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# Fluctuations in the Josephson–pancake combined vortex lattice

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**Abstract**

We study vortex fluctuations in strongly anisotropic layered superconductors placed in tilted magnetic fields where there are two alignments of vortices: pancake vortices (PV's) oriented along the *c*-axis, and Josephson vortices (JV's) aligned along the *ab*-plane. For low enough out-of-plane magnetic fields $H_c$, the JV sublattice pins some PV's, confining several of their degrees of freedom. This can result in the suppression of PV thermal fluctuations and a weak increase in the out-of-plane vortex lattice melting field $B_z^{\mathrm{melt}}$ with increasing in-plane magnetic field $H_{ab}$.


*Keywords:* Layered superconductor; Tilted magnetic fields; Vortex lattice melting; Thermal fluctuations of the vortex chain-lattice

Direct visualization [1–3] as well as magnetic [4,5] and transport [6,7] measurements show that oblique magnetic fields penetrate a strongly anisotropic layered superconductor as two perpendicular alignments of vortices. The pancake vortex (PV) lattice, oriented originally along the *c*-axis, is locally bent by the in-plane current generated by Josephson vortex (JV), which are confined in-between $CuO_2$ layers. Such an interaction between both vortex subsystems somewhat links the degrees of freedom of PV's and JV's, giving a quite unusual shape for the vortex lattice melting transition line on the $H_c$–$H_{ab}$ phase diagram [4–7]. For instance, the experimentally observed linear decay of the out-of-plane melting field $H_c^{\mathrm{melt}}$ with the in-plane magnetic field $H_{ab}$ [4–7] was thermodynamically interpreted [8] via the linear increase of the free energy of the crossing vortex lattices with $H_{ab}$. The fluctuation mechanism of this linear decay is probably related to the softening of the elastic constants of the PV sublattice by the *c*-axis component of the JV current. With further increasing the in-plane magnetic field $H_{ab}$, the out-of-plane melting field component shows a plateau-like dependence or even a slow increase [5–7]. The origin of this dependence was attributed to the trapping of some PV's by JVs [7,9]. In this paper we demonstrate how the sticking of PV's to JV's influences the PV thermal fluctuations and the vortex lattice melting transition.

Here we consider weak, $H_c < \Phi_0/(\gamma^2 s^2)$, out-of-plane magnetic fields $H_c$, when any *z*–*x* wall formed by JV's can effectively interact with only one PV row placed directly on the wall (see Fig. 1) [1] In this case, the strongly pinned PV chains coexist with the very weakly deformed PV lattice settled between JV's [8]. However, the pinned PV rows (chains) restrict the degrees of

[1] For high out-of-plane magnetic fields, $H_c > \Phi_0/(\gamma^2 s^2)$, the pinning of PV's by JV's is weak. In that case, the elasticity of the PV lattice is suppressed by interactions with the out-of-plane currents generated by JV's, resulting in increasing PV fluctuations and a linear decay of $B_z^{\mathrm{melt}}$ with increasing $H_{ab}$ [10].

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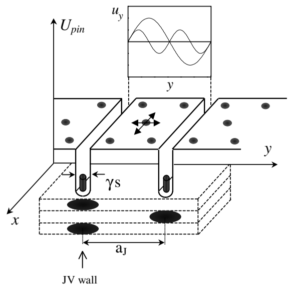


Fig. 1. Sketch of the mutual JV–PV sticking potential, trapping PV chains in the potential wells produced by the JV's. The very weakly deformed PV lattice confined between wells has less degrees of freedom: the $y$-displacement waves with nodes at the potential wells (examples plotted in the inset) can only propagate on the PV lattice.

freedom of the remaining PV's, fixing their $y$-displacement as $u_y(na_J) = 0$, with $a_J$ equal to the distance between nearest JV walls, and $n$ an integer. Thus, the "free" $x$-displacement $u_x$ can be expanded via the usual Fourier integral as $u_x = \int_{BZ} u_x(\mathbf{k}) \exp(i\mathbf{kr})\, d^3\mathbf{k}/(2\pi)^3$, where the integration in $\mathbf{k}$-space is done over the first Brillouin zone (BZ): $k_x^2 + k_y^2 < 4\pi B_z/\Phi_0$ ($B_z$ is the magnetic induction along the $c$-axis), $|k_z| < k_z^{max} = \min(1/\xi_c, 1/s)$ with the $c$-axis coherence length $\xi_c$ and distance between $CuO_2$ planes $s$. However, the $y$-displacement $u_y$ of PV's confined between two neighboring PV chains (Fig. 1) can be expressed as $u_y = \int((d\mathbf{k}_{xz}\, dq_y)/(4\pi^3))u_y(\mathbf{k}_{xz}, q_y)\sin(q_y y)\exp(i\mathbf{k}_{xz}\mathbf{r}_{xz})$, with $\mathbf{k}_{xz} = k_x\mathbf{e}_x + k_z\mathbf{e}_z$, $\mathbf{r}_{xz} = x\mathbf{e}_x + z\mathbf{e}_z$, and unit vectors $\mathbf{e}_x$ and $\mathbf{e}_z$ along $x$ and $z$ axes. The wave vectors in the last Fourier expansion are restricted as $\sqrt{B_x/(\gamma\Phi_0)} \sim 1/a_J < q_y < 1/a_p$, $q_y^2 + k_x^2 < 4\pi B_z/\Phi_0$, and $|k_z| < k_z^{max}$. Here, $B_x$ is the in-plane magnetic induction. The $y$-component $q_y$ of the PV wave vector is restricted from below because only waves with nodes on the neighboring JV walls can propagate in the PV lattice.

