Comparison of the sensitivity to systematic errors between nonadiabatic non-Abelian geometric gates and their dynamical counterparts

Shi-Biao Zheng,[1,2] Chui-Ping Yang,[1,3] and Franco Nori[1,4]
[1]*CEMS, RIKEN, Wako-shi, Saitama 351-0198, Japan*
[2]*Department of Physics, Fuzhou University, Fuzhou 350116, China*
[3]*Department of Physics, Hangzhou Normal University, Hangzhou 310036, China*
[4]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*
(Received 24 September 2015; published 9 March 2016)

We investigate the effects of systematic errors of the control parameters on single-qubit gates based on nonadiabatic non-Abelian geometric holonomies and those relying on purely dynamical evolution. It is explicitly shown that the systematic error in the Rabi frequency of the control fields affects these two kinds of gates in different ways. In the presence of this systematic error, the transformation produced by the nonadiabatic non-Abelian geometric gate is not unitary in the computational space, and the resulting gate infidelity is larger than that with the dynamical method. Our results provide a theoretical basis for choosing a suitable method for implementing elementary quantum gates in physical systems, where the systematic noises are the dominant noise source.

DOI: 10.1103/PhysRevA.93.032313

I. INTRODUCTION

A quantum system undergoing a cyclic evolution will acquire a geometric phase, in addition to a dynamical phase [1–6]. In contrast with the dynamical phase, the geometric phase is only determined by some geometric features of the closed circuit that the system traces in the projective Hilbert space, independent of the rate at which the circuit is followed. In the case of a cyclic adiabatic evolution of parameters in the Hamiltonian, this additional phase is the well-known Berry phase [1], which is proportional to the area spanned in parameter space. When the subspace of the Hilbert space traversed by the system is nondegenerate, the geometric phase is a real number. On the other hand, the cyclic evolution of a degenerate subspace results in a matrix-valued transformation, known as the non-Abelian holonomy [5,6]. In addition to its fundamental interest, the geometric phase is considered to be a candidate for the coherent control of quantum systems. Quantum gates based on Berry phases have been investigated both theoretically and experimentally [7–11].

The use of Berry phases for implementing fault-tolerant quantum computation has stimulated interest in their behavior in the presence of noise [12–18]. In particular, the effect of the fluctuation noise in the classical control parameters on the Berry adiabatic phase of a spin-1/2 system has been analyzed [12]. It was shown [12] that the contribution of the Berry phase to both the phase variance and dephasing vanishes for a long evolution time, because the effect of noise on the global features of the evolution path is averaged out in the adiabatic limit. This has been demonstrated experimentally using ultracold neutrons [17] and circuit quantum electrodynamics [18]. The main obstacle for the implementation of adiabatic geometric phase gates is that the time scale associated with the adiabatic evolution should be much longer than the dynamical one, which implies that these geometric gates operate very slowly, compared to the dynamical process. This makes the system vulnerable to open-system effects, which would result in the loss of coherence. To overcome this problem, schemes based on nonadiabatic Abelian geometric phases have been proposed [19,20], whose sensitivity to noise has also been analyzed [21–23].

Recently, a method has been proposed for implementing holonomic quantum computation based on nonadiabatic non-Abelian geometric phases [24]. To realize a single-qubit gate, the two computational basis states of a qubit are coupled to an auxiliary state by using two classical fields, which drive the degenerate subspace spanned by the two basis states to undergo a cyclic evolution. These kinds of geometric gates have received considerable interest, and have been experimentally demonstrated in circuit QED [25], NMR systems [26], and solid-state spins associated with a diamond nitrogen-vacancy (NV) center [27,28]. The effects of decoherence and noise on this gate have also been investigated [29]. However, an open problem remains whether this gate is more robust against errors in control parameters than the dynamical ones. This is the aim of the present work. In the manuscript we compare the fidelities of quantum logic gates relying on nonadiabatic non-Abelian holonomies and the corresponding dynamical gates in the presence of systematic errors. Here the systematic errors refer to the errors of the corresponding control parameters that fluctuate slowly compared with the system evolution, so that they approximately keep constant during the gate operation. Our results show that the systematic error in the Rabi frequency of the driving fields affects the nonadiabatic non-Abelian geometric gate and the dynamical one in different ways: for the former, this error results in the leakage of the qubit's population into the noncomputational space, while for the latter it causes a deviation of the transition probability from the expected value in the computational space. As a result, the nonadiabatic non-Abelian geometric gate is more sensitive to the Rabi frequency systematic error than the dynamical gate; in particular, for the geometric implementation of a Hadamard gate, the infidelity induced by this systematic error is increased by one order of magnitude, compared to the dynamical method. Furthermore, the imperfect control of the amplitude ratio of the two driving fields for realizing the non-Abelian holonomies introduces an additional error.

