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Tunable photonic crystal for THz radiation in layered superconductors: Strong magnetic-field dependence of the transmission coefficient

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Abstract

Josephson plasma waves are scattered by the Josephson vortex lattice. This scattering results in a strong dependence, on the in-plane magnetic-field H_{ab}, of the reflection and transmission of THz radiation propagating in layered superconductors. In particular, a tunable band-gap structure (THz photonic crystal) occurs in such a medium. These effects can be used, by varying H_{ab}, for the selective frequency-filtering of THz radiation.

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1. Introduction

Strongly-anisotropic high-T_c superconductors (HTS), as well as artificial multilayered heterostructures, are commonly considered as stacks of Josephson junctions (SJJ). When a magnetic-field H_{ab} is applied parallel to the layers, Josephson vortices (JVs) penetrate the sample and form a triangular lattice. The pinning of JVs is weak and the JV lattice is near perfect.

It is known that the Josephson plasma frequency, ω_J, of HTS layered structures is in the THz-range, which is of particular interest for applications [1–4]. The radiation produced by moving JVs, as well as Josephson plasma waves, were analyzed by many authors [1,3–11]. The influence of a JV lattice on the propagation of THz electromagnetic waves (EMW) was studied in [10]. Here we extend the recent analysis [10] of forbidden gaps in the frequency spectrum (THz photonic crystal) and the magnetic-field dependence of the transmission, T, and reflection, $R = 1 - T$, coefficients of EMW in layered superconductors.

2. Model

We consider SJJ with layers in the xz plane and the y-axis across the layers. The in-plane field H_{ab} and, thus, the JVs are parallel to the z-axis. The distance d_x between JVs in the lattice along the x-direction is much larger than along the y-direction, d_y, and $d_x/d_y = \gamma$ ($\gamma = 300$–600 for HTS). The gauge-invariant phase difference φ_n in SJJ can be described by a set of coupled sine-Gordon equations [12]. We consider the EMW $\boldsymbol{H}(x, y, t) = \hat{\mathbf{z}} H_0(x) \exp(\mathrm{i}qy - \mathrm{i}\omega t)$, propagating through the sample, where $\hat{\mathbf{z}}$ is the unit vector along z. If we neglect the electric field component along superconducting layers, then, the y

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component of the electric field obeys the Maxwell equation $-E_y'(x) = \mathrm{i}\omega H/c$. We assume that the amplitude H_0 is small compared to H_{ab} and the gauge-invariant phase can be obtained perturbatively as $\varphi_n = \varphi_n^0 + \varphi_n^1$, where φ_n^0 corresponds to the steady JV lattice and $|\varphi_n^0| \gg |\varphi_n^1|$. For moderate H_{ab}, φ_n^0 can be approximated as a sum, $\varphi_n^0 = \sum_m \varphi^0(x - x_{mn})$, of solitons [13] $\varphi_0 = \pi + 2\tan^{-1}(x/l_0)$, where $2l_0 = \gamma s$. Here x_{nm} is the position of the mth JV in the nth layer and s is the interlayer distance.

Substituting $\varphi^1(x, y, t) = \psi(x)\exp(\mathrm{i}qy - \mathrm{i}\omega t)$ into the set of corresponding sine-Gordon equations and averaging over y for $|q| < \pi/s$, we derive in the linear approximation

$$\psi''(\eta) - \kappa_0^2(q)[\tilde{\omega}_\mathrm{J}^2(\eta) - \tilde{\omega}^2]\psi(\eta) = 0, \tag{1}$$

where the following dimensionless variables were introduced: $\eta = x/\gamma s$, $\tilde{\omega} = \omega/\omega_\mathrm{J}$, $\tilde{\omega}_\mathrm{J}(\eta) = \omega_\mathrm{J}(\eta)/\omega_\mathrm{J}$,

$$h_{ab} = \frac{\gamma s^2 H_{ab}}{2\phi_0}, \quad \kappa_0^2(q) = \left(\frac{s}{\lambda_{ab}}\right)^2 (1 + q^2\lambda_{ab}^2), \tag{2}$$

