Unified single-photon and single-electron counting statistics: From cavity QED to electron transport

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A key ingredient of cavity QED is the coupling between the discrete energy levels of an atom and photons in a single-mode cavity. The addition of periodic ultrashort laser pulses allows one to use such a system as a source of single photons—a vital ingredient in quantum information and optical computing schemes. Here we analyze and time-adjust the photon-counting statistics of such a single-photon source and show that the photon statistics can be described by a simple transport-like nonequilibrium model. We then show that there is a one-to-one correspondence of this model to that of nonequilibrium transport of electrons through a double quantum dot nanostructure, unifying the fields of photon-counting statistics and electron-transport statistics. This correspondence empowers us to adapt several tools previously used for detecting quantum behavior in electron-transport systems (e.g., super-Poissonian shot noise and an extension of the Leggett-Garg inequality) to single-photon-source experiments.

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I. INTRODUCTION

Cavity QED studies the interaction between a two-level atom and a single-mode cavity (see, e.g., [1–4]). Vacuum Rabi oscillations, the coherent excitation transfer between atoms and cavity photons, can occur if the atom-photon coupling strength g overwhelms both the loss rate (κ) of the cavity photons and the emission rate (γ) into other modes, as shown schematically in Fig. 1. To observe such a quantum oscillation, a velocity-selected atomic beam is passed through an open Fabry-Perot resonator to control the interaction time t_i. The probability $P_e(t_i)$ that the atom remains in the excited state $|e\rangle$ at time t_i can be written [1–3] as $P_e = (1 + \cos 2gt_i)/2$. This coherent coupling can be observed by examining the so-called vacuum Rabi splitting (VRS) in the transmission spectrum of the cavity. Clear evidence of VRS has been demonstrated not only in atomic systems [2–4] but also in semiconductor self-assembled quantum dots [5] and circuit QED [6–8] systems.

In several recent experiments (see, e.g., [1–4]), a cavity-QED system was used as a source of single photons by deterministically exciting the atom via periodic ultrashort laser pulses. Normally, one interprets the total photon statistics from such an experiment as the ensemble average of a single event: The atom is excited at $t = 0$, interacts with the cavity, and eventually, the cavity photon is emitted at some later time. All of the recorded single-photon-detection events are then combined to give the ensemble average of this single situation.

Here we propose a simple alternative method of analyzing the photon-detection events of this kind of experiment [1–4] that we term time-adjusted photon counting. We show that the photon emission spectrum can then be modeled via a Markovian master equation which has a one-to-one correspondence to a well-studied model of a double quantum dot (DQD) in the large-bias, Coulomb-blockade regime. This allows us to reinterpret data from existing (and future) cavity-QED single-photon-source experiments as a continuous transport-like phenomenon, unifying photon and electron statistics. A summary of this correspondence can be found in Table I.

DQDs are artificial atoms in a solid [9–11]. A variety of powerful tools have been developed to study transport through such devices. These tools have revealed unique features like Coulomb blockade [12], the Kondo effect [12], and coherent oscillations [13,14]. Our main result here is that the electron-counting statistics developed for the DQD model (e.g., current, current noise, and higher order cumulants) can be observed in existing photon-counting cavity-QED experiments. We will use the analogy between these two apparently unrelated systems to show that the photon statistics have a nonnegative shot-noise feature, complementary to their sub-Poissonian antibunching statistics, that indicates VRS. Moreover, we calculate the second-order correlation functions and show that these violate an extended form [15] of the Leggett-Garg (LG) inequality [16]. For completeness, we also consider violations of this inequality by the nonadjusted statistics.

II. STANDARD PHOTON COUNTING

A way to produce [1] single photons from a cavity is the following: Ultrashort laser pulses with a given time constant are applied to the atom. The single-photon detector records the arrival time t_n of a photon decaying out of the cavity with respect to each pulse, as shown Fig. 1. A normalized histogram of detection times reveals [1] photon antibunching and oscillations because of the atom-cavity coupling.

Neglecting the emission rate γ into other modes, the normal cavity-QED system [4] can be described by the Markovian master equation (setting $\hbar = 1$ throughout)

$$\dot{\rho} = \mathcal{W}_c[\rho] = -i[H_c,\rho] + L_c[\rho], \tag{1}$$

where

$$L_c[\rho] = \kappa a\rho a^\dagger - \frac{\kappa}{2}[a^\dagger a\rho + \rho a^\dagger a], \tag{2}$$

$$H_c = \nu a^\dagger a + \frac{\omega}{2}\sigma_z + g(\sigma_- a^\dagger + \sigma_+ a). \tag{3}$$

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TABLE I. Comparison between the properties of the cavity-QED system studied here and a double quantum dot.

