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Quantum metamaterials: Electromagnetic waves in Josephson qubit lines

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We consider the propagation of a classical electromagnetic wave through a transmission line, formed by identical superconducting charge qubits inside a superconducting resonator. Since the qubits can be in a coherent superposition of quantum states, we show that such a system demonstrates interesting new effects, such as a "breathing" photonic crystal with an oscillating bandgap. Similar behaviour is expected from a transmission line formed by flux qubits. The key ingredient of these effects is that the optical properties of the Josephson transmission line are controlled by the quantum coherent state of the qubits.

The progress in experimental and theoretical investigation of mesoscopic structures, in particular superconducting qubits [1], made it possible now to shift focus to a more interesting business of building an "artificial matter", a structure containing a large enough number of qubits, which maintain quantum coherence for a long enough time to demonstrate certain new properties. One can consider a working quantum computer an ultimate example of such a structure. Nevertheless we don't need to go that far. For starters, we can consider the propagation of electromagnetic wave through a large set of qubits, considering the latter as an effective medium. Such media built of classical artificial components are known as metamaterials. Depending on their structure and parameters, they can demonstrate some strikingly unusual properties (e.g., refraction of the electromagnetic wave in so-called left-handed materials [2]). We will therefore call a medium built of qubits a *quantum metamaterial*.

A simple example of a quantum metamaterial if put a large number of qubits in a one-dimensional resonator. Of course, a chain of interacting qubits would do as well, but we have already seen that putting qubits in resonators provides several advantages, such as a convenient control and readout and increased decoherence time, and is therefore relevant from the experimental point of view.

The propagation of an electromagnetic wave along this sytem will be affected by the coherent quantum dynamics of qubits. Should one expect anything unusual?

Let us consider an infinite number of superconducting charge qubits placed at equal intervals l between two massive superconductors separated by a distance D of the same order (Fig. 1) [3]. The superconductors form a waveguide, and qubits – an effective medium filling it. The magnetic field of the electromagnetic wave propagating in this structure must be parallel to the superconducting banks. If neglect – as we do – the energy losses, it must be also normal to the direction of the wave propagation. Therefore $\mathbf{H} = H_y \mathbf{e}_y$, and it is conveniently described by the vector potential $\mathbf{A} = A_z \mathbf{e}_z$. We will see that if we choose realistic parameters, the wavelength of a propagating wave is much larger than interqubit distance, and therefore one can neglect all dependence on the transverse coordinates. More-

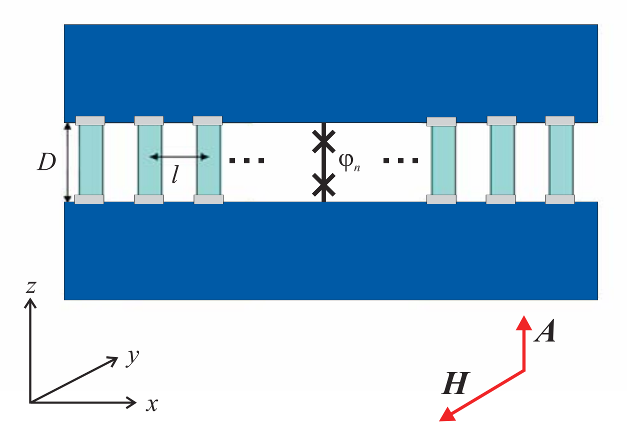

Figure 1 An example of 1D quantum metematerial. Identical charge qubits are placed at equal intervals l between bulk superconductors separated by a distance D. Control circuits of the qubits are not shown.

over, the vector potential within a single cell (say, between the nth and $(n+1)$st qubits) is approximately constant and can be denoted as A_{zn}.