Here λ_{ab} is the London penetration depth across the layers, $\omega_\mathrm{J} = \sqrt{8\pi e s J_\mathrm{c}/\hbar\varepsilon}$ is the Josephson plasma frequency, ε is the dielectric constant, and J_c is the critical current density across the SJJ. We also neglect the relaxation term in Eq. (1). The function

$$\tilde{\omega}_\mathrm{J}^2(\eta) = \tilde{\omega}_\mathrm{J}^2(x/\gamma s) = \left\langle \sum_m \varphi^0(x - x_{mn}) \right\rangle_n \tag{3}$$

has a period d_x along the x direction, where $\langle \cdots \rangle_n$ denotes an average over the layers. This averaging is valid for $qs < \pi\sqrt{h_{ab}}$ or at any q for $h_{ab} = 0$. The physical meaning of the spatial modulation in $\tilde{\omega}_\mathrm{J}(\eta)$ is that the effective critical current is modulated due to the current suppression near the JV cores. We can approximate the dependence $\tilde{\omega}_\mathrm{J}^2(\eta)$ by a stepwise function:

$$\tilde{\omega}_\mathrm{J}^2(\eta) = 1 - \pi\sqrt{h_{ab}} \sum_m F\left(\eta - \frac{m}{\sqrt{h_{ab}}}\right), \tag{4}$$

where $F(\eta) = 1$ if $|\eta| < 1$, and $F = 0$ if $|\eta| > 1$, and we use the relation $2\phi_0/(d_x d_y) = H_{ab}$. The approximate formula (4) is improved as compared with that in Ref. [10]. The relations between φ and the electromagnetic-field in the geometry considered can be presented [14] as $H = -\phi_0\varphi'(x)/2\pi s$, and $E_y = \mathrm{i}\omega\phi_0\varphi(x)/2\pi sc$. The continuity conditions apply to $\varphi(\eta)$ and $\varphi'(\eta)$ in the sample.

3. Tunable photonic crystal

Forbidden zones in the $\omega(k)$ dependence (photonic crystal), can occur when the EMW propagates through a periodically modulated structures [15]. The dimensionless period of the JV lattice is $1/\sqrt{h_{ab}}$. We obtain the solution of Eq. (1) in the form of the Bloch wave $\psi(x) = u(x, k)\exp(\mathrm{i}kx)$, where $u(x, k)$ is a periodic function with period $1/\sqrt{h_{ab}}$ and the wave vector k is in the first Brillouin zone, $-\pi\sqrt{h_{ab}} < k < \pi\sqrt{h_{ab}}$. The solution of Eq. (1), within one elementary cell j, is a sum of exponential terms multiplied by constants C_j. Using the continuity of ψ and ψ' and the periodicity of these functions, we obtain a set of homogeneous linear equations for C_j. The non-trivial solution of these equations exists only if the determinant of the set of equations is zero. Then, we find the dispersion equation for $\omega(k)$ in the form

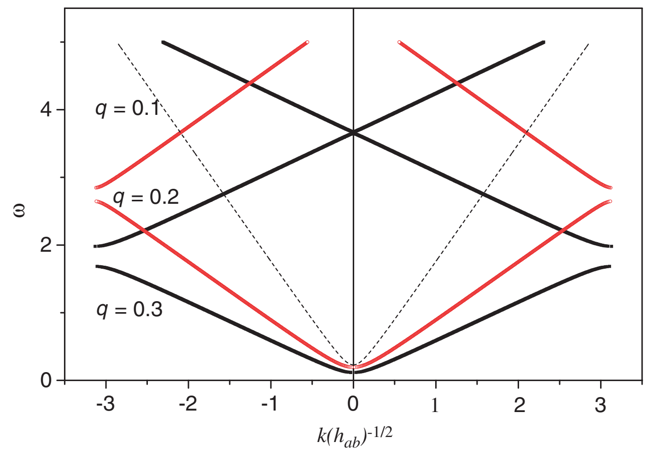

Fig. 1. Band-gap structure: $\tilde{\omega}$ versus $k(h_{ab})^{-1/2}$ at $h_{ab} = 0.3$ and for $qs = 0.3\pi$, $qs = 0.2\pi$, and $qs = 0.1\pi$. The frequency gap (forbidden frequency range) between the first and the second zone is $\Delta\tilde{\omega} \approx 0.2$. The gap diminishes for smaller q ($q = y$-axis wave-vector). Here, we use $s = 15$ Å, $\lambda_{ab} = 2000$ Å, $\gamma = 600$.

$$\cos(\kappa_1 b)\cos(\kappa_2) - \frac{\kappa_1^2 + \kappa_2^2}{2\kappa_1\kappa_2}\sin(\kappa_1 b)\sin(\kappa_2) = \cos[k(b+1)],$$

$$b = 1/\sqrt{h_{ab}} - 1, \tag{5}$$

$$\kappa_1 = \kappa_0(\tilde{\omega}^2 - 1)^{1/2},$$

$$\kappa_2 = \kappa_0(\tilde{\omega}^2 + \pi\sqrt{h_{ab}} - 1)^{1/2}. \tag{6}$$

The dependence $\tilde{\omega}(k)$ is shown in Fig. 1. The forbidden gap $\Delta\tilde{\omega}$ is suppressed when decreasing H_{ab} or the transverse wave vector q.