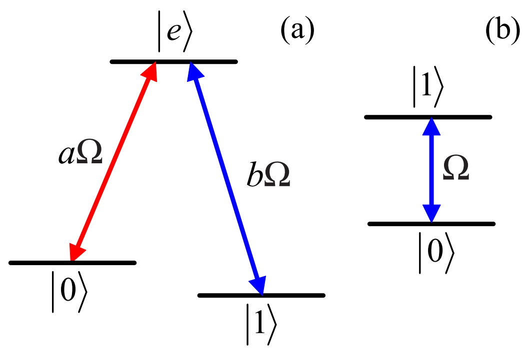

FIG. 1. System level configuration and excitation scheme. (a) Non-Abelian nonadiabatic gate: the qubit is represented by two ground states $|0\rangle$ and $|1\rangle$ of a Λ-type three-level system, with the transitions $|0\rangle \rightarrow |e\rangle$ and $|1\rangle \rightarrow |e\rangle$ being resonantly driven by two classical fields with the couplings $a\Omega$ and $b\Omega$, respectively, with $|a|^2 + |b|^2 = 1$. Ω is the Rabi frequency characterizing the transition between the bright state $|\phi_{b,0}\rangle = a^*|0\rangle + b^*|1\rangle$ and the auxiliary excited state $|e\rangle$. (b) Dynamical gate: the transition $|0\rangle \rightarrow |1\rangle$ is directly coupled by a classical field with Rabi frequency Ω.

This paper is organized as follows. In Sec. II, we analyze the effect of the systematic errors of the control fields on the nonadiabatic non-Abelian geometric gate. The result shows that the qubit has a probability of leaking out of the computational space in the presence of the Rabi frequency fluctuation. In Sec. III, we calculate the infidelity of the dynamical gate caused by the systematic errors. Since the qubit remains in the computational space, the relation between the infidelity and the parameters of the expected gate transformation matrix is different from that for the nonadiabatic non-Abelian geometric gate. In Sec. IV, we compare the infidelities of the two gates, demonstrating that the geometric gate is more sensitive to the Rabi frequency systematic error than the dynamical one. Conclusions appear in Sec. V.

II. FIDELITY OF NONADIABATIC NON-ABELIAN GEOMETRIC GATES

We first consider the infidelity of the nonadiabatic non-Abelian geometric operation in the presence of systematic errors in the control parameters. Recently, it has been shown [24] that a universal holonomic single-qubit gate based on the nonadiabatic non-Abelian geometric transformation can be realized in a three-level system, with the two lower states, $|0\rangle$ and $|1\rangle$, representing the logic states and one higher state $|e\rangle$ acting as the auxiliary state. The transitions $|0\rangle \rightarrow |e\rangle$ and $|1\rangle \rightarrow |e\rangle$ are resonantly driven by two classical fields, as shown Fig. 1(a). The Hamiltonian in the interaction picture is given by

$$H_g = \hbar\Omega(a|e\rangle\langle 0| + b|e\rangle\langle 1| + \text{H.c.}), \tag{1}$$

where $|a|^2 + |b|^2 = 1$, Ω is the Rabi frequency characterizing the transition between the bright state, $|\phi_b\rangle = a^*|0\rangle + b^*|1\rangle$, and the excited state $|e\rangle$, and H.c. represents the Hermitian conjugate. The three eigenstates of the system are

$$\begin{aligned} |\phi_d\rangle &= (b|0\rangle - a|1\rangle), \\ |\phi_{b,\pm}\rangle &= \frac{1}{\sqrt{2}}(|\phi_b\rangle \pm |e\rangle), \end{aligned} \tag{2}$$