By now we have learned enough about charge qubits to directly write the energy per unit cell

$$\mathcal{E} = \frac{E_J}{2\omega_J^2}\left[\left(\frac{2\pi D\dot{A}_{zn}}{\Phi_0}+\dot{\varphi}_n\right)^2+\left(\frac{2\pi D\dot{A}_{zn}}{\Phi_0}-\dot{\varphi}_n\right)^2\right] - E_J\left\{\cos\left[\varphi_n + \frac{2\pi D A_{zn}}{\Phi_0}\right] + \cos\left[\varphi_n - \frac{2\pi D A_{zn}}{\Phi_0}\right]\right\} + \frac{Dl}{8\pi}\left(\frac{A_{zn+1}-A_{zn}}{l}\right)^2. \quad (1)$$

Here the dot denotes the time derivative, and we assume that the charge qubit has two identical Josephson junctions with $\omega_J = eI_c/\hbar C$ each. We took into account that, in the presence of the vector potential, the superconducting phase differences across the junctions of the nth qubit, $\pm\varphi_n$, acquire a gauge term, $\alpha_n = 2\pi D A_{zn}/\Phi_0$.

Introducing the dimensionless units $E = \mathcal{E}l/E_J$, and $t \to \omega_J t$, we can rewrite Eq. (1) as

$$E = \dot{\varphi}_n^2 + \dot{\alpha}_n^2 - 2\cos\alpha_n\cos\varphi_n + \beta^2(\alpha_{n+1}-\alpha_n)^2, \quad (2)$$

where

$$\beta^2 = \frac{1}{8\pi l D E_J}\left(\frac{\Phi_0}{2\pi}\right)^2 \equiv \frac{E_{\mathrm{EM}}}{E_J} \quad (3)$$

characterizes the ratio (E_{EM}/E_J) of electromagnetic and Josephson energies.

Eventually we will consider such properties of a quantum metamaterial as lasing, which would require a quantum description of the electromagnetic field. At this point, though, we can treat the field classically, and therefore assume that its amplitude is small, i.e., $\alpha_n \ll 1$. The physical sense of the latter inequality is that the magnetic flux per unit cell area $H_y D \times l$ is much smaller than Φ_0. Under such assumptions, the Hamiltonian for a single qubit is

$$\mathcal{H} = -\left(\frac{\partial}{\partial\varphi_n}\right)^2 - \alpha_n^2\cos\varphi_n. \quad (4)$$

As usual, we restrict the states of each qubit to either its ground state $|0\rangle$, with energy E_0, or excited state $|1\rangle$ with energy E_1. We can also introduce the Heisenberg basis states

$$\{\,|0\rangle\exp(i\varepsilon t/2),\ |1\rangle\exp(-i\varepsilon t/2)\}\,, \quad (5)$$

where $\varepsilon = \frac{E_1-E_0}{\hbar\omega_0}$ is the dimensionless qubit excitation energy.

The wave function of the nth qubit, Ψ_n, is a sum

$$\Psi_n = C_0^n\,|0\rangle\,e^{i\varepsilon t/2} + C_1^n\,|1\rangle\,e^{-i\varepsilon t/2}. \quad (6)$$

Using the standard time-dependent perturbation theory, we see from Eq. (4) that in the presence of the field

$$i\frac{dC_k^n}{dt} = \alpha_n^2\sum_{m=1,2}V_{km}^n(t)\,C_m^n(t) \quad (7)$$

with some initial conditions $C_k^n(t=0) = C_k^{n0}$. Here

$$V_{km}^n(t) = \langle k|\cos\varphi_n|m\rangle$$

are matrix elements of the field–qubit interaction calculated in the Heisenberg basis.

Varying the energy (2), we obtain the equation for the electromagnetic field in the linear approximation:

$$\ddot{\alpha}_n - \beta^2(\alpha_{n+1}+\alpha_{n-1}-2\alpha_n) + \alpha_n\langle\Psi_n|\cos\varphi_n|\Psi_n\rangle = 0. \quad (8)$$

This equation together with Eqs. (7) allows to determine the behaviour of both the field and the qubits, but it is not yet what we need. Indeed, this equation still contains the states of individual qubits, while we strive to look at the big picture. If, as we assumed, the field wavelength exceeds the length of a single unit cell, it will not "see" separate qubits. Then, instead of a difference equation (8) for $\alpha_n(t)$ and $\Psi_n(t)$, we can write a differential equation for $\alpha(x,t)$ and $\Psi(x,t)$:

$$\ddot{\alpha} - \beta^2\frac{\partial^2\alpha}{\partial x^2} + V_0\alpha = 0,\ \ V_0 = \langle\Psi(x)|\cos\varphi(x)|\Psi(x)\rangle. \quad (9)$$

Here $n\cdot l$ was replaced by x.

