IL NUOVO CIMENTO

rivista internazionale di fisica

fondata a Pisa nel 1855 da C. MATTEUCCI e R. PIRIA
dal 1897 Organo della Società Italiana di Fisica

pubblicata
sotto gli auspici del Consiglio Nazionale delle Ricerche

Direzione e Redazione Via degli Andalò, 2 40124 Bologna

Loop-Space Quantum Formulation of Free Electromagnetism.

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(ricevuto il 28 Settembre 1983)

PACS. 03.65. – Quantum theory; quantum mechanics.

Summary. – A procedure for direct quantization of free electromagnetism in the loop-space is proposed. Explicit solutions for the loop-dependent vacuum and the Wilson loop-average are given. It is shown that elementary lines of magnetic field appear as extremals in the vacuum state as a result of the regularization procedure.

The relevance of path-dependence to gauge theories was first recognized by MANDELSTAM [1] in his search of a gauge-invariant quantization procedure for the electrodynamic interaction. The description by WU and YANG [2,3] in terms of the non-integrable phase factor was also intended to understand gauge theories from a geometric and gauge-invariant point of view. A direct approach to this geometric view follows from the consideration of the group of loops [4]. It may be shown that the kinematics of any Yang-Mills theory can be obtained from simple geometric constructions involving loops and open paths without reference to any specific gauge group [5]. The attractive proposition by POLYAKOV [6] to consider the Yang-Mills field as a chiral field in the loop-space is a good example of the possibilities offered by the group of loops also at the dynamical level. There is at present considerable interest in nonperturbative approaches to gauge theories based on the language of loops. MAKEENKO and MIGDAL [7,8]

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(1) S. MANDELSTAM: *Ann. Phys.*, **19**, 1 (1962).
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have proposed a loop-dependent equation for Wilson's loop average and established the correspondence with the planar graphs of the theory in the $N \to \infty$ limit. A different method of quantization has been suggested by 'T HOOFT [9,10] and MANDELSTAM [11] in terms of operators creating lines of electric flux and complementary operators associated with vortices of magnetic flux. The geometry of the loop-space is then a most promising framework to surpass the perturbative level in classical and quantum gauge theories.

In this paper a complete gauge-invariant quantization program is carried out in the Abelian case. Starting from an appropriate algebra of canonical equal-time Poisson brackets involving loop-dependent variables one directly quantizes in the loop-space. The relevant equations for the basic states are proposed and explicit solutions are found. Several questions concerning regularization and measure are also discussed. A similar program was also considered in ref. [12], but there the whole method was based on the knowledge of the exact propagator of the theory. This has been avoided with the present procedure which is, therefore, generalizable to the non-Abelian case [13].

Let us then introduce classical variables given by the Wilson loop

$$w(C) = \exp\left[ie\oint_C \mathrm{d}y^i A_i(y)\right] \tag{1}$$

and its temporal loop derivatives

$$w_i(x, C) = -\frac{i}{e}\Delta_{0i}(x)w(C) = f_{0i}(x)w(C)\,. \tag{2}$$

Where only spatial loops entirely contained in $t = \text{const}$ are considered.

From the conventional equal-time canonical Poisson brackets of the Lagrangian formulation of free electromagnetism it is straightforward to obtain the algebra

$$\begin{cases} \{w(C), w(C')\}_{\mathrm{PB}} = 0\,, \\ \{w(C), w_i(x, C')\}_{\mathrm{PB}} = -iew(C)w(C')\,\delta_i(x-C)\,, \\ \{w_i(x, C), w_j(x', C')\}_{\mathrm{PB}} = ie\delta_i(x-C')w(C)w_j(x', C') - ie\delta_j(x'-C)w(C')w_i(x, C)\,, \end{cases} \tag{3}$$

where the distributional tangent of the loop is defined by

$$\delta_i(x-C) = \oint_C \mathrm{d}y_i\,\delta^3(y-x)\,. \tag{4}$$

Other gauge-invariant PB are

$$\{f_{0i}(x), w(C)\}_{\mathrm{PB}} = iew(C)\,\delta_i(x-C)\,, \tag{5}$$

$$\{E, w(C)\}_{\mathrm{PB}} = ie\oint_C \mathrm{d}y_i w_i(y, C)\,, \tag{6}$$

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where E is the electric part of the Hamiltonian

$$E = \tfrac{1}{2}\int \mathrm{d}^3x f_{0i}(x) f_{0i}(x)\,. \tag{7}$$

One may then propose from (3) a gauge-invariant quantization procedure in the loop-space replacing classical variables by operators and PB by commutators. Moreover, a simple geometric realization of (3) may be obtained in a Hilbert space spanned by a « co-ordinate » basis of kets $|C\rangle$. Here C refers to Abelian loops entirely contained in $t = \text{const}$. The operator $W(C)$ is then defined as

$$W(C)|C'\rangle = |C + C'\rangle\,, \tag{8}$$

where one has written «+» for the operation in the group of loops to emphasize its Abelian character in the present case. Clearly $W(C)$ and $W(C')$ commute and the constraint

$$W(C + C') = W(C)\,W(C') \tag{9}$$

is satisfied.