If, in addition, the $c$-axis magnetic field satisfies the inequality $H_c < \Phi_0/\lambda_{ab}^2$, the main contribution to the elastic free energy is related to the electromagnetic tilt rigidity of PV's:

$$F = \frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\overline{U}_{44}u_x(\mathbf{k})u_x(-\mathbf{k}) + \frac{1}{2}\int \frac{d\mathbf{k}_{xz}\, dq_y}{4\pi^3}\overline{U}_{44}u_y(\mathbf{k}_{xz}, q_y)u_y(-\mathbf{k}_{xz}, q_y), \qquad (1)$$

where the elastic stiffness is the $\mathbf{k}$-independent constant $\overline{U}_{44} = \Phi_0 \ln(1 + 4\lambda_{ab}^2/c_L^2 a_p^2)/(32\pi^2\lambda_{ab}^4)$ as long as $|k_z| < k_z^* = \min(\pi/s, \gamma\sqrt{\ln(1 + 4\lambda_{ab}^2/c_L^2 a_0^2)}/\lambda_{ab})$. By using Eq. (1), the mean square displacements of PV's from their equilibrium positions associated with thermal fluctuations can be estimated as $\langle u^2 \rangle = (T/\overline{U}_{44})(N_x + N_y)$, where $N_x$ and $N_y$ are the number of the degrees of freedom attributed to PV displacements along the $x$ and $y$ axes: $N_x = \int_{BZ} d^3\mathbf{k}/(2\pi^3) \approx k_z^* B_z/(\pi\Phi_0)$, while $N_y = \int d^2\mathbf{k}_{xz}dq_y/(4\pi^3) \sim (k_z^*/\pi)\sqrt{B_z/\Phi_0}[\sqrt{B_z/\Phi_0} - \sqrt{B_x/(\gamma\Phi_0)}]$.

In order to understand qualitatively how the freezing of some degrees of freedom of PV lattice influences the vortex lattice melting transition, we can use the Lindemann criterion $\langle u^2 \rangle = c_L^2 a_p^2 = c_L^2 \Phi_0/B_z$ with the interpancake distance $a_p$ and the Lindemann number $c_L$. The last equation can be approximately rewritten in the form $B_z^{melt} - \frac{1}{2}\sqrt{B_z^{melt}B_x/\gamma} = B_0$, with the melting field $B_0$ if $\mathbf{H}$ is applied along the $c$-axis: $B_0 = c_L^2\Phi_0^3 \ln(1 + 4\lambda_{ab}^2/(c_L^2 a_p^2))/(64\pi T\lambda_{ab}^2 k_z^*)$. If an applied in-plane magnetic field satisfies the inequality $H_{ab} \ll 16\gamma B_0$, the dependence of $B_z^{melt}$ on the in-plane field $B_x$ is expressed as

$$B_z^{melt} \approx B_0 + \frac{1}{2}\sqrt{\frac{B_x B_0}{\gamma}}. \qquad (2)$$

The shift of the vortex lattice melting transition to higher out-of-plane magnetic fields by applying an in-plane field is related to the growth of the number of the mutual JV–PV pinning centers, resulting in the increase of the fraction of the frozen degrees of freedom of PV's. Nevertheless, an almost constant $B_z^{melt}$ versus $B_x$ is usually observed [5,7]. A possible reason for this discrepancy is that the transition line separating the lattice-chain state from the phase with the unpinned PV and JV sublattices does not depend on $H_{ab}$ [9]. Thus, the growth of $B_z^{melt}$ may be suppressed by defrosting the PV degrees of freedom, giving an almost constant dependence $B_z^{melt}(B_x)$. The increase of $B_z^{melt}$ with increasing $H_{ab}$ observed in [4] could be related to the commensurability of the Josephson vortex lattice and the pancake vortex lattice; this keeps the PV's trapped by JV's with increasing $H_c$ and $H_{ab}$.

In conclusion, the fluctuations of the pancake vortices in the mixed chain-lattice phase of the PV sublattice in the crossing vortex lattice structure are studied. We show that the linear decay of the out-of-plane vortex lattice melting field with increasing in-plane magnetic field is stopped due to the freezing of the PV degrees of freedom.

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