Staying within the perturbation theory approach, we split the electromagnetic wave into the larger incident wave, α_0, and a smaller scattered wave α_1. A quantum state of the system is now described by the wave function

$$\Psi(x,t) = C_0(x,t)|0\rangle e^{i\varepsilon t/2} + C_1(x,t)|1\rangle e^{-i\varepsilon t/2}. \quad (10)$$

In the unperturbed state the coefficients in this equation are $C_i^0(x)$. Splitting, in their turn, the coefficients $C_i(x,t)$ into the unperturbed solution $C_i^0(x)$ and a small perturbation $C_i^1(x,t)$, $|C_i^1| \ll 1$, and using Eq. (7), we derive

$$\begin{aligned} iC_0^1 &= \int_0^t dt' \alpha_0^2 \left(V_{00}\, C_0^0 + V_{01}\, C_1^0 e^{-i\varepsilon t'} \right) \\ iC_1^1 &= \int_0^t dt' \alpha_0^2 \left(V_{11}\, C_1^0 + V_{10}^*\, C_0^0 e^{i\varepsilon t'} \right). \end{aligned} \tag{11}$$

Here the matrix elements $V_{ik} = \langle i|\cos\varphi|k\rangle$ are calculated using the unperturbed wave functions. Obviously $V_{10}^* = V_{01}$.

For the unperturbed EM wave α_0, we obtain from Eq. (9) a usual-looking wave equation

$$\ddot{\alpha}_0 - \beta^2 \frac{\partial^2 \alpha_0}{\partial x^2} + V_0\, \alpha_0 = 0. \tag{12}$$

Nevertheless the specifics of a quantum metamaterial is hidden in the quantity $(V_0)^{1/2}$

$$V_0 = |C_0^0|^2\, V_{00} + |C_1^0|^2\, V_{11} + C_0^0 C_1^{0*} e^{i\varepsilon t}\, V_{10} + \text{h.c.}, \tag{13}$$

which plays the role of the Josephson plasma frequency. Now we see directly that the "optics" of electromagnetic wave propagation through the system is determined by the quantum state and quantum dynamics of the qubits, *which are supposed to be under our direct control*.

Before proceeding, it is wise to check whether the various assumptions we made are consistent. To begin with, we completely neglected both dephasing and relaxation in qubits, and losses in the transmission line formed by the bulk superconductors. Of course, this only holds as long as the characteristic time scale of the effect we investigate is short enough. If use typical enough experimental data on charge qubits in a superconducting resonator [4], the dephasing rate of a qubit is ~ 5 MHz, the photon loss rate from the resonator 0.57 MHz, while the Josephson energy (which provides the scale for the effects we are considering here) is ~ 6 GHz, which is orders of magnitude higher. Therefore our assumptions hold water. Moreover, for this choice of parameters and sensible dimensions of the system $D \sim l \sim 10\mu$m the parameter $\beta \sim 30$. Since β is the velocity in the dimensionless wave equation (12), i.e. the number of unit cells per wavelength, our second assumption, the one which allowed us to derive (12), is also valid.

Now we can safely consider some characteristic solutions of the above equations. For simplicity, we assume that α_0 is a standing wave, $\alpha_0 = A\cos(\omega t)\cos[k(\omega)x]$.