with the corresponding eigenenergies given by

$$\begin{aligned} E_d &= 0, \\ E_{b,\pm} &= \pm\Omega. \end{aligned} \tag{3}$$

The dark state $|\phi_d\rangle$ is decoupled from the Hamiltonian and undergoes no transition during the application of the driving fields. Without loss of generality, we here set $a = \sin(\theta/2)e^{i\phi}$ and $b = \cos(\theta/2)$, with $0 \leqslant \theta \leqslant \pi$ decided by the ratio between the amplitudes of the two driving fields and $0 \leqslant \phi \leqslant 2\pi$ depending on the relative phase of these fields. The evolution of the initial basis states $|k\rangle$ $(k = 0,1)$ are given by $|\psi_k(t)\rangle = \exp(-i\int_0^t H\, dt/\hbar)|k\rangle$. When a/b remains unchanged during the interaction, the evolution satisfies the parallel-transport condition $\langle\psi_k(t)|H|\psi_j(t)\rangle = 0$, and hence is purely geometric. For $\int_0^T \Omega\, dt = \pi$, the degenerate qubit space undergoes a cyclic evolution (the system returns to the subspace spanned by qubit logic states $|0\rangle$ and $|1\rangle$), leading to a non-Abelian holonomic transformation. As a consequence, the final evolution operator in the computational basis $\{|0\rangle, |1\rangle\}$ is

$$U_g = \begin{pmatrix} \cos\theta & -\sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}, \tag{4}$$

which can be used to realize any single-qubit rotation.

When the fluctuations in the amplitudes and phases of the driving fields are considered, the Hamiltonian becomes

$$H_g^{'} = \hbar\Omega^{'}[\sin(\theta^{'}/2)e^{i\phi^{'}}|e\rangle\langle 0| + \cos(\theta^{'}/2)|e\rangle\langle 1|] + \text{H.c.}, \tag{5}$$

where $\Omega^{'} = \Omega + \delta\Omega$, $\theta^{'} = \theta + \delta\theta$, and $\phi^{'} = \phi + \delta\phi$, with $\delta\Omega$, $\delta\theta$, and $\delta\phi$ being the deviations of the corresponding parameters from the expected values due to the presence of systematic errors of the driving fields. We note that $\delta\theta$ arises from the imperfect control of the ratio of the amplitudes of the two fields. We here consider the case where the control parameters fluctuate slowly as compared to the gate speed, so that $\delta\Omega$, $\delta\theta$, and $\delta\phi$ can be regarded as constants during the geometric operation. This is the case in experiments performed in certain physical systems [17,18,30]. In terms of the basis states $\{|0\rangle, |1\rangle, |e\rangle\}$, the evolution operator can be expressed as

$$U_g^{'} = U_{g,0}^{'} + U_{g,1}^{'}, \tag{6}$$

where

$$U_{g,0}^{'} = \begin{pmatrix} \cos\theta^{'} & -e^{-i\phi^{'}}\sin\theta^{'} & 0 \\ -e^{i\phi^{'}}\sin\theta^{'} & -\cos\theta^{'} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

$$U'_{g,1} = \begin{pmatrix} [1-\cos(\delta\Omega T)]\sin^2(\theta'/2) & \frac{1}{2}[1-\cos(\delta\Omega T)]\sin\theta' e^{-i\phi'} & i\,\sin(\delta\Omega T)\sin(\theta'/2)e^{i\phi'} \\ \frac{1}{2}[1-\cos(\delta\Omega T)]\sin\theta' e^{i\phi'} & [1-\cos(\delta\Omega T)]\cos^2(\theta'/2) & i\,\sin(\delta\Omega T)\cos(\theta'/2) \\ i\,\sin(\delta\Omega T)\sin(\theta'/2)e^{-i\phi'} & i\,\sin(\delta\Omega T)\cos(\theta'/2) & -\cos(\delta\Omega T) \end{pmatrix}. \tag{8}$$

This result shows that the Rabi frequency fluctuation causes the leakage of the qubit's population out of the computational space.

