Producing Cluster States in Charge Qubits and Flux Qubits

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We propose a method to efficiently generate cluster states in charge qubits, both semiconducting and superconducting, as well as flux qubits. We show that highly entangled cluster states can be realized by a "one-touch" entanglement operation by tuning gate bias voltages for charge qubits. We also investigate the robustness of these cluster states for nonuniform qubits, which are unavoidable in solid-state systems. We find that quantum computation based on cluster states is a promising approach for solid-state qubits.

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One-way quantum computing [1], which is based on a series of one-qubit measurements starting from cluster states of a qubit array, is an intriguing alternative to the widely studied approach using unitary quantum gates. Here the power of quantum mechanics, such as quantum parallelism and entanglement, is already stored in the initial cluster state. Cluster states are highly entangled states that involve all qubits and act as a universal resource for quantum computing.

Because of their unique importance, cluster states have been studied in a variety of physical systems. They have been extensively explored in optical quantum computers both theoretically [2] and experimentally [3]. By incorporating cluster states, optical quantum computing can achieve substantially simpler operations [2] compared to the original linear optics quantum computing proposal [4]. Cluster states have also been studied in solid-state qubits. In particular, processes of generating cluster states for single and encoded spin qubits have been proposed [5,6] using the Heisenberg exchange interaction.

The existing methods of generating cluster states all require multiple steps because of the types of interaction involved (in the case of photonic qubits, a large number of optical elements is also required). Here we describe theoretically an efficient method to create scalable cluster states in charge qubits [7–13] and flux qubits [14–16], using existing Ising-like interactions. Our key result is that cluster states in charge qubits can be created by applying a single gate bias pulse, right after preparing an initial product state $|\Psi_0\rangle \equiv |\Psi(t=0)\rangle = \Pi_{i=1}^{N}|+\rangle_i$, where $|\pm\rangle_i = (|0\rangle_i \pm |1\rangle_i)/\sqrt{2}$. Here $|0\rangle_i$ and $|1\rangle_i$ are the two states of the ith qubit in an N-qubit system. We also calculate the time-dependent fidelity of the cluster states in charge qubits using a quantum dot (QD) system with decoherence produced by the measurement backaction and explore the effects of nonuniformity among qubits, which is a realistic characteristics for all solid-state qubits.

Cluster states in charge qubits.—The Hamiltonian for an array of charge qubits with nearest-neighbor interactions is described by

$$H_{\rm cq} = \sum_i(\Omega_i\sigma_{ix} + \epsilon_i\sigma_{iz}) + \sum_{i<j}J_{ij}\sigma_{iz}\sigma_{jz}, \qquad (1)$$

with Pauli matrices σ_{ix} and σ_{iz} for the ith qubit. Ω_i is either the inter-QD tunnel coupling for coupled QD systems [7–9] or half of the Josephson energy for superconducting charge qubits [11–13], respectively. For either semiconducting or superconducting charge qubits, ϵ_i is the charging energy and corresponds to the energy difference between $|0\rangle_i$ and $|1\rangle_i$ for each qubit. The coupling constants J_{ij} are derived from the capacitance couplings. In one-way quantum computing [1], the σ_{ix} term needs to be switched off during the creation of the cluster state and then switched on when measurements are carried out. From this perspective, charge qubits with tunable Ω_i [17] are desirable. However, tunability can produce decoherence and cross talk between qubits themselves and between qubits and the environment. In addition, for some qubit systems, once the qubit array is made, Ω_i and J_{ij} are fixed, and only ϵ_i is controllable via the gate voltage bias (we call these "simple-design qubits"). Practically, such a simple design is preferable for solid-state qubits so as to simplify fabrication and enhance scalability. Thus, our goal here is to generate cluster states for simple-design qubits. Hereafter, without loss of generality and for convenience, we focus on qubits of coupled QDs. $|0\rangle_i$ and $|1\rangle_i$ refer to the two states in which the excess charge is localized (see Ref. [9]).

