langle g|)e^{i(\delta_a - \delta_b)t} + \text{H.c.}]. \end{aligned} \tag{A3}$$

When the levels $|e\rangle$ and $|f\rangle$ are not occupied, the effective Hamiltonian H_{eff} reduces to

$$\begin{aligned} H_{\text{eff}} &= -\left(\frac{g_a^2}{\Delta_a} + \frac{\tilde{g}_a^2}{\delta}\right)\hat{a}^{\dagger}\hat{a}|g\rangle\langle g| - \frac{g_b^2}{\delta}\hat{b}^{\dagger}\hat{b}|g\rangle\langle g| \\ &\quad + \lambda(\hat{a}\hat{b}^{\dagger} + \hat{a}^{\dagger}\hat{b})|g\rangle\langle g|, \end{aligned} \tag{A4}$$

where $\lambda = \tilde{g}_a g_b/\delta$ and we have set $\delta_a = \delta_b = \delta$.

2. Master equation and parameters used in the numerical simulation

After taking dissipation and dephasing into account, the system dynamics is determined by the master equation

$$\begin{aligned} \frac{d\rho}{dt} &= -i[H',\rho] + \kappa_a\mathcal{L}[\hat{a}] + \kappa_b\mathcal{L}[\hat{b}] \\ &\quad + \sum_{j=g,g',e}\gamma_{fj}\mathcal{L}[\sigma_{fj}^{-}] + \sum_{k=g,g'}\gamma_{ek}\mathcal{L}[\sigma_{ek}^{-}] + \gamma_{g'g}\mathcal{L}[\sigma_{g'g}^{-}] \\ &\quad + \sum_{j=g',e,f}\gamma_{\varphi,l}(\sigma_{ll}\rho\sigma_{ll} - \sigma_{ll}\rho/2 - \rho\sigma_{ll}/2), \end{aligned} \tag{A5}$$

where $\mathcal{L}[\Lambda] = \Lambda\rho\Lambda^{+} - \Lambda^{+}\Lambda\rho/2 - \rho\Lambda^{+}\Lambda/2$ (with $\Lambda = \hat{a},\hat{b},\sigma_{g'g}^{-},\sigma_{eg}^{-},\sigma_{eg'}^{-},\sigma_{fg}^{-},\sigma_{fg'}^{-},\sigma_{fe}^{-}$), $\sigma_{g'g'} = |g'\rangle\langle g'|$, $\sigma_{ee} = |e\rangle\langle e|$, and $\sigma_{ff} = |f\rangle\langle f|$. In addition, κ_a (κ_b) is the decay rate of resonator a (b); $\gamma_{g'g}$, γ_{eg}, $\gamma_{eg'}$, γ_{fg}, $\gamma_{fg'}$, and γ_{fe} are the energy relaxation rates for $|g'\rangle \to |g\rangle$, $|e\rangle \to |g\rangle$, $|e\rangle \to |g'\rangle$, $|f\rangle \to |g\rangle$, $|f\rangle \to |g'\rangle$, and $|f\rangle \to |e\rangle$, respectively, and $\gamma_{\varphi,g'}$, $\gamma_{\varphi,e}$, and $\gamma_{\varphi,f}$ are the dephasing rates of levels $|g'\rangle$, $|e\rangle$, and $|f\rangle$.

The parameters used in the numerical simulation are (i) $\Delta_a/2\pi = 1.5$ GHz, $\Delta/2\pi = 1.25$ GHz, (ii) $\delta_b/2\pi = 0.25$ GHz, (iii) $\gamma_{\varphi,g'}^{-1} = \gamma_{\varphi,e}^{-1} = \gamma_{\varphi,f}^{-1} = 15$ μs, (iv) $\gamma_{g'g}^{-1} = 60$ μs, $\gamma_{eg'}^{-1} = 40$ μs, $\gamma_{fe}^{-1} = 30$ μs, $\gamma_{eg}^{-1} = \gamma_{fg'}^{-1} = \gamma_{fg}^{-1} = 100$ μs [59], and (v) $\kappa_a^{-1} = \kappa_a^{-1} = 20$ μs. We choose $g_{12} = 0.1\max\{g_a, g_b\}$. Here, we consider a rather conservative case for both the interresonator cross talk and the decoherence time of flux qudits because the interresonator cross-talk strength can be smaller by at least one order of magnitude [29] and decoherence time ranging from 70 μs to 1 ms has been reported for a superconducting qudit [60–63].

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