System	Double quantum dot	Cavity QED
Carrier	Electrons	Photons
Ground state	$\|R\rangle$ = electron in the right dot	$\|g,1\rangle = \|$ground state atom, 1 photon$\rangle$
Excited state	$\|L\rangle$ =electron in the left dot	$\|e,0\rangle = \|$excited atom, 0 photons$\rangle$
Energy difference $\triangle E$	$E_L - E_R$	$\delta/2 \ = \ (\omega-\mu)/2$
Rabi rate	Tunneling amplitude T	Atom-Photon coupling g
Input rate	Tunneling rate $\Gamma_L \to \infty$	Laser pulses with time-adjusted shift
Output rate	Tunneling rate Γ_R	Cavity loss rate κ
Quantum noise signature	Super-Poissonian $F_e(\omega \to 0) > 1$	Non-negative $F_{\rm ph}(\omega \to 0) > 0$
Extended LG inequality	$\|2\langle I(t+\tau)I(t)\rangle - \langle I(t+2\tau)I(t)\rangle\| \leqslant \Gamma_R \langle I(t)\rangle$	$\|2g^{(2)}(t,t+\tau) - g^{(2)}(t,t+2\tau)\| \leqslant \langle a^\dagger(t)a(t)\rangle^{-1}$

Here ω is the atomic level splitting, ν is the cavity frequency, and κ is the cavity loss rate by which we acquire photons. Here $a^\dagger$ (a) denotes the creation (annihilation) operator of a cavity photon. The atomic operators are defined as $\sigma_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_- \equiv |g\rangle\langle e|$, and $\sigma_+ \equiv |e\rangle\langle g|$, where $|e\rangle$ and $|g\rangle$ denote the excited and ground states, respectively. The atomic polarization decay γ can be easily included in this analysis, but for simplicity, we neglect it here. Furthermore, we omit variations in the coupling strength g that can occur between each pulse. This could be an important factor, which can be dealt with by numerically integrating our final result over a Gaussian spread in g or by including an additional dephasing term in the master equation.

III. TIME-ADJUSTED PHOTON COUNTING

Now, rather than collating data in the manner shown in Fig. 1(b), we propose to perform a time-adjusted analysis of the photon data, as shown in Fig. 1(c). Namely, the time between a photon-detection event and the next laser pulse (shown in green in the figure) is eliminated, moving the time of each laser pulse to the time of the previous photon count, and any periods of time with no photon detection are eliminated from the data set. Thus the system can then be viewed as one with instantaneous feedback that maintains one excitation in the combined atom-cavity basis.

A. Effective feedback formalism

To show that the time-adjusted data set is governed by a simple master equation in the single-occupation basis, we employ an effective feedback formalism based on Refs. [17,18]. First, we describe the measurement of a single photon (which has leaked from the cavity and is incident on a photodetector) as

$$\mathcal{K}(dt)_1 = \sqrt{\kappa\, dt}\, a. \tag{4}$$

The complementary operator to this one, which is applied when no photon is detected during the duration dt, is

$$\mathcal{K}(dt)_0 = 1 - \left(iH_c + \frac{\kappa}{2}a^\dagger a\right) dt. \tag{5}$$

Following the reasoning in Refs. [17,18], the nonselective evolution under this measurement is given by

$$\rho(t+dt) = \sum_{\alpha=0,1} \mathcal{K}(dt)_\alpha\, \rho(t)\, \mathcal{K}^\dagger(dt)_\alpha, \tag{6}$$

which is equivalent to the normal master equation [Eqs. (1)–(3)].

As shown in Figs. 1(b) and 1(c), in the time-adjusted frame, if a photon is observed, then the atom is suddenly excited by the ultrashort laser pulse. This is an immediate and instantaneous feedback effect. In other words, we assume that the laser pulse is so fast that the system does not evolve while it is being applied (apart from the resulting excitation of the atom), unlike the more general feedback elucidated in Refs. [17,18]. Such an assumption is already implicit in the analysis of Ref. [1]. In this case, we can describe this effective feedback (which is applied to the system following a photon detection and time adjustment) as

$$\mathcal{O}[\cdots] \ = \ |e\rangle\langle g| \cdots |g\rangle\langle e|; \tag{7}$$

that is, the atom is instantaneously and incoherently projected into its excited state. In general, this is not a trace-preserving evolution as it is not a Liouvillian evolution and, as mentioned, is thus different from the class of feedback mechanisms described in Refs. [17,18]. Alternatively, one could assume a full Liouvillian evolution according to the coherent dynamics of a laser pulse causing π-pulse/Rabi oscillations of the atom from its ground to excited state, via, for example, an operator like $\exp -i\mathcal{Z}$, where $\mathcal{Z} = [\sigma_x,\ldots]$. However, in the limit when this transition is faster than all other dynamics, when the cavity (κ) and atomic (γ) decay terms are faster than the time between laser pulses, and when we are in the single-excitation manifold, it is equivalent to $\mathcal{O}[\cdots]$.