Typically, cluster states are generated by an Ising-like Hamiltonian $H_{\rm cs} = (g/4)\sum_{i<j}(1-\sigma_{iz})(1-\sigma_{jz})$, where i and j are nearest-neighbor sites, starting from the initial state $|\Psi_0\rangle$. Preparing a unitary evolution $U_{\rm cs}(t) = \exp(-itH_{\rm cs})$ (we use $\hbar = 1$) at $gt = (2n_{\rm I}+1)\pi$, where $n_{\rm I}$ is an integer, is the first step for one-way quantum computing. Since simple-design charge qubits have Ising-like interactions, all we need to do to get $H_{\rm cs}$ out of Eq. (1) is to turn off the effect of the σ_x terms in (1). This

can be achieved in the high-bias regime where $\epsilon_i \gg \Omega_i > J_{ij}$, by applying a canonical transformation to Eq. (1):

$$H_{\rm cq}^{\rm (eff)} = e^S H_{\rm cq} e^{-S} = \sum_i E_i \sigma_{iz} + \sum_{i<j} J'_{ij}\sigma_{iz}\sigma_{jz} + H_{\rm uw}, \tag{2}$$

where $S = i\sum_{i=1}^N (\alpha_i/2)\sigma_{iy}$, with $\tan\alpha_i = \Omega_i/\epsilon_i \ll 1$. Also $E_i = (\epsilon_i + \Omega_i^2/\epsilon_i)\cos\alpha_i$ and $J'_{ij} = J_{ij}\cos\alpha_i\cos\alpha_j$. $H_{\rm uw}$ is an unwanted interaction term given by $H_{\rm uw} = \sum_{i<j} J'_{ij}\{-(\Omega_i/\epsilon_i)\sigma_{jz}\sigma_{ix} - (\Omega_j/\epsilon_j)\sigma_{iz}\sigma_{jx} + (\Omega_i\Omega_j)/(\epsilon_i\epsilon_j)\sigma_{ix}\sigma_{jx}\}$. As long as $H_{\rm uw}$ is sufficiently small and can be neglected, we can periodically generate cluster states in the tilted frame $|\tilde\Psi(t)\rangle = e^S|\Psi(t)\rangle$ after a time $t_{\rm cs}$, if both $J' t_{\rm cs} = \pi/4 + 2n_J\pi$ and $E_i t_{\rm cs} = -(\pi/4)\bar n_i + 2n_E\pi$ are satisfied [$\bar n_i$ is the number of qubits connected to the ith qubit; n_J (≥ 0) and n_E are arbitrary integers, and J'_{ij} should be uniform: $J'_{ij} = J'$]. These two equalities lead to the relation $J'(8n_E - \bar n_i)/(8n_J + 1) = E_i \equiv E_{i,t_{\rm cs}}$. Thus, to generate a cluster state, the gate bias voltage ϵ_i for the ith qubit needs to be set at

$$\epsilon_i = \epsilon_i^{\rm cs} = \sqrt{E_{i,t_{\rm cs}}^2 - \Omega_i^2} \tag{3}$$

during $t_{\rm cs} = \pi(8n_J + 1)/(4J')$. A solution for $\epsilon_i^{\rm cs}$ exists when $n_E - (\Omega_i/J')n_J > (\Omega_i/J' + \bar n_i)/8$. Hereafter, we assume these conditions and consider terms up to second order in α_i, neglecting terms $\sim(\Omega_i/\epsilon_i)^3$ or higher order. In such a case, $\alpha_i = \Omega_i/\epsilon_i$ and $J' = J$. We also choose to treat the case of $n_J = 0$, which corresponds to the shortest possible time to generate cluster states. To further ensure the high-bias regime, where $\epsilon_i \gg \Omega_i$, we require $8n_E - \bar n_i \gg \sqrt{2}\Omega_i/J$. An initial product state $|\tilde\Psi(0)\rangle = |\Psi_0\rangle$ has to be prepared for qubits other than the input qubits [1]. $|\Psi(0)\rangle = \Pi_i(\cos[(\Omega_i/(2\epsilon_i) + \pi/4]|0\rangle_i + \sin[(\Omega_i/(2\epsilon_i) + \pi/4]|1\rangle_i)$ is the corresponding state in the original $\{|0\rangle_i, |1\rangle_i\}$ basis, which can also be adjusted by the gate bias on each qubit.