For any input state $|\psi_i\rangle$, the fidelity of the output state is defined as

$$\mathcal{F}_{g,\psi,1} = |\langle\psi_d|\psi_r\rangle|^2, \tag{9}$$

where $|\psi_d\rangle = U_g|\psi_i\rangle$ is the desired output state, and $|\psi_r\rangle = U'_g|\psi_i\rangle$ is the real output state with the amplitude and phase errors of the driving fields being considered. Setting $|\psi_i\rangle = \cos\frac{\alpha}{2}|0\rangle + \sin\frac{\alpha}{2}e^{i\beta}|1\rangle$ [23,31], to second order in $\delta\theta$, $\delta\phi$, and $\delta\Omega T$, we obtain

$$\begin{aligned}\mathcal{F}_{g,\psi} \simeq\ & 1-(\delta\theta)^2[1-\sin^2\alpha\,\sin^2(\beta-\phi)] \\ & -(\delta\phi)^2\big\{\sin^2\theta - \big[\cos\alpha\,\sin^2\theta \\ & +\tfrac{1}{2}\sin\alpha\,\sin(2\theta)\cos(\beta-\phi)\big]^2\big\} \\ & -2\delta\theta\,\delta\phi\,\sin\alpha\,\sin(\beta-\phi)\big[\cos\alpha\,\sin^2\theta \\ & +\tfrac{1}{2}\sin\alpha\,\sin(2\theta)\cos(\beta-\phi)\big] \\ & -(\delta\Omega T)^2|\cos\alpha\,\sin(\theta/2)e^{i\phi}+\sin\alpha\,\cos(\theta/2)e^{i\beta}|^2.\end{aligned} \tag{10}$$

For fixed parameter errors, the fidelity depends on the initial qubit state. Averaging over all the input states, the average fidelity is calculated to be

$$\begin{aligned}\mathcal{F}_g &= \frac{1}{4\pi}\int_0^\pi \sin\alpha\,d\alpha\int_0^{2\pi} d\beta\,\mathcal{F}_{g,\psi} \\ &= 1-\frac{1}{3}\left(\pi\frac{\delta\Omega}{\Omega}\right)^2[1+\cos^2(\theta/2)] - \frac{2}{3}(\delta\theta)^2 \\ &\quad -(\delta\phi)^2\left[\frac{2}{3}\sin^2\theta - \frac{1}{12}\sin^2(2\theta)\right].\end{aligned} \tag{11}$$

We here have used the relation $\Omega T = \pi$. The result shows that the gate infidelity arising from the Rabi frequency systematic error (the second term of $\mathcal{F}_g$) decreases as the parameter θ increases.

III. FIDELITY OF THE DYNAMICAL GATE

Let us now consider the effects of systematic errors of the control parameters on gates based on purely dynamical evolution, which can be achieved by resonantly driving the transition between the two logic states $|0\rangle$ and $|1\rangle$ with a classical field, as shown in Fig. 1(b). The corresponding Hamiltonian is given by

$$H_d = \hbar(\Omega\,e^{-i\phi}|0\rangle\langle 1| + \Omega\,e^{i\phi}|1\rangle\langle 0|). \tag{12}$$

With this Hamiltonian one can obtain an evolution operator similar to the geometric gate

$$U_d = \begin{pmatrix} \cos\theta & -i\,e^{-i\phi}\sin\theta \\ -i\,e^{i\phi}\sin\theta & \cos\theta \end{pmatrix}, \tag{13}$$

where $\theta = \Omega\tau$, with τ being the gate duration. Any single-qubit transformation can be obtained through this kind of evolution with $0 < \theta \leqslant \pi/2$. For example, with the choice $\phi = 3\pi/2$ this evolution yields the gate $U_y(\phi_y) = \exp(i\phi_y\sigma_y)$ with $\phi_y = \theta$. Two sequential evolutions $U_d(\theta = \pi/2, \phi = \phi_z/2)$ and $U_d(\theta = \pi/2, \phi = -\phi_z/2)$ produce the gate $U_z = \exp(i\phi_z\sigma_z)$ plus a trivial global π-phase shift. The rotations $U_y(\phi_y)$ and $U_z(\phi_z)$ with $0 < \phi_y < \pi$ and $0 < \phi_z < \pi$ form a universal set of single-qubit operations. The same single-qubit operation can be achieved using a similar combination of nonadiabatic non-Abelian geometric gates [27]. In this sense, the effect of the dynamical gate U_d is equivalent to that of the geometric gate U_g.