The unnormalized density matrix $\tilde{\rho}_1$ following the detection of a photon at time t, and evolution because of the laser-pulse feedback, is now

$$\tilde{\rho}_1(t+dt) = \kappa\, \mathcal{O}\, a\, \rho(t)\, a^\dagger dt.$$

Our time-adjustment scheme implies that the time delay is far smaller (effectively zero) than the cavity decay time. Then, the (nonselective) evolution is described by

$$\rho(t+dt) = \tilde{\rho}_1(t+dt) + \tilde{\rho}_0(t+dt), \tag{8}$$

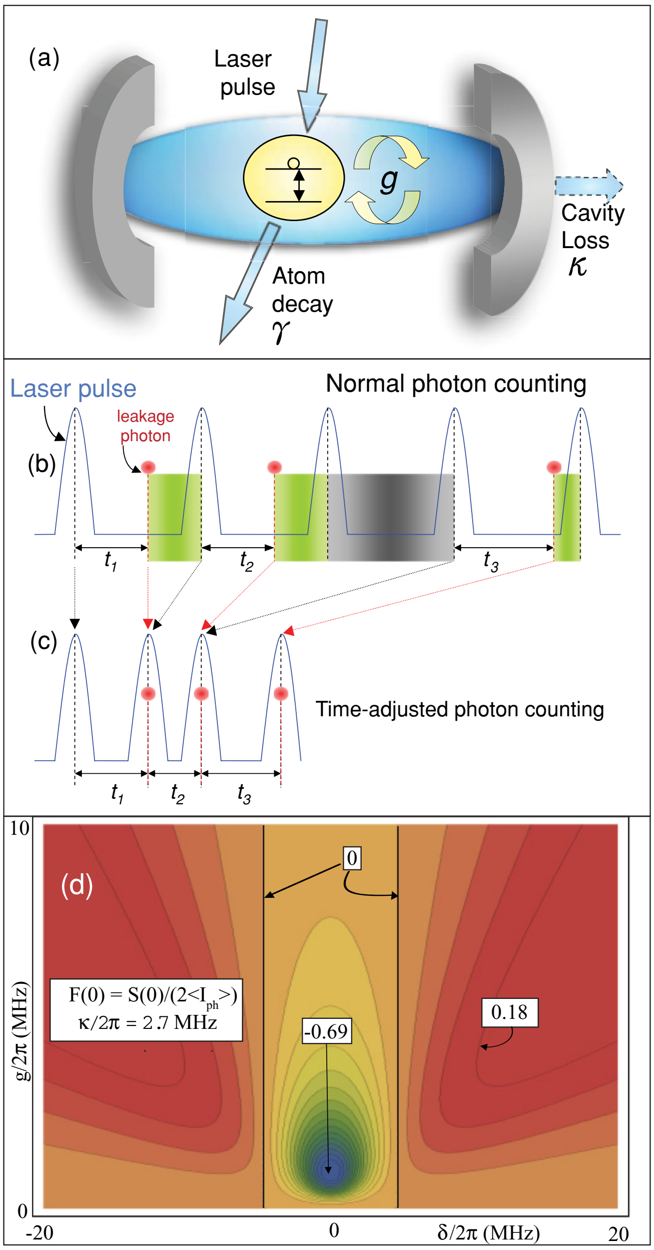

FIG. 1. (Color online) (a) Schematic of a QED system. Vacuum Rabi oscillations can occur if the atom-photon coupling strength g overwhelms the loss rate κ of cavity photons and the emission rate γ into other modes. (b) Normal photon counting: Periodic ultrashort laser pulses excite the atom faster than all other time scales. The single-photon detector records the arrival time t_n of a photon decaying out of the cavity with respect to each pulse. The gray area in (b) means that no photon is detected because of detector inefficiency. (c) Time-adjusted photon counting: The time (shown in green) between a photon-detection event (in red) and the subsequent laser pulse is eliminated, moving the time of each laser pulse to the time of the previous photon count. Any periods (in gray) of no photon detection are eliminated. In this manner, the system has an effective instantaneous feedback: Once a photon is detected, a laser pulse immediately drives the atom to the excited state. (d) The zero-frequency component of the power spectrum $F(0)$ is positive in the presence of coherent VRS, even if the coupling g is very small. In (d), the color brown is maximum, blue is minimum, and the black vertical lines show $F(0) = 0$.

and thus one can assume the master equation

$$\dot{\rho} = -i[H_c,\rho] + \kappa\mathcal{O}a\rho a^\dagger - \frac{\kappa}{2}[a^\dagger a\rho + \rho a^\dagger a].$$

As the feedback is via the operator $\mathcal{O}[\cdots]$, which does not have the form of a Liouville superoperator, this is not a trace-preserving equation of motion. However, if we restrict ourselves to the single (lowest) excitation manifold, only the $|e,0\rangle$, $|g,1\rangle$, and $|g,0\rangle$ states are important, where 1 (0) represents a single (no) photon in the cavity. Then, the feedback term becomes

$$\mathcal{O}a\rho a^\dagger = |e,0\rangle\langle g,1|\rho|g,1\rangle\langle e,0| \tag{9}$$

and, in this truncated basis $\mathcal{O}[\cdots]$, becomes trace preserving, as the state $|e,1\rangle$ is decoupled from the equation of motion (implying $\langle e,1|\rho|e,1\rangle = 0$), giving

$$\begin{aligned}\mathrm{Tr}[a\rho a^\dagger] &= \langle e,1|\rho|e,1\rangle + \langle g,1|\rho|g,1\rangle \\ &= \langle g,1|\rho|g,1\rangle \\ &= \mathrm{Tr}[\mathcal{O}a\rho a^\dagger].\end{aligned} \tag{10}$$