First, assume that *all* the qubits are initially in the ground state $|0\rangle$, i.e., $C_0^0 = 1$ and $C_1^0 = 0$. In this case $V_0 = V_{00}$ and the dispersion law

$$k(\omega) = \frac{1}{\beta}\sqrt{\omega^2 - V_{00}}. \tag{14}$$

Thus, the metamaterial is transparent for the waves with frequencies exceeding $(V_{00})^{1/2}$, which can be interpreted as the "ground state" plasma frequency of the medium. From Eq. (11) we obtain

$$\begin{aligned} \frac{C_0^1(x,t)}{V_{00}} &= -\frac{iA^2\cos^2(kx)}{2}\left\{t + \frac{\sin(2\omega t)}{2\omega}\right\} \\ \frac{C_1^1(x,t)}{V_{01}} &= -\frac{A^2\cos^2(kx)}{2}\left\{\frac{e^{i\varepsilon t}-1}{\varepsilon}\right. \\ &\quad \left. + \frac{\varepsilon + e^{i\varepsilon t}\left[2i\omega\sin(2\omega t) - \varepsilon\cos(2\omega t)\right]}{4\omega^2 - \varepsilon^2}\right\} \end{aligned} \tag{15}$$

The initial disturbance of the wave function, in its turn, produces a disturbance α_1 in the propagating wave. For this perturbation, using Eq. (9), we derive

$$\ddot{\alpha}_1 - \beta^2 \frac{\partial^2 \alpha_1}{\partial x^2} + V_{00}\alpha_1 + \Delta V_0 \alpha_0 = 0, \tag{16}$$

ΔV_0 being the perturbation of the field–qubit coupling.

By means of Eqs. (15) we find

$$\begin{aligned} \Delta V_0(t) &= -|V_{01}|^2 A^2 \cos^2(kx) \\ &\times \left\{\frac{1}{\varepsilon} - \frac{2(2\omega^2 - \varepsilon^2)\cos(\varepsilon t) + \varepsilon^2\cos(2\omega t)}{\varepsilon\,(4\omega^2 - \varepsilon^2)}\right\}. \end{aligned} \tag{17}$$

We see that the electromagnetic wave is in resonance with the qubit line if its frequency is half the inter-level distance, $\omega = \varepsilon/2$. This is due to the term proportional to α^2 in the Hamiltonian (4). Obviously, this result does not hold near the resonance, since the condition $|C_i^1| \ll 1$ is no longer valid.

An almost identical result is obtained if *all* qubits are initially in the excited state $|1\rangle$. We should only exchange $0 \leftrightarrow 1$ and $\varepsilon \leftrightarrow -\varepsilon$ in Eqs. (14)–(17), which leads to

$$k(\omega) = \frac{1}{\beta}\sqrt{\omega^2 - V_{11}}, \tag{18}$$

$$\begin{aligned} \frac{C_1^1(x,t)}{V_{11}} &= -\frac{iA^2\cos^2(kx)}{2}\left\{t + \frac{\sin(2\omega t)}{2\omega}\right\} \\ \frac{C_0^1(x,t)}{V_{10}} &= -\frac{A^2\cos^2(kx)}{2}\left\{\frac{1-e^{-i\varepsilon t}}{\varepsilon}\right. \\ &\quad \left. + \frac{-\varepsilon + e^{-i\varepsilon t}\left[2i\omega\sin(2\omega t) + \varepsilon\cos(2\omega t)\right]}{4\omega^2 - \varepsilon^2}\right\}. \end{aligned} \tag{19}$$

For the electromagnetic wave one gets

$$\ddot{\alpha}_1 - \beta^2 \frac{\partial^2 \alpha_1}{\partial x^2} + V_{11}\alpha_1 + \Delta V_1 \alpha_0 = 0, \tag{20}$$

where

$$\Delta V_1(t) = -\Delta V_0(t),$$

and $\Delta V_0(t)$ is given by Eq. (17).

There is, though, an important difference. If all qubits are initially in the state $|1\rangle$, the metamaterial is an *active medium*, with the complete population inversion. One should therefore expect a resonant amplification of the electromagnetic wave as it propagates along. However, our formulae predict $\Delta V_1 \rightarrow 0$ at $2\omega \rightarrow \varepsilon$ (and $\Delta V_0 \rightarrow 0$ at $2\omega \rightarrow \varepsilon$ as well). What is wrong?