With the field amplitude and phase deviations $\delta\Omega$ and $\delta\phi$ being included, the qubit evolution operator can now be written as

$$U'_d = \begin{pmatrix} \cos\theta' & -i\,e^{-i\phi'}\sin\theta' \\ -i\,e^{i\phi'}\sin\theta' & \cos\theta' \end{pmatrix}, \tag{14}$$

where $\theta' = \theta + \delta\Omega\tau$ and $\phi' = \phi + \delta\phi$. This result shows that the Rabi frequency fluctuation leads to the imperfect control of the probability for the qubit's transition between the computational states $|0\rangle$ and $|1\rangle$. For the input state $|\psi_i\rangle = \cos\frac{\alpha}{2}|0\rangle + \sin\frac{\alpha}{2}e^{i\beta}|1\rangle$, to second order in $\delta\Omega\tau$ and $\delta\phi$, the fidelity of the output state becomes

$$\begin{aligned}\mathcal{F}_{d,\psi} \simeq\ & 1-(\delta\Omega\tau)^2[1-\sin^2\alpha\,\cos^2(\beta-\phi)] \\ & -(\delta\phi)^2\big\{\sin^2\theta - \big[\cos\alpha\,\sin^2\theta \\ & +\tfrac{1}{2}\sin\alpha\,\sin(2\theta)\sin(\beta-\phi)\big]^2\big\} \\ & -2\delta\phi\,\delta\Omega\tau\,\sin\alpha\,\cos(\beta-\phi)\big[\cos\alpha\,\sin^2\theta \\ & -\tfrac{1}{2}\sin\alpha\,\sin(2\theta)\sin(\beta-\phi)\big].\end{aligned} \tag{15}$$

Averaging over all the input states and using the relation $\theta = \Omega\tau$, we obtain the average fidelity for the dynamical gate

$$\mathcal{F}_d \simeq 1-\frac{2}{3}\left(\theta\frac{\delta\Omega}{\Omega}\right)^2 - (\delta\phi)^2\left[\frac{2}{3}\sin^2\theta - \frac{1}{12}\sin^2(2\theta)\right]. \tag{16}$$

The result shows that the gate infidelity induced by the Rabi frequency systematic error (the second term of $\mathcal{F}_d$) increases quadratically with θ, which is in distinct contrast with the case for the nonadiabatic non-Abelian geometric gate.

IV. COMPARISON

The error of the dynamical gate due to the Rabi frequency fluctuation $\delta\Omega$ is proportional to the square of the parameter θ of the expected gate transformation, which is in stark contrast with the nonadiabatic non-Abelian geometric gate, whose infidelity associated with $\delta\Omega$ is proportional to $1+\cos^2(\theta/2)$. This difference is due to the fact that the required operation time $\tau=\theta/\Omega$ is proportional to θ for the dynamical evolution, while the geometric operation time does not depend upon θ for a given Rabi frequency Ω. In other words, the condition $\Omega T=\pi$ is always required to implement any nonadiabatic non-Abelian geometric gate. As a result, the error of the dynamical gate shows much stronger dependence on θ than that of the geometric gate. We note that the fluctuation $\delta\Omega$ affects the nonadiabatic non-Abelian geometric gate and the dynamical one in different ways. For the nonadiabatic non-Abelian geometric gate, this fluctuation leads to the leakage of the population from the bright state $|\phi_b\rangle$ to the auxiliary state $|e\rangle$ after the operation, so that the resulting transformation in the computational space $\{|0\rangle,|1\rangle\}$ is not unitary. In contrast, for the dynamical method this fluctuation results in inaccurate control of the transition probability between the two logic states $|0\rangle$ and $|1\rangle$; these two states span the whole state space and the resulting transformation in the computational space remains unitary. It should be noted that the second term of Eq. (16) can be expressed as $-2(\delta\theta)^2/3$, which has the same form as the third term of Eq. (11). However, the reasons for the occurrence of the error $\delta\theta$ are completely different for the nonadiabatic non-Abelian geometric gate and the dynamical one. For the former, this error is caused by imperfect control of the ratio between the amplitudes of the two fields driving the transitions $|0\rangle\rightarrow|e\rangle$ and $|1\rangle\rightarrow|e\rangle$, and is equal to $\delta\theta=2\frac{\delta r}{1+r^2}$, where $r=|a/b|$, with $|a|\Omega$ and $|b|\Omega$ characterizing the amplitudes of the two driving fields, as shown in Eq. (1). For the latter, $\delta\theta$ is due to the Rabi frequency fluctuation, and is given by $\delta\theta=\delta\Omega\tau$.