Now the action of the instantaneous feedback is clear such that the state $|g,0\rangle$ in the photon-decay terms in the master equation is decoupled from the single-excitation manifold. We can now write our equation of motion purely in the pseudospin two-state basis [defined as $\tilde{\sigma}_z = |e,0\rangle\langle e,0| - |g,1\rangle\langle g,1|$],

$$\begin{aligned}\dot{\rho} = &-i\left[\frac{\nu}{2} + \frac{\delta}{2}\tilde{\sigma}_z + g\tilde{\sigma}_x,\rho\right] \\ &+ \kappa\tilde{\sigma}_+\rho\tilde{\sigma}_- - \frac{\kappa}{2}[\tilde{\sigma}_-\tilde{\sigma}_+\rho + \rho\tilde{\sigma}_-\tilde{\sigma}_+],\end{aligned} \tag{11}$$

where $\delta = \omega - \nu$ is the detuning between the atom and the cavity. This restricted-basis equation of motion is equivalent to a two-level atom undergoing resonance fluorescence in free space [19]. Here the two-level atom is represented by the combined atom-cavity states $|g,1\rangle$ and $|e,0\rangle$. The coherent input field is the natural atom-cavity interaction, and the time-adjusted photon counting gives an effective decay from $|g,1\rangle$ to $|e,0\rangle$ by eliminating the no-excitation state $|g,0\rangle$. The time-adjusted data set then represents a single trajectory in the ensemble described by this new equation of motion.

To achieve the preceding, we note the following points. First, the delay between measurement and feedback is instantaneous. In our case, the delay is zero, as dictated by our adjustment of time intervals in the data set. Second, the feedback action (i.e., laser pulse) instantaneously projects the system into the $|e,0\rangle$ state. This has recently been achieved by Bochmann *et al.* [1] using ultrashort laser pulses. Third, the photon detection is here assumed to be 100% efficient. This can be effectively achieved by simply omitting the time intervals where no photons are detected. The only issue remaining, then, is if the cavity has not decayed before a subsequent laser pulse is applied. This has the possibility of forcing us to leave the single-excitation basis. This can be neglected in the limit when the time between pulses is much bigger than the cavity decay time.

From this simple two-state model, the ensemble-averaged measurements of the photon output from the cavity can be easily calculated. In particular, the second-order correlation function,

$$g^{(2)}(t,t+\tau) = \frac{\langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle}{\langle a^\dagger(t)a(t)\rangle^2}, \quad (12)$$

is found to be

$$g^{(2)} = \frac{e^{-\alpha t}}{8\Theta}[3\kappa - 4\Theta + 8\Theta e^{\alpha t} - 4\alpha e^{2\Theta t}], \quad (13)$$

where $\alpha = 3\kappa/4 + \Theta$, $\Theta = \sqrt{\kappa^2/16 - 4g^2}$ and $\delta = 0$ for convenience. It is important to point out that one cannot define a correct first-order correlation function $G^{(1)}(t,t+\tau) = \langle a^\dagger(t)a(t+\tau)\rangle$ with this two-state model unless one performs a full numerical simulation using a trace-preserving feedback operator or retains the incoherent transition through the $|g,0\rangle$ state in the equation of motion.

IV. ANALOGY WITH ELECTRON TRANSPORT

As discussed earlier, our goal is to show that this simple model is equivalent to the electron transport through a solid-state double quantum dot and then to take advantage of common tools from transport theory. In the transport regime, a DQD is connected to electronic reservoirs with tunneling rates Γ_L and Γ_R. If one assumes the strong Coulomb-blockade regime, that is, that the charging energy is much larger than other parameters, then one only needs to consider a single level in each dot. One can then define the three-state basis: $|L\rangle$, $|R\rangle$, and $|0\rangle$, representing an electron in the left-dot, the right-dot, and the empty state, respectively. The DQD Hamiltonian is written as

$$H_d = E_L|L\rangle\langle L| + E_R|R\rangle\langle R| + T|L\rangle\langle R| + T|R\rangle\langle L|, \quad (14)$$

where E_L (E_R) is the energy for the left-dot (right-dot) level and T is the coherent tunneling amplitude between them.

The density matrix $\rho(t)$ of the DQD satisfies

$$\frac{d}{dt}\rho(t) = \mathcal{W}_d[\rho(t)] = -i[H_d,\rho(t)] + L_d[\rho(t)]. \quad (15)$$

The L_d term contains the transport properties and dissipation within the device:

$$L_d[\rho(t)] = -\frac{\Gamma_L}{2}[s_L s_L^\dagger \rho(t) - 2s_L^\dagger \rho(t) s_L + \rho(t) s_L s_L^\dagger] - \frac{\Gamma_R}{2}[s_R^\dagger s_R \rho(t) - 2 s_R \rho(t) s_R^\dagger + \rho(t) s_R^\dagger s_R], \quad (16)$$

where

$$s_L = |0\rangle\langle L|, \quad s_L^\dagger = |L\rangle\langle 0|, \quad s_R = |0\rangle\langle R|, \quad s_R^\dagger = |R\rangle\langle 0|.$$

One can calculate the current of electrons leaving the device using a current superoperator (e.g., for the junction on the right and setting the electric charge $e = 1$ throughout):

$$\widehat{I}_R\rho(t) = \Gamma_R|0\rangle\langle R|\rho(t)|R\rangle\langle 0|. \quad (17)$$