To realize the electric-field operator in this space we choose

$$F_{0i}(x)|0\rangle = 0 \tag{10}$$

and then it follows from (5)

$$F_{0i}(x)|C\rangle = -e\delta_i(x-C)|C\rangle\,, \tag{11}$$

which shows that $|C\rangle$ are eigenstates of $F_{0i}(x)$ the corresponding eigenvalues being elementary lines of electric flux. The choice (10) leading to (11) ensures the kinematical constraint

$$\partial_i F_{0i}(x) = 0\,. \tag{12}$$

The electric part of the Hamiltonian acts in this space by

$$E_{op}|C\rangle = \frac{e^2}{2}\oint_C \mathrm{d}y_i \oint_C \mathrm{d}y'_i\,\delta^3(y-y')|C\rangle\,. \tag{13}$$

It is also immediate to see that the operator $W_i(x, C)$ must be realized by the symmetric expression

$$W_i(x, C) = \tfrac{1}{2}W(C)F_{0i}(x) + \tfrac{1}{2}F_{0i}(x)\,W(C) \tag{14}$$

in order to be compatible with (5).

The magnetic-field operator is defined by

$$F_{ij}(x) = -\frac{i}{e}\Delta_{ij}(x)\,W(C)\Big|_{C=0} \tag{15}$$

and according to (9) is then the generator for infinitesimal translations in the loop-space. Accordingly its eigenstates

$$F_{ij}(x)|A\rangle = f_{ij}(x)|A\rangle \tag{16}$$

must conform the basis dual to $|C\rangle$. (16) may be written as

$$\langle C|F_{ij}(x)|A\rangle = -\frac{i}{e}\,\Delta_{ij}(x)\langle C|A\rangle = f_{ij}(x)\langle C|A\rangle \tag{17}$$

its general solution being

$$\langle C|A\rangle = \exp\left[ie\oint_C \mathrm{d}y^i A_i(y)\right], \qquad f_{ij}(x) = A_{j,i}(x) - A_{i,j}(x)\,, \tag{18}$$

which are then the plane waves of the formulation.

Let us now consider the Schrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle\,, \tag{19}$$

which according to (13) and (15) may be written in co-ordinate representation as

$$\left\{-\frac{1}{4e^2}\int \mathrm{d}^3x\,\Delta_{ij}(x)\,\Delta_{ij}(x) + \frac{e^2}{2}\oint_C \mathrm{d}y_i \oint_C \mathrm{d}y_i'\,\delta^3(y-y')\right\}\Psi(C) = E\Psi(C)\,, \tag{20}$$

which admits as its vacuum solution

$$\Psi_0(C) = \exp\left[-\frac{e^2}{2}\oint_C \mathrm{d}y_i \oint_C \mathrm{d}y_i' D_1(y-y')\right], \tag{21}$$

where

$$D_1(y-y') = \frac{1}{(2\pi)^3}\int \mathrm{d}^3q\,\frac{1}{|q|}\exp\left[-iq\,(y-y')\right] \tag{22}$$

is the spatial restriction of the symmetric propagator of free electromagnetism.

In the Abelian case there is a one-to-one correspondence between loops and tangents (5). Therefore, integration in the loop-space may be performed as functional integration with the tangent (5) as variable. We choose to do it in the holomorphic representation by introducing

$$C_i(q) = \frac{1}{(2\pi)^{\frac{3}{2}}}\,\frac{e}{\sqrt{q}}\oint_C \mathrm{d}y_i \exp\left[iq\,y\right]. \tag{23}$$

Then for instance

$$\int \mathrm{D}C\Psi_0^*(C)\,\Psi_0(C) = \int \frac{\mathrm{D}C_i\mathrm{D}C_i^*}{2\pi i}\exp\left[-\int \mathrm{d}^3q C_i(q)\,C_i^*(q) = 1\right]. \tag{24}$$

Having established the measure, one may now compute the expectation values of the formulation. For instance the Wilson loop average for spatial loops may be obtained by performing a convolution in the loop-space:

$$\langle W(C)\rangle = \langle \Psi_0|W(C)|\Psi_0\rangle = \int \mathrm{D}C'\langle \Psi_0|W(C)|C'\rangle\langle C'|\Psi_0\rangle = \int \mathrm{D}C'\,\Psi_0(C')\,\Psi_0^*(C'+C)\,. \tag{25}$$

The integration is readily performed using (24) and one obtains

$$\langle W(C)\rangle = \exp\left[-\tfrac{1}{4}\int \mathrm{d}^3q\,C_i(q)\,C_i^*(q)\right], \tag{26}$$

which differs from (21) by an extra factor of $\frac{1}{2}$ in the exponent. This reflects the change from the symmetric propagator in Ψ_0 to the Feynman propagator in $\langle W(C)\rangle$ when evaluated at equal-time arguments. Wilson's loop average for arbitrary loops in R^4 may be obtained by solving the Schrödinger equation with an external current term in the Hamiltonian.