Fortunately, the "paradox" only reflects the limitations of the first order perturbation approximation, where $|C_i^0 + C_i^1|^2 = |C_i^0|^2$. In other words, to first order, the qubit energy simply does not change. To describe the signal amplification, the higher order terms are essential. On the other hand, a proper description of such an effect – essentially, a lasing – anyway requires a quantum description of the electromagnetic field. We will deal with it later on.

Finally, let us prepare the qubits in a superposition state, $C_0 = C_1 = 1/2$. In this case all of them *'rotate'*, and the matrix element in Eq. (9) is now

$$V_0(t) = \frac{1}{4}\left[V_{00} + V_{11} + 2V_{01}\cos(\varepsilon t)\right]. \tag{21}$$

If assume, for simplicity, that the frequency of the electromagnetic wave is high, $\omega \gg \varepsilon$, then its wave vector is a slowly oscillating function of time

$$k(\omega, t) \approx \sqrt{\omega^2 - \frac{V_{00} + V_{11} + 2|V_{01}|\cos(\varepsilon t)}{4\beta^2}} \tag{22}$$

Therefore, if the wave frequency ω is close to the threshold

$$\omega_c = \sqrt{V_{00} + V_{11}}/2\beta,$$

the metamaterial will alternate between transparent and reflecting state with a frequency ε, as the wave vector $k(t)$ switches between real and imaginary values. In addition, it will generate electromagnetic waves with frequencies ε and $\omega \pm \varepsilon$.

The last example – a transparency which senses coherent transitions between the qubit states – gives us a taste of what new effects appear in a quantum metamaterial. This effect is due to the effective plasma frequency of the uniform medium being explicitly dependent on the quantum state of qubits, as can be seen from Eqs. (14,18,22).

Now let us look at a little more advanced arrangement – a *quantum metamaterial photonic crystal*. In a photonic crystal [5] a periodic modulation of the refractive constant (i.e., the light propagation vspeed) produces a frequency gap in the spectrum of electromagnetic wave, similar to the gap in the eletronic spectrum in a periodic lattice. It is therefore natural to expect the same if the qubits in our system are prepared in spatially periodic states. The question is, will there be anything else?

Suppose the qubits are in either $|\gamma\rangle$ or $|\delta\rangle$ state, with a spatial period $2L$. The wave then obeys the equation

$$\ddot{\alpha} - \beta^2\alpha_{xx} + V_{\gamma\gamma}\,\alpha = 0 \tag{23}$$

or

$$\ddot{\alpha} - \beta^2\alpha_{xx} + V_{\delta\delta}\,\alpha = 0. \tag{24}$$

The $|\gamma\rangle$ or $|\delta\rangle$ can be either stationary states (eigenstates of the qubit Hamiltonian), or their superpositions. In the latter case, the photonic crystal discussion makes sense only if the quantum beat frequency is small compared to the frequency of the propagating wave, that is, $\varepsilon^2 \ll |V_{00}|$ and $|V_{11}|$.

Following the usual band-theory approach for electrons in a crystal lattice [6], we seek the solution of Eq. (23,24) in the form of a Bloch wave $\alpha(t,x) = u(x,k)\exp(ikx - i\omega t)$, where $u(x,k)$ is a periodic function of x with the period $2L$, and the dimensionless wave vector k is in the first Brillouin zone, $-\pi/L < k < \pi/L$. Consider the jth elementary cell of our periodic structure: for $x_j < x < x_j + L$ all the qubits are in state $|\gamma\rangle$, and for $x_j + L < x < x_j + 2L$ in the state $|\delta\rangle$. In both regions, the solution $\alpha(t,x)$ of Eq. (23,24) is a sum of exponential terms multiplied by constants C_j. Using the continuity of α and $\partial\alpha/\partial x$ at the boundaries of different regions and the periodicity of the Bloch functions $u(x,k)$, we obtain a set of homogeneous linear equations for C_j. The nontrivial solution of these equations exists only if the determinant of the set of equations is zero. Then, after straightforward algebra, we obtain the dispersion equation for the frequency $\omega(k)$ in the form