For example, to obtain the two-qubit cluster state $|\Psi\rangle_{C_2} = (|0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2)/\sqrt{2}$, where $\bar n_1 = \bar n_2 = 1$, we can use Eq. (3) and choose a gate bias ($\epsilon_1^{\rm cs} = \epsilon_2^{\rm cs}$) from $\{6.71J, 14.9J, \ldots\}$ ($n_E \geq 1$) for $\Omega = 2J$ and $\{5.74J, 14.5J, \ldots\}$ ($n_E \geq 1$) for $\Omega = 4J$, etc.

Our approach is valid as long as the unwanted term $H_{\rm uw}$ can be neglected. We can estimate a lifetime, beyond which we lose the cluster state due to the presence of $H_{\rm uw}$, by calculating the fidelity defined by $F(t) = |\langle\Psi_0|e^{iH_{\rm cs}t}e^{-i(H_{\rm cs}+H_{\rm uw})t}|\Psi_0\rangle|^2 \approx |1 - it\langle\Psi_0|H_{\rm uw}|\Psi_0\rangle + (1/2)(it)^2\langle\Psi_0|[H_{\rm uw}, H_{\rm cs}] + H_{\rm uw}^2|\Psi_0\rangle|^2$. For a d-dimensional N-qubit array,

$$F(t) \approx 1 - N(dJt)^2\sin^2 2\alpha. \tag{4}$$

Thus, the lifetime of the cluster state is limited by $t < t_{\rm uw} \equiv (2dJ(\Omega/\epsilon)\sqrt{N})^{-1}$. Furthermore, the constraint $t_{\rm cs} < t_{\rm uw}$ imposes a limit on the number of clustered qubits: $N_{\max} < (2\epsilon/(\pi\Omega d))^2$. For example, consider a one-dimensional qubit chain with $\Omega_i = 4J$ and $\bar n_i = 2$. For $n_E = 2$ ($\epsilon_i^{\rm cs}/J \approx 13.4$), $N_{\max} = 4$. For $n_E = 6$ ($\epsilon_i^{\rm cs}/J \approx 45.8$), $N_{\max} = 57$. Indeed, we can choose an infinite number of bias conditions for each set of fixed Ω_i and J. These are closely related to the possible number $N_{\max}$ of clustered qubits and to the scalability of the system. Various kinds of errors, as discussed in Ref. [2], should be taken into account for more detailed estimates.

As shown in Ref. [9], J_{ij} and Ω_i are determined by the distances between QDs. J_{ij} is basically a linear function of the distances between QDs. Ω_i depends exponentially on the distances between two QDs in a qubit. With current technology, it is quite difficult to fabricate an array of QDs with very uniform Ω_i. For superconducting charge qubits, the situation is similar. However, notice that our approach does not require Ω_i to be extremely uniform, since we can adjust ϵ_i according to Ω_i in order to obtain an appropriate E_i in Eq. (2). In addition, as noted above, n_E can be selected arbitrarily, which adds flexibility to our approach. Thus, the one-touch cluster state generation method we discuss here should work with any charge qubit architecture. In short, although in general charge qubits have shorter decoherence times compared with spin qubits, our simpler and faster generation method could make them competitive with spin qubits in the context of cluster states, since several steps are required to generate cluster states for spin qubits [5,6].