One may also consider the Schrödinger equation in the dual representation associated with the basis $|A\rangle$:

$$\left\{-\frac{1}{2}\int \mathrm{d}^3x\,\frac{\delta}{\delta A_i(x)}\,\frac{\delta}{\delta A_i(x)} + \frac{1}{4}f_{ij}(x)f_{ij}(x)\right]\Psi\{A\} = E\Psi\{A\}\,, \tag{27}$$

which has the vacuum solution

$$\Psi_0\{A\} = \exp\left[-\tfrac{1}{4}\int \mathrm{d}^3y\,\mathrm{d}^3y' f_{ij}(y)\,D_1(y-y')\,f_{ij}(y')\right] = \exp\left[-\tfrac{1}{2}\int \mathrm{d}^3q\,q a_i^{\mathrm{T}}(q)\,a_i^{*\mathrm{T}}(q)\right], \tag{28}$$

where $a_i^{\mathrm{T}}(q)$ is the Fourier transform of the transverse potential:

$$q_i a_i^{\mathrm{T}}(q) = 0\,. \tag{29}$$

Let us recall that the functional in (21) is divergent ([12]) due to the self-energy term (13) in the Hamiltonian. A suitable regularization procedure consists in modifying the propagator in (21) to

$$\bar{D}_1(y-y') = \frac{1}{(2\pi)^3}\int \mathrm{d}^3q\,\frac{r(q\varepsilon)}{q}\exp\left[iq\,(y-y')\right], \qquad r(0) = 1\,. \tag{30}$$

It may be shown that the necessary and sufficient condition for the removal of the self-energy singularity in $\Psi_0(C)$ is

$$\int_0^\infty \mathrm{d}x\,r(x) = 0\,. \tag{31}$$

Possible choices are

$$r(q\varepsilon) = (1-q\varepsilon)\exp\left[-q\varepsilon\right], \quad r(q\varepsilon) = (1-\tfrac{1}{2}q\varepsilon)\,\theta(1-q\varepsilon), \quad \text{etc.} \tag{32}$$

It is apparent that $r(x)$ cannot be positive definite and develops negative values at high frequencies. In ref. ([12]) the properties of the regularized $\Psi_0(C)$ were extensively discussed. Here we shall consider the dual vacuum (28). If one replaces D_1 by $\bar{D}_1$ in (28), the exponent is no longer negative definite and nontrivial extremals may be obtained. To see this it is convenient to choose some fixed reference loop C, define $C_i(q)$ as in (23) and expand the transverse potential in the basis

$$a_i^{\mathrm{T}}(q) = A(q)\,q_i + B(q)\,C_i(q) + C(q)\,\varepsilon_{ijk}q_j C_k(q)\,. \tag{33}$$

Then extremals of the exponent in (28) may be found with

$$B(q)\,\alpha\,\delta\left(q-\frac{1}{\varepsilon}\right), \tag{34}$$

while $A(q)$ is 0 by the transversality condition (29) and $C(q)$ is depressed relative to $B(q)$ by a factor of ε. In the limit $\varepsilon \to 0$, this extremal corresponds in direct space to

$$A_i(x)\,\alpha\oint \mathrm{d}y_i\,\delta^3(y-x) \tag{35}$$

and elementary lines of magnetic flux appear in the dual vacuum as a by-product of the regularization procedure.

LETTERE AL NUOVO CIMENTO VOL. 38, N. 15 10 Dicembre 1983

Nonleptonic Weak Decays of Bottom Mesons with Soft-Pion Emission.

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(ricevuto il 7 Settembre 1983)

PACS. 13.90. – Other topics in specific reactions and phenomenology of elementary particles.

Summary. – Inclusive soft-meson theorems are derived for the nonleptonic decay of the bottom meson within the framework of the Kobayashi-Maskawa six-quark model.

The discovery of the narrow upsilon states Υ, Υ' and Υ'' (1) and its confirmation in e^+e^- annihilation at the DORIS (2) storage ring at DESY strongly suggest that a new quark called bottom, b, with charge $Q = -\frac{1}{3}$ exists. A fourth broad upsilon state Υ''' has been discovered at the Cornell Electron Storage Ring and the broad width of this new resonance suggests that it decays strongly into mesons containing a b-quark which subsequently undergoes weak decay. The measurement of the charged multiplicity on and off the Υ'''-resonance and charged and neutral kaon content of Υ''' decays give further information on the properties of B-mesons (3).

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