$$\cos(\kappa_\gamma L)\cos(\kappa_\delta L) - \frac{\kappa_\gamma^2 + \kappa_\delta^2}{2\kappa_\gamma\kappa_\delta}\sin(\kappa_\gamma L)\sin(\kappa_\delta L) = \cos(2kL), \tag{25}$$

where

$$\kappa_\gamma^2 = \frac{\omega^2 - V_{\gamma\gamma}}{\beta^2}, \qquad \kappa_\delta^2 = \frac{\omega^2 - V_{\delta\delta}}{\beta^2}. \tag{26}$$

This equation predicts the spectrum $\omega(k)$ with gaps if the difference between κ_γ and κ_δ is large enough; that is, $|\kappa_\gamma^2 - \kappa_\delta^2| \gtrsim 1$, or

$$|V_{\gamma\gamma} - V_{\delta\delta}| \gtrsim \beta^2. \tag{27}$$

Thus, in order to form a photonic crystal in the qubit line, the Josephson energy E_J must be large compared to the magnetic energy or, according to Eq. (3),

$$E_J \gg \frac{1}{8\pi}\left(\frac{\Phi_0}{\pi D\, l}\right)^2. \tag{28}$$

This condition is again consistent with our basic assumptions.

The gap in the photonic spectrum depends on the *externally controllable* quantum state of the qubits. We are therefore dealing with a *quantum photonic crystal*: Changing the microscopic quantum state of the qubits changes the macroscopic electromagnetic response of the system at will!

Suppose now that one or both of the qubit states are not the eigenstates $|0\rangle$ or $|1\rangle$. The system then would exhibit *quantum beats* between the two. E.g., let $|\gamma\rangle = |0\rangle$ and

$$|\delta\rangle = \left\{|0\rangle e^{i\varepsilon t} + |1\rangle e^{-i\varepsilon t}\right\}/2.$$

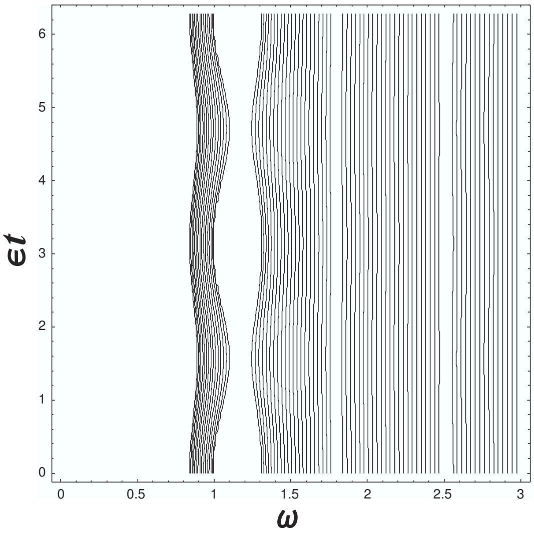

Figure 2 Breathing photonic crystal: contour curves of the wave vector k as a function of ω and εt in the situation described by Eq. (29). The parameters used here are $V_{00} = V_{01} = 1$, $V_{11} = 2$ (units of the qubit Josephson energy E_J), $\beta = 0.5$, $L = 2$. The time-dependent gaps in the spectrum are clearly seen.

