Zhang *et al.* Reply: The preceding Comment [1] admitted and confirmed that the calculations in our Letter [2] are correct. The Comment [1] uses the Caldeira-Leggett model for a harmonic oscillator to question the physical relevance of an unbounded total Hamiltonian for only one of three examples explored in Ref. [2]. Below we show how important and well-known aspects of the physical relevance of the Hamiltonian used in Ref. [2] address the question.

The total Hamiltonian used in Ref. [2], $H_{\rm tot} = \sum_i \varepsilon_i a_i^\dagger a_i + \sum_k \omega_k b_k^\dagger b_k + \sum_{ik} V_{ik}(a_i^\dagger b_k + b_k^\dagger a_i)$ (valid for both fermion and boson systems), is indeed the Fano-Anderson (FA) Hamiltonian in condensed matter physics [3,4]. It is also referred to as the Lee-Friedrichs Hamiltonian in atomic physics [5,6]. This is the basic Hamiltonian for investigating quantum resonances and the decay of impurities or discrete states embedded in a continuum, and it is widely used in atomic physics, quantum optics, condensed matter, as well as quantum field theory. The physical relevance of the FA Hamiltonian includes the famous Fano effect, Anderson localization, the Dicke effect and superradiance, resonant tunnelings, etc., just to name a few examples. The localized modes (or dressed bound states) existing outside and inside of the environment spectra are a general consequence of this Hamiltonian, which can be obtained by simply solving the Schrödinger equation given in many textbooks and review articles, e.g., see Refs. [4,6,7], and have been observed experimentally in different physical systems [8]. Thus, very surprisingly, the Comment [1] questions the physical relevance of a well-established and experimentally justified Hamiltonian.

When the environment is initially in thermal equilibrium, the Schrödinger equation is no longer valid because the whole system is initially not in a pure state. Then the nonequilibriun dynamics of open systems arises, and the master equation approach becomes fundamental. We derived, for the first time, the exact master equation for the FA Hamiltonian, for both fermion systems [9] and boson systems [10]. In our Letter [2], within our exact master equation formalism, we show that localized modes (dressed bound states) lead to dissipationless nonequilibrium dynamics, a general long-lived non-Markovian behavior for open systems. Thus, the system cannot reach equilibrium with its environment (see the detailed proof given in Ref. [11]). This agrees with what Anderson pointed out in his seminal paper [3]: that localization does not allow the system to approach equilibrium.

In Ref. [2], we consider three very different examples to illustrate the generality of the non-Markovian dynamics. One of the localized-mode energies, given in the first example in Ref. [2], is negative but finite (close to zero, see the 2nd inset in Fig. 1 of Ref. [1]) in the strong-coupling regime. The Comment [1] argues that this negative energy could cause the total Hamiltonian to be unbounded (if the total particle number goes to infinity). We have shown rigorously that because the total Hamiltonian conserves the total particle number, the system has a well-defined ground state in each fixed-particle-number state space [12]. Also, within our exact master equation formalism, the total energy is always positive definite [12]. The unboundedness of the total Hamiltonian is irrelevant to the nonequilibrium dynamics investigated in Ref. [2], and the Comment [1] is incorrect when stating that this unboundedness casts serious doubts on the usefulness of the approach.

The authors of the Comment [1] use the Caldeira-Leggett (CL) model [13] to argue that any residual counterrotating terms will cause the system to tend towards a state with infinite negative energy (thermodynamically unstable). The counterrotating terms in the CL model originate from $H_{\rm int} = \sum_k c_k x q_k = \sum_k c'_k (a^\dagger b_k + a b_k^\dagger + a^\dagger b_k^\dagger + a b_k)$ in which the last two terms, i.e., the counterrotating terms, correspond to the processes of generating or annihilating two quanta of energy out of the blue, from nothing. One should be aware that the CL model is a semiempirical one; its physical motivation is to derive the classical dissipation motion of a Brownian particle from quantum mechanics. This is valid in a high-temperature environment in the weak-coupling limit, as specified in Ref. [13], where it is well known that the counterrotating terms have no physical contribution. In the strong-coupling regime, if adding the counterrotating terms would cause the system to tend towards the ground state with infinite negative energy, as the Comment [1] suggests, then according to renormalization theory, the counterrotating terms must contain unphysical features and should be properly subtracted away from the Hamiltonian, similar to the subtraction introduced by Caldeira and Leggett themselves to ensure the potential stability in the original CL Hamiltonian [13]. This is a problem in the CL Hamiltonian, and it has nothing to do with the FA Hamiltonian.

In summary, the physical relevance of the FA Hamiltonian used in our Letter [2] is well established in the literature. The unboundedness of this Hamiltonian questioned in Ref. [1] is irrelevant to the nonequilibrium dynamics investigated in Ref. [2]. At the same time, the physical relevance of the CL Hamiltonian used in Ref. [1] for a harmonic oscillator should always be examined when one goes beyond the physical conditions spelled out in Ref. [13].

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