Measurement scheme in charge qubits.—In one-way quantum computing, calculations are carried out by a series of local measurements in the σ_{ix} and σ_{iy} eigenbasis. For most charge qubits, however, the measurement is carried out in the σ_{iz} eigenbasis $\{|0\rangle_i, |1\rangle_i\}$ by simply applying a large gate bias and using field-effect detectors such as quantum point contacts (QPCs) or single-electron transistors. Thus, for charge qubits the σ_{ix} and σ_{iy} basis measurements should be converted into σ_{iz} measurements after rotating the frame via $\pm(\pi/2)_y$ and $\pm(\pi/2)_x$ pulses. These pulses can be generated by applying ac gate biases such as $\epsilon_i(t) = \epsilon_{0i}\cos(\omega_{ic}t + \phi_i)$. In a rotating frame of $U_{\rm rw}(t) = \exp(-i\sum_i(\omega_{ic}t/2)\sigma_{ix})$ [18], the wave function is given by $|\tilde\Psi(t)\rangle = U_{\rm rw}^\dagger|\Psi(t)\rangle$, and the Hamiltonian on resonance ($\omega_{ic} = 2\Omega_i$) is given by:

$$H_{\rm rw} \approx \sum_i \frac{\epsilon_{i0}}{2}(\sigma_{iz}\cos\phi_i - \sigma_{iy}\sin\phi_i) + H_{yz}, \tag{5}$$

where $H_{yz} = \sum_{ij}(J_{ij}/2)(\sigma_{iy}\sigma_{jy} + \sigma_{iz}\sigma_{jz})$ is an unwanted term here. In order to realize $|\Psi(\tau)\rangle = e^{i\sum_i\theta_i\sigma_{iy}}|\Psi(0)\rangle$ at a time $t = \tau$, we should have $e^{-i\sum_i\Omega_i\tau\sigma_{ix}}e^{-iH_{\rm rw}\tau/\hbar} \propto e^{i\sum_i\theta_i\sigma_{iy}}$. First, to exclude a prefactor e^{-S} in the cluster state wave function, we need to choose the phase $\phi_i = \pi/2$ and the voltage amplitude $\epsilon_{i0} = \Omega^2/(\pi\epsilon_{\rm cs})$ at $\tau = t_{\rm ini} = \pi/\Omega$. For the σ_{ix} measurement, we need a measurement time $t_m = (\pi/\Omega)l_1$, the phase $\phi_i = \pi/2 + l_2\pi$, and the voltage amplitude $\epsilon_{i0} = (\Omega/(2l_1))(\pm 1 + 4l_3)$, if l_2 is even, and $\epsilon_{i0} = (\Omega/(2l_1))(\mp 1 + 4l_3)$, if l_2 is odd [l_1, l_2, and l_3 are arbitrary integers ($l_1 \neq 0$)]. If we take $t_m = (\pi/\Omega)(l_1 \mp 1/4)$ and $\epsilon_{i0} = (2\Omega l_2)/(l_1 \mp 1/4)$ such that $l_1 + l_2$ is even, we realize the σ_{iy} measurement. By applying a gate bias at the time t_m following $t_{\rm ini}$, σ_x and σ_y measurements are processed. This scheme works well for the region where $\epsilon_{i0}/J \gg 1$.

For the experiment in Ref. [13], $\Omega = 20.5\ \mu$eV and $J = 95\ \mu$eV, and thus $t_{\rm cs} = \pi/(4J) = 34.2$ psec, $\epsilon_{\rm cs} \sim 664\ \mu$eV from Eq. (3) at $n_E = 1$ and $t_{\rm uw} = \epsilon_{\rm cs}/(2\sqrt{2}J\Omega) = 0.5$ ns for a dephasing time of the order of $T_2 \sim 5$ ns. Thus, our approach can be applied, if a smaller J ($<\Omega$) is prepared, for instance, by increasing the distance between qubits.

Cluster states in flux qubits.—For flux qubits, $\Omega > \epsilon$ in Eq. (1), so that here we cannot directly use the above-mentioned one-touch approach for charge qubits. Here we show a method to generate cluster states for flux qubits by applying an oscillating magnetic field. Consider two inductance-coupled flux qubits working at the optimal bias [16] with the Hamiltonian:

$$H_{\rm fq} = \epsilon_1\sigma_{1z} + \epsilon_2\sigma_{2z} + \Omega_1^R\cos(\omega_1^{\rm rf}t + \phi_1)\sigma_{1x} + \Omega_2^R\cos(\omega_2^{\rm rf}t + \phi_2)\sigma_{2x} + J_{xx}\sigma_{1x}\sigma_{2x}, \tag{6}$$