The steady state current of the right junction is then [20,21]

$$\langle I_s\rangle = \mathrm{Tr}(\widehat{I}_R\rho_0) = \frac{\Gamma_R T^2}{\Gamma_R^2/4 + \varepsilon^2 + T^2(2 + \Gamma_R/\Gamma_L)}, \quad (18)$$

where $\varepsilon = E_L - E_R$ is the energy difference between the two dots. The shot noise $S_e(\omega)$ of this device [22,23] can also be easily obtained:

$$S_e(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau}\langle[\delta I_R(t),\delta I_R(t+\tau)]_+\rangle_{t\to\infty} + 2\langle I_s\rangle\delta(\tau), \quad (19)$$

where the fluctuating right-junction current is

$$\delta I_R(t) = \widehat{I}_R(t) - \langle I_s\rangle \quad (20)$$

and the self-correlation term $2\langle I_s\rangle\delta(\tau)$ represents the correlation of a tunneling event with itself [20]. The shot noise (zero-frequency noise) is found to be [22,23]

$$F_e = S_e(0)/2e\langle I_s\rangle = \left\{1 - 8T^2\Gamma_L \frac{4\varepsilon^2(\Gamma_R - \Gamma_L) + 3\Gamma_L\Gamma_R^2 + \Gamma_R^3 + 8\Gamma_R T^2}{[\Gamma_L\Gamma_R^2 + 4\Gamma_L\varepsilon^2 + 4T^2(\Gamma_R + 2\Gamma_L)]^2}\right\}. \quad (21)$$

To reduce the DQD problem from a three-state basis to a two-state one, we take the limit of

$$\Gamma_L \gg \Gamma_R, T, \varepsilon. \quad (22)$$

This allows us to eliminate the $|0\rangle$ empty state so that as an electron tunnels out of the right junction, an electron immediately tunnels through the left one, mimicking the effective two-level behavior of the cavity-QED system. Interestingly, this limit gives exact results for all the stationary currents but only returns the correct time dependence for the right-junction current, in analogy with the inability for the restricted-basis cavity-QED model to correctly construct $G^{(1)}$. In this case, the superoperator for the right-junction current becomes

$$\widehat{I}_R\rho(t) = \Gamma_R|L\rangle\langle R|\rho(t)|R\rangle\langle L|. \quad (23)$$

Similarly, the zero state is eliminated from the L_d term so that only one tunneling rate, Γ_R, remains. In this limited basis, Eqs. (14) and (15) are equivalent to Eq. (11). Table I lists how the various parameters correspond to one another in the two different systems.

Of particular interest to us is how the right-junction second-order current-correlation function in the large Γ_L limit is equivalent to the second-order photon correlation function $g^{(2)}(t,t+\tau)$ discussed earlier. This is because (in the limit $\Gamma_L \gg \Gamma_R, T, \varepsilon$)

$$\langle I_R(t+\tau)I_R(t)\rangle = \mathrm{Tr}[\hat{I}_R e^{\mathcal{W}_d\tau}\hat{I}_R\rho(t)] = \Gamma_R^2\,\mathrm{Tr}\,[|L\rangle\langle R|\{e^{\mathcal{W}_d\tau}|L\rangle\langle R|\rho(t)|R\rangle\langle L|\}|R\rangle\langle L|], \quad (24)$$

where each superoperator acts on those to the right. The corresponding correlation function for $g^{(2)}(t,t+\tau)$ for the cavity-QED system, in the reduced basis we discussed earlier, is defined as

$$g^{(2)}(t,t+\tau)\langle a^\dagger(t)a(t)\rangle^2 = \langle a^\dagger(t)a^\dagger(t+\tau)a(t+\tau)a(t)\rangle = \mathrm{Tr}\,[\tilde{\sigma}_-\tilde{\sigma}_+ e^{\mathcal{W}_c\tau}\tilde{\sigma}_+\rho(t)\tilde{\sigma}_-], \quad (25)$$

where we have adopted the traditional input-output formalism to define the photon intensity in terms of the internal pseudospin operators $\tilde{\sigma}$. It is easy to see that the current-correlation measurement can be made equivalent to the second-order photon-intensity measurement simply by multiplying by a factor of κ^2.

In summary, the ensemble-averaged photon statistics from a periodically pulsed cavity-QED system, following appropriate time adjustments, have the same properties as the transport of electrons through a DQD. As an example of the power of this apparently simple analogy, summarized in Table I, we examine two tests for quantum behavior in DQDs (nonnegative shot noise and a special case of the LG inequality) and show how these two tests can be applied to the time-adjusted cavity-QED system.