Since $\theta\leqslant\pi/2$, the maximum infidelity induced by the error $\delta\Omega$ for the dynamical operation U_d is $\frac{1}{6}(\pi\frac{\delta\Omega}{\Omega})^2$, which is only one half of the minimum of the corresponding infidelity for the nonadiabatic non-Abelian geometric operation U_g, when the values of the relative error $\delta\Omega/\Omega$ are the same for both gates. More importantly, for the dynamical method this infidelity decreases quadratically as the parameter θ decreases. For example, for the dynamical implementation of the Hadamard transformation this infidelity is reduced by one order of magnitude, compared to the geometric method. Since only one field is needed for realizing U_d, the error associated with the fluctuation in the ratio between the amplitudes of the two fields used for implementing U_g does not appear in the dynamical method. For the same θ and $\delta\phi$, the errors due to the field phase fluctuation are the same for both cases. Therefore, when the systematic errors are the main error source, the fidelity of the non-Abelian nonadiabatic geometric operation is lower than the corresponding dynamical one.

We further note that the required dynamical gate time θ/Ω is shorter than $\pi/(2\Omega)$ as $\theta\leqslant\pi/2$, while the geometric gate duration is always π/Ω. Due to the longer operation time, the nonadiabatic non-Abelian geometric gate is more sensitive to decoherence effects, arising from the system-reservoir coupling, than the dynamical gate.

V. CONCLUSION

In conclusion, we have analyzed the effects of systematic errors in the control parameters on the nonadiabatic non-Abelian geometric gate and on its dynamical equivalent. We show that the systematic error of the Rabi frequency of the driving fields affects these two gates in different ways. This systematic error causes the qubit's population to leak to the noncomputational space after the nonadiabatic non-Abelian geometric gate is performed, while the qubit always remains in the computational space when the gate is implemented by the dynamical method. As a consequence, the nonadiabatic non-Abelian geometric gates are more sensitive to the Rabi frequency systematic error than the purely dynamical ones. For implementation of the Hadamard gate, the infidelity induced by this error in the nonadiabatic non-Abelian geometric method is one order of magnitude larger than that in the dynamical one. Our results are useful for choice of optimal methods for implementing elementary quantum gates under the condition that the fluctuations of the control parameters are slow compared with the gate speed.

ACKNOWLEDGMENTS

S.B.Z. acknowledges support from the Major State Basic Research Development Program of China under Grant No. 2012CB921601 and the National Natural Science Foundation of China under Grant No. 11374054. C.P.Y. acknowledges support from the National Natural Science Foundation of China under Grant No. 11374083. F.N. is partially supported by the RIKEN iTHES Project, the MURI Center for Dynamic Magneto-Optics via the AFOSR Grant No. FA9550-14-1-0040, the IMPACT program of JST, and a Grant-in-Aid for Scientific Research (A).

[1] M. V. Berry, Quantal phase-factors accompanying adiabatic changes, Proc. R. Soc. A **392**, 45 (1984).

[2] J. Anandan, The geometric phase, Nature (London) **360**, 307 (1992).

[3] Y. Aharonov and J. Anandan, Phase-change During a Cyclic Quantum Evolution, Phys. Rev. Lett. **58**, 1593 (1987).

[4] S. Oh, X. Hu, F. Nori, and S. Kais, Singularity of the time-energy uncertainty in adiabatic perturbation and cycloids on a Bloch sphere, Sci. Rep. **6**, 20824 (2016).

[5] F. Wilczek and A. Zee, Appearance of Gauge Structure in Simple Dynamical Systems, Phys. Rev. Lett. **52**, 2111 (1984).

[6] J. Anandan, Non-adiabatic non-Abelian geometric phase, Phys. Lett. A **133**, 171 (1988).

[7] P. Zanardi and M. Rasetti, Holonomic quantum computation, Phys. Lett. A **264**, 94 (1999).

[8] G. Falci, R. Fazio, G. M. Palma, J. Siewert, and V. Vedral, Detection of geometric phases in superconducting nanocircuits, Nature (London) **407**, 355 (2000).