4. Strong magnetic-field dependence of the THz transmission coefficient

Now we consider a wave with $\omega > \omega_\mathrm{J}$ propagating from the vacuum to the sample edge across the ab planes. The value of qs is small in the THz-range since in vacuum $k^2 + q^2 = \omega^2/c^2$, while s is in the nanometer range. The solution of Eq. (1), for the jth cell of the magnetic structure, can be expressed as a vector $\psi_\alpha^j = \{C_{1\alpha}^j \exp(\mathrm{i}\kappa_\alpha x); C_{2\alpha}^j \exp(-\mathrm{i}\kappa_\alpha x)\}$, where $\alpha = 1, 2$. We impose the continuity of ψ and ψ' at any discontinuity of the function $\tilde{\omega}^2(\eta)$. As a result, we obtain a set of linear equations relating ψ_α^{j-1} and ψ_α^j. The solution of these equations can be presented as $\psi_\alpha^j = \widehat{L}\psi_\alpha^{j-1}$, where $\widehat{L}$ is a 2×2 matrix. We use a linear transformation $\widehat{G}$ that diagonalizes $\widehat{L}$. By applying N times such a procedure, we find the linear transformation that propagates the solution from the 0th to Nth elementary cell. This is also known as a transfer-matrix approach. Imposing the continuity of both H and E_y at the sample

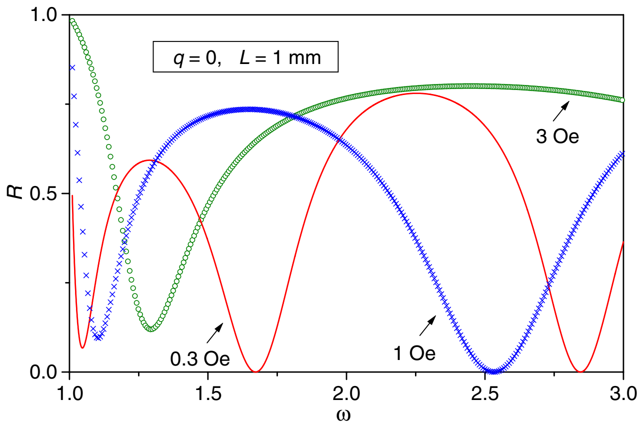

Fig. 2. The reflection coefficient R versus $\tilde{\omega}$ for an EMW propagating from the vacuum at $q = 0$, for $l = 1000\ \gamma s \approx 1$ mm, $H_{ab} = 3$ Oe (open circles), $H_{ab} = 1$ Oe (crosses), $H_{ab} = 0.3$ Oe (solid line). Here, we use $s = 15$ Å, $\lambda_{ab} = 2000$ Å, $\gamma = 600$.

surface, we find the expression for the amplitude r of the reflected wave

$$
\begin{aligned}
r &= \frac{1 + Z(\tilde{\omega})D(\tilde{\omega})\exp(-2\mathrm{i}\kappa_1 b)}{Z(\tilde{\omega}) + D(\tilde{\omega})\exp(-2\mathrm{i}\kappa_1 b)},\\
D &= \frac{\beta_1(Z+\beta_2)M_2^N - (1+\beta_1 Z)M_1^N}{(Z+\beta_2)M_2^N - \beta_2(1+\beta_1 Z)M_1^N}, \qquad (7)\\
Z &= (\kappa_1 - \tilde{\omega}g)/(\kappa_1 + \tilde{\omega}g), \;\; g = s/(\sqrt{\varepsilon}\lambda_{ab}\sqrt{1 - q^2c^2/\omega^2}),\\
M_{1,2} &= \left[\cos\kappa_2(b-1) \pm \mathrm{i}\frac{\kappa_1^2+\kappa_2^2}{2\kappa_1\kappa_2}\sin\kappa_2(b-1)\right]\mathrm{e}^{\mp\mathrm{i}(b-1)\kappa_1}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{1,2} &= \pm\frac{M_2 - M_1 + \sqrt{(M_2-M_1)^2 + 4L_1L_2}}{2L_{1,2}} \qquad (8)\\
L_{1,2} &= \pm\mathrm{i}\frac{\kappa_1^2-\kappa_2^2}{2\kappa_1\kappa_2}\sin\kappa_2(b-1)\mathrm{e}^{\pm\mathrm{i}(b-1)\kappa_1}.
\end{aligned}
$$

The frequency dependence of the reflection coefficient $R = |r|^2$ is shown in Fig. 2. The transparency (transmission T) of the crystal increases when increasing the frequency $\tilde{\omega}_J$ and when decreasing either the sample length $l = \gamma s N/\sqrt{h_{ab}}$ or H_{ab}, due to the decrease of the number of scattering layers. The oscillation in the transition and reflection coefficients occurs due to the interference of the scattered and transmitted waves on JVs and sample boundaries. Varying the applied magnetic-field H_{ab} tunes the reflection at a given frequency from 0 to 1.

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