V. TESTS OF QUANTUMNESS

A. Super-Poissonian shot noise

It has been argued that super-Poissonian shot noise can be observed in DQDs only if there is coherent quantum tunneling between the two dots [24,25]. If Γ_L is larger than Γ_R, the second term in Eq. (21) becomes negative and produces a super-Poissonian value: $S(0)/2e\langle I_s\rangle > 1$. This was thought to have been observed experimentally [24,25], but the exact source of the large super-Poissonian noise in these experiments is still open to alternative interpretation [26]. In our analogy, Γ_L is always much larger than Γ_R (it is effectively infinite). Using the correspondence between electron current and photon intensity, we can define an effective fluctuating photon-intensity noise spectrum:

$$S_{\rm ph}(\omega) = 2{\rm Re}\left[\int_0^\infty d\tau\ e^{i\omega\tau}\langle I_{\rm ph}\rangle^2(g^{(2)}(t,t+\tau)-1)\right],$$

where the effective photon current is

$$\langle I_{\rm ph}\rangle = \kappa\langle a^\dagger(t)a(t)\rangle \quad (26)$$

and the subscript ph represents the photon analogy to typical electron-transport measurements. In the photon case, there is no self-correlation term. We can easily calculate a photon-current Fano factor using the same technique used for DQDs and find that

$$F_{\rm ph} = \frac{S_{\rm ph}(0)}{2\langle I_{\rm ph}\rangle} = -\frac{8g^2(3\kappa^2-\delta^2)}{(8g^2+\kappa^2+\delta^2)^2}. \quad (27)$$

From this equation, one can easily see that super-Poissonian noise in the transport case corresponds to positive noise in the photon case ($F_{\rm ph} > 0$). This can occur if $3\kappa^2 < \delta^2$; otherwise, the shot noise for photons is negative. As mentioned earlier, in the DQD electron-transport case, the super-Poissonian noise is only obtained for coherent coupling between the two dots. For classical sequential tunneling between two dots, the result turns out to be solely sub-Poissonian [24,25]. For the time-adjusted single-photon cavity-QED system we consider here, coherent Rabi oscillations between the atom and cavity photon states produce a positive zero-frequency component in the shot-noise spectrum. This is clearly indicated in Fig. 1(d), using parameters akin to those in the experiment in Ref. [1]. Even for $g \leqslant \kappa$, the spectrum remains positive. Only for $\kappa \gg g$ does the whole range of $F_{\rm ph}(0)$, as a function of δ, become negative.

Typically, one can call the light observed from a single-photon source nonclassical because of its antibunching statistics. Equation (27) implies a secondary criterion for the quantumness of the observed light from such a single-photon source: If the correlation function is conditioned by quantum coherent oscillations (VRS), one should see a positive value for the zero-frequency limit of the photon noise spectrum.

B. Extension of the LG inequality

To further clarify the quantum signatures in these correlation functions, we turn to an extension [15] of the LG inequality [16]. Recently, we derived our extended inequality based on measurements of the fluctuating current through a DQD device [15]. Since the current is essentially an invasive measurement, we showed that the inequality only applies for systems with a state space of three or less states and under the assumption of irreversible transport into the right reservoir (see the Appendix). For the cavity-QED system, these two assumptions become that of being in the single-excitation manifold and the irreversible loss of photons once they leave the cavity, respectively. Thus the current inequality of Ref. [15],

$$|L_I(t)| \equiv |2\langle I(t+\tau)I(t)\rangle - \langle I(t+2\tau)I(t)\rangle| \leqslant \Gamma_R\langle I(t)\rangle, \quad (28)$$

becomes a second-order photon-correlation inequality:

$$|L_g(t)| \equiv |2g^{(2)}(t,t+\tau) - g^{(2)}(t,t+2\tau)| \leqslant \langle a^\dagger(t)a(t)\rangle^{-1}. \quad (29)$$

Superficially, this inequality is equivalent to the LG inequality in the stationary limit [16]. However, as mentioned earlier, the

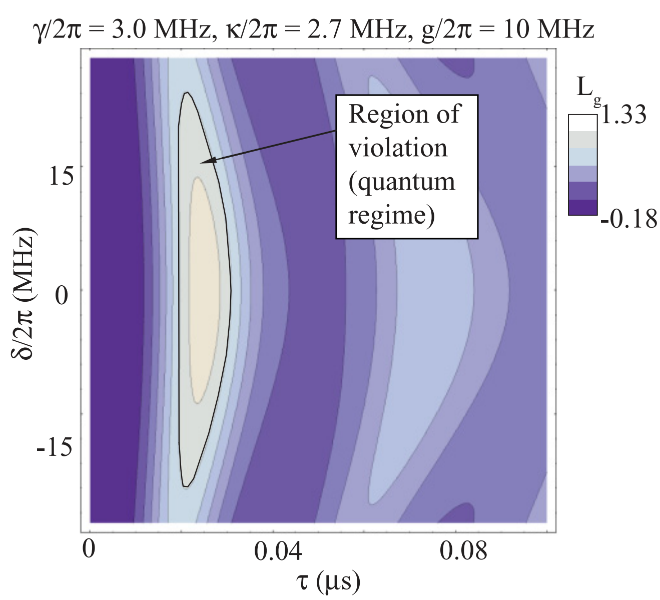

FIG. 2. (Color online) A violation of the extended LG inequality [Eq. (29)] for typical parameters in single-photon cavity-QED experiments [2,3] using time-adjusted photon statistics. The parameters used here are $\kappa/2\pi = 2.7$ MHz, $g/2\pi = 10$ MHz, and the variation of the detuning $\delta/2\pi$ can be up to 20 MHz. The violation of the inequality is indicated by the gray island inside the black contour line.