Then $V_{\gamma\gamma} = V_{00}$ and

$$V_{\delta\delta}(t) = [V_{00} + V_{11} + 2V_{01}\cos(\varepsilon t)]/4.$$

We see that the photonic crystal arises if any of the matrix elements is of the order of unity. The frequency gap is modulated by the value of $V_{01}/2$ with the period $\Delta t = 2\pi/\varepsilon$. If $V_{01} \sim 1$, then the modulation is significant. If

$$|\gamma\rangle = \left\{|0\rangle e^{i\varepsilon t} - |1\rangle e^{-i\varepsilon t}\right\}/2$$

and

$$|\delta\rangle = \left\{|0\rangle e^{i\varepsilon t} + |1\rangle e^{-i\varepsilon t}\right\}/2,$$

then

$$V_{\gamma\gamma}(t) = [V_{00} + V_{11} - 2V_{01}\cos(\varepsilon t)]/4,$$
$$V_{\delta\delta}(t) = [V_{00} + V_{11} + 2V_{01}\cos(\varepsilon t)]/4. \qquad (29)$$

If $V_{01} \sim 1$, we obtain the gap which is strongly modulated in time from zero, at $t = (2n+1)\pi/2\varepsilon$, to its maximum value, when $t = n\pi/\varepsilon$ (here n is an integer). This is a *"breathing"* photonic crystal (Fig. 2).

Is it possible to replace charge qubits with flux qubits? One can see no reason why not, but the equations are slightly different. Consider a set of flux qubits inductively coupled to a transmission line (i.e., placed inside a strup line resonator). It is convenient to represent such a system using the approximation of lumped elements (Fig. 3). Following the approach of Ref. [7], we assign to each node a flux variable φ_n, such that, e.g., the current between the nodes n and $n+1$ is given by $(\varphi_{n+1} - \varphi_n)/L$, L being the self-inductance of the section between the nodes. The coupling to the qubits takes place due to the fluxes $\phi_n = M\langle \hat{I}_p\rangle$ sent by the n-th qubit through the corresponding section of the line, M being their mutual inductance, and $\hat{I}_p$ the persistent current operator in the qubit loop. (We could easily include the direct qubit–qubit interaction and take into account the inhomogeneity of the structure, but these are not important for the current consideration). It is straightforward now to write the equations of motion for the (classical) field in the line,

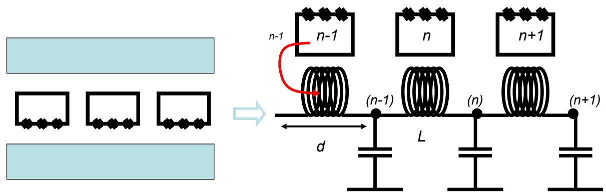

Figure 3 Flux qubits in a resonator: Representation with lumped circuit elements. The length of a single section is d, its self-inductance L, and capacitance C. The phase velocity in the unperturbed line is thus $s = d\Omega$, where $\Omega^2 = 1/LC$.

$$\ddot{\varphi}_n + \Omega^2(2\varphi_n - \varphi_{n+1} - \varphi_n) = \Omega^2(\phi_n - \phi_{n-1}). \qquad (30)$$

In the continuous limit, this equation becomes

$$\ddot{\varphi}(x,t) - s^2\frac{\partial^2\varphi(x,t)}{\partial x^2} = s\Omega\frac{\partial\phi(x,t)}{\partial x}. \qquad (31)$$

the qubit wave function, Eq. (6), determines the right-hand side via

$$\phi(x,t) = MI_0\langle\Psi(x,t)|\hat{\sigma}_z|\Psi(x,t)\rangle. \qquad (32)$$

This equation will be completed by Eqs. (7) for $\Psi(x,t)\rangle$, but with a different set of V_{km}. Note that unlike Eq. (9) for charge qubits, the unperturbed spectrum of Eq. (31) is gapless.

What conclusions can we make from the above results? An extended structure built of qubits behaves with respect to the propagation of a *classical* electromagnetic wave in a way, which reveals quantum coherence of its constituting elements. Not only one can tweak the parameters of this medium by changing the qubit states – this is, arguably, no different from controlling parameters in a classical transmission line. Of more fundamental importance is the result that the classical field propagation through a quantum metamaterial will reveal such essentially quantum effects as quantum beats due to the system being prepared in a coherent superposition of quantum states. One can think of this as an inversion of the double-slit experiment. There a quantum particle interacted with a classical scatterer. Here the situation is reversed. An exciting part is, that this allows us a direct glimpse into the very boundary between quantum and classical worlds.

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