[9] L. M. Duan, J. I. Cirac, and P. Zoller, Geometric manipulation of trapped ions for quantum computation, Science **292**, 1695 (2001).
[10] S. B. Zheng, Unconventional geometric quantum phase gates with a cavity QED system, Phys. Rev. A **70**, 052320 (2004).
[11] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Geometric quantum computation with NMR, Nature (London) **403**, 869 (2000).
[12] G. De Chiara and G. M. Palma, Berry Phase for a Spin-1/2 Particle in a Classical Fluctuating Field, Phys. Rev. Lett. **91**, 090404 (2003).
[13] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Geometric Phase in Open Systems, Phys. Rev. Lett. **90**, 160402 (2003).
[14] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Spin-1/2 Geometric Phase Driven by Decohering Quantum Fields, Phys. Rev. Lett. **92**, 020402 (2004).
[15] S. B. Zheng, Manifestation of a nonclassical Berry phase of an electromagnetic field in atomic Ramsey interference, Phys. Rev. A **85**, 022128 (2012).
[16] P. J. Leek, J. M. Fink, A. Blais, R. Bianchetti, M. Göppl, J. M. Gambetta, D. I. Schuster, L. Frunzio, R. J. Schoelkopf, A. Wallraff, Observation of Berry's phase in a solid state qubit, Science **318**, 1889 (2007).
[17] S. Filipp, J. Klepp, Y. Hasegawa, C. Plonka-Spehr, U. Schmidt, P. Geltenbort, and H. Rauch, Experimental Demonstration of the Stability of Berry's Phase for a Spin-1/2 Particle, Phys. Rev. Lett. **102**, 030404 (2009).
[18] S. Berger, M. Pechal, A. A. Abdumalikov, C. Eichler, L. Steffen, A. Fedorov, A. Wallraff, and S. Filipp, Exploring the effect of noise on the Berry phase, Phys. Rev. A **87**, 060303(R) (2013).
[19] X. B. Wang and M. Keiji, Nonadiabatic Conditional Geometric Phase Shift with NMR, Phys. Rev. Lett. **87**, 097901 (2001).
[20] S. L. Zhu and Z. D. Wang, Implementation of Universal Quantum Gates Based on Nonadiabatic Geometric Phases, Phys. Rev. Lett. **89**, 097902 (2002).
[21] A. Nazir, T. Spiller, and W. J. Munro, Decoherence of geometric phase gates, Phys. Rev. A **65**, 042303 (2002).
[22] A. Blais and A.-M. S. Tremblay, Effect of noise on geometric logic gates for quantum computation, Phys. Rev. A **67**, 012308 (2003).
[23] J. T. Thomas, M. Lababidi, and M. Tian, Robustness of single-qubit geometric gate against systematic error, Phys. Rev. A **84**, 042335 (2011)
[24] E. Sjöqvist, D. M. Tong, L. M. Andersson, B. Hessmo, M. Johansson, and K. Singh, Non-adiabatic holonomic quantum computation, New J. Phys. **14**, 103035 (2012).
[25] A. A. Abdumalikov, , J. M. Fink, K. Juliusson, M. Pechal, S. Berger, A. Wallraff, and S. Filipp, Experimental realization of non-Abelian non-adiabatic geometric gates, Nature (London) **496**, 482 (2013).
[26] G. Feng, G. Xu, and G. Long, Experimental Realization of Nonadiabatic Holonomic Quantum Computation, Phys. Rev. Lett. **110**, 190501 (2013).
[27] C. Zu, W.-B. Wang, L. He, W.-G. Zhang, C.-Y. Dai, F. Wang, and L.-M. Duan, Experimental realization of universal geometric quantum gates with solid-state spins, Nature (London) **514**, 72 (2014).
[28] S. Arroyo-Camejo, A. Lazariev, S. W. Hell, and G. Balasubramanian, Room temperature high-fidelity holonomic single-qubit gate on a solid-state spin, Nat. Commun. **5**, 4870 (2014).
[29] M. Johansson, E. Sjöqvist, L. M. Andersson, M. Ericsson, B. Hessmo, K. Singh, and D. M. Tong, Robustness of non-adiabatic holonomic gates, Phys. Rev. A **86**, 062322 (2012).
[30] J. Benhelm, G. Kirchmair, C. F. Roos, and R. Blatt, Towards fault-tolerant quantum computing with trapped ions, Nat. Phys. **4**, 463 (2008).
[31] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).