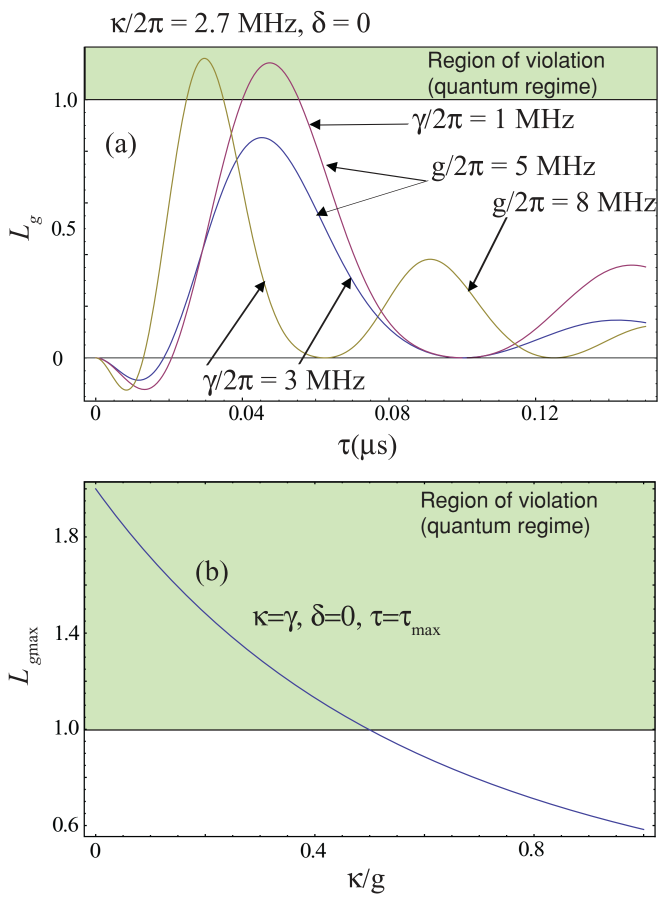

FIG. 3. (Color online) (a) The nonadjusted photon-counting statistics also exhibit a violation of the LG inequality [Eq. (29)]. The combined effect of the atom polarization decay and the cavity decay rates ($\gamma/2\pi = 3$ MHz and $\kappa/2\pi = 2.7$ MHz) prevents a violation being seen in the data in Ref. [1]. However, a slight decrease in the polarization decay rate γ or an increase in the coupling strength g, combined with the ultrashort laser pulses in Ref. [1], should reveal a violation of the inequality [Eq. (29)]. (b) Using the same model (without time-adjusted counting), one can estimate the ratio between the coupling strength g and the dissipation (κ,γ) needed to observe a violation of Eq. (29). This is approximately given by VRS $= 4g/(\kappa + \gamma) > 4$. In (b), we have chosen $\kappa = \gamma$.

observation of a photon is an invasive measurement in terms of the cavity-atom state, and thus, strictly speaking, this inequality no longer can be discussed in terms of distinguishing theories obeying macroscopic realism from quantum mechanics: the original goal of the work of Leggett and Garg. Here this inequality only distinguishes quantum dynamics from those given by a classical rate equation (see the Appendix and Ref. [15] for more details). Like the non-negative shot-noise feature, this inequality reveals a more nuanced way of understanding whether certain photon statistics have quantum characteristics beyond those indicated by antibunching alone.

In Fig. 2, we show how the violation of this inequality occurs for a typical cavity-QED experiment using [1–4] $\kappa/2\pi = 2.7$ MHz, $\gamma/2\pi = 3$ MHz, and $g = 10$ MHz. As seen in Fig. 2, the violations of the inequality are easily observable and appear in a wide range of detuning δ.

C. Extended inequality with standard photon statistics

One can also apply the extended [15] LG inequality to the photon statistics without time adjustment. In this case, a histogram of the photon counts as a function of time (after the initial excitation of the atom) is equivalent to the second-order correlation function of the atom-cavity system with one excitation and cavity decay but no further time-dependent excitations. See Refs. [1–4] for clear examples of such statistics.

Using a simple model of Bochmann *et al.* experiment [1] [using Eqs. (1)–(3) and the initial state $\rho_0 = |g,1\rangle\langle g,1|$], we have found that their experiment does not violate the inequality (29) (the bound is now set by the choice of initial state; see the Appendix). A factor of 2 decrease in the atomic polarization decay rate, or a correspondingly stronger coupling strength, should reveal a violation. We illustrate this in Fig. 3(a), again using their parameters, though we omit dephasing because of variations in coupling g. It is easy to see that a violation of Eq. (29) should be possible with minor improvements in system parameters. To give a more general bound for parameters which can cause a violation, in Fig. 3(b), we show the magnitude of the violation versus the cavity and atomic losses. This gives a bound on the vacuum Rabi splitting parameters needed to observe a violation of

$$2g/[(\gamma + \kappa)/2] > 4. \tag{30}$$

Many realizations of cavity QED have parameters which exceed this (see, e.g., [5]). We therefore think that this inequality [Eq. (29)] is a useful addition to the toolbox one can use to test for quantum behavior in optical systems (see Refs. [27,28] for reviews of other common tests).

VI. CONCLUSIONS

We have shown how a simple adjustment of the output photon-detection statistics of a periodically excited cavity-QED system can be described by a nonequilibrium model, with an exact analogy to electron-transport properties through a DQD. This represents a unification of the fields of photon-counting statistics and electron-transport statistics. We then adapted several recent results from transport theory to describe or test the quantum nature of the photon statistics being emitted from the cavity.

We emphasize that not only the current noise but also the higher order moments [29,30] can be examined with this time-adjusted scenario. Moreover, we point out that the same features could be observed in a circuit-QED system, where the artificial atom (qubit) is periodically excited by some external means and photons are detected with a microwave photon counter [31]. Finally, there are many more complex interacting light-matter systems (see, e.g., [32,33]) which could benefit from a similar analysis and which represent a field ripe for future study.

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APPENDIX: DERIVATION OF THE EXTENDED LG INEQUALITY

In Ref. [15], we extended the LG inequality to work under the conditions of invasive measurement but with additional restrictions. Here we summarize and reformulate the proof of that inequality but now using the language of cavity QED.

For the cavity-QED case, we posit that any photon-intensity measurements not conditioned by quantum dynamics obey

$$|L_g(t,t+\tau)| \equiv |2g^{(2)}(t,t+\tau) - g^{(2)}(t,t+2\tau)| \leqslant \langle a^\dagger(t)a(t)\rangle^{-1}. \quad (A1)$$

In the language of an effective photon current, we can write this as

$$|L_I(t,t+\tau)| \equiv |2\langle I_{\rm ph}(t+\tau)I_{\rm ph}(t)\rangle - \langle I_{\rm ph}(t+2\tau)I_{\rm ph}(t)\rangle| \leqslant \kappa\langle I_{\rm ph}\rangle, \quad (A2)$$

where κ is the rate of photon leak from the cavity, $I_{\rm ph}(t) \equiv I_{\rm ph}(t=0)$, and $\langle I_{\rm ph}(t)\rangle$ is the average photon current of the initial state. Hereafter we omit the t variable. In the master equation approach, the current operator translates into a jump superoperator, and Eq. (A2) thus represents an inequality concerning transitions in the system and not static properties. Thus it is obviously suitable for application to single-photon measurements, which give us information about a change in the state of the cavity-QED system. As described in the text, the photon current superoperator acts as before, $\hat{I}_{\rm ph}[\rho] = \kappa\tilde{\sigma}^+\rho\tilde{\sigma}^-$, such that the average current is $\langle I_{\rm ph}\rangle = {\rm Tr}\{\hat{I}_{\rm ph}\rho\}$ and the correlation function of interest is obtained as

$$\langle I_{\rm ph}(\tau)I_{\rm ph}\rangle = {\rm Tr}\{\hat{I}_{\rm ph}\exp[\mathcal{L}\tau]\hat{I}_{\rm ph}\rho_0\}. \quad (A3)$$

For our time-adjusted case, the stationary distribution is chosen as the initial state. For the non-time-adjusted photon statistics, one chooses $\rho_0 = |g,1\rangle\langle 1,g|$.

In these terms, the inequality expression can be written as

$$L_I(\tau) = {\rm Tr}\{\hat{I}_{\rm ph}(2\exp[\mathcal{L}\tau] - \exp[2\mathcal{L}\tau])\hat{I}_{\rm ph}\rho_0\}. \quad (A4)$$

If the cavity-QED system contains no coherent quantum dynamics, $\hat{I}_{\rm ph}$ is the 3×3 matrix with elements $\hat{I}_{{\rm ph}_{\alpha\beta}} = \kappa\delta_{\alpha,0}\delta_{\beta,\mathcal{P}}$, where the indices $0 = |g,0\rangle, \mathcal{P} = |g,1\rangle$, $\mathcal{A} = |e,0\rangle$. Thus, using the Chapman-Kolgomorov equation, we have

$$L_I(\tau) = \kappa^2 P_{\mathcal{P}}(0)[\Omega_{\mathcal{P}0}(2 - \Omega_{00} - \Omega_{\mathcal{P}\mathcal{P}}) - \Omega_{\mathcal{P}\mathcal{A}}\Omega_{\mathcal{A}0}],$$

where Ω represents the matrix elements of the propagator. For a general Markov stochastic matrix, Ω, the maximum of L_I is

$$\max\{L_I\} = 2\kappa^2 P_{\mathcal{P}}(0).$$

However, the rate equation form $\Omega(\tau) = \exp\mathcal{L}\tau$ furnishes us with a further constraint. Maximizing $L_I(\tau)$ with respect to time, from $\dot{L}_I = 0$ and $\dot{\Omega} = \mathcal{L}\Omega$, we find that the maximum of L_I occurs when $\Omega_{00} + \Omega_{\mathcal{P}\mathcal{P}} = 1$ and $\Omega_{\mathcal{P}0} = 1$, giving

$$\max\{L_I\} = \kappa^2 P_{\mathcal{P}}(0) = \kappa\langle I_{\rm ph}\rangle.$$

This result relies on the form of the jump operator and the absence of reabsorbtion of photons by the cavity, that is, $\mathcal{L}_{\mathcal{P}0} = 0$. These requirements mean that we must always be in the single-excitation regime and that once a photon leaves the cavity and is measured, it cannot return. Fortunately, this is implicit in the definition of destructive photon measurement.

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