where Ω_i^R and $\omega_i^{\rm rf}$ are the half amplitude and the frequency, respectively, of the applied classical field. At the optimal bias point, the system is immune, up to first order, to variations on the control parameters and is thus robust against decoherence. This Hamiltonian is a good starting point for generating cluster states. In the rotating wave approximation for two identical qubits ($\Omega_1 = \Omega_2$), we have $\tilde{H}_{\rm fq} = H_0 + H_{xy}$, with $H_0 = \sum_{i=1}^2(\Omega_i^R/2)\times(\sigma_{ix}\cos\phi_i + \sigma_{iy}\sin\phi_i)$ and $H_{xy} = J_{xx}(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y})$. The operator to generate cluster states $U_{\rm cs}$ is produced by switching on and off the resonant field of Ω_i^R and controlling the phase ϕ_i similarly to the conventional conditional phase gate operation. For example, if we define $R_{i\nu}(\theta) \equiv \exp(i\theta\sigma_{i\nu})$ ($\nu = x, y$) and $U_{xy}(\theta) \equiv \exp(i\theta(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}))$, we have:

$$\tilde{U}_{\rm cs} = R_{1x}(\theta_1)R_{2x}(\theta_2)U_{xy}(\theta_3)R_{1x}(\theta_4)U_{xy}(\theta_5)R_{1x}(\theta_6) = \exp(-i\pi(\sigma_{1x} + \sigma_{2x} - \sigma_{1x}\sigma_{2x})/4), \tag{7}$$

with $\theta_1 = \theta_2 = -\pi/4$, $\theta_3 = \theta_5 = \pi/8$, $\theta_4 = \pi/2$, and $\theta_6 = -\pi/2$. After rotating $\tilde{U}_{\rm cs}$ around the y axis, we recover the original cluster state generator $U_{\rm cs}$. In the case of many flux qubits, cluster states for the entire system are generated by simultaneously applying Eq. (7) to all of the neighboring qubit pairs. Note that Eq. (6) also describes the rotating wave approximation for the charge qubits in Eq. (5). Thus, this method of generating cluster states is also applicable to charge qubits and not only to flux qubits.

The time required for the creation of a cluster state in flux qubits is $T_{\rm flux} = 5\pi/(2\Omega_1^R) + \pi/(4\Omega_2^R) + \pi/(4J_{xx})$. Taking $\Omega_1^R \sim \Omega_2^R \sim J_{xx} \sim 0.5$ GHz, we obtain $T_{\rm flux} \sim 18$ ns ($T_2 \sim 200$ ns [15]). The effect of imperfect pulses can be estimated by substituting $\theta_j \to \theta_j + \delta_j$ in Eq. (7). If we take the deviation from a perfect pulse as $\delta_1 = \delta_2$, $\delta_3 = \delta_5$, $\delta_4 = \delta_6$, for the initial state $|\Psi_0\rangle$, the fidelity is given by

$$F(t) \approx 1 - \delta_1^2 - 4\delta_3^2 + \delta_4^2\sin^2(\pi/8). \tag{8}$$

Thus, the fidelity $F(t)$ remains close to 1, up to second order in the pulse shape error, even when the pulse shape has defects.

Effect of nonuniformity in cluster states.—Cluster states are highly entangled states involving all qubits and can be decohered by various kinds of *local* fluctuations. Here we investigate the effect of nonuniform qubit parameters on cluster states in semiconductor QDs, from Eq. (1), using a measurement setup, which produces decoherence in the double dot qubits through backaction. We analyze a capacitively coupled detector (such as a QPC), whose shot noise constitutes a random charge fluctuation on the qubits.

We use a density matrix (DM) to describe up to four qubits (inset in Fig. 1) [19]. The DM equations for the qubits and the QPC detector are derived in Ref. [19]:

$$\frac{d\rho_{z_1,z_2}}{dt} = i[K_{z_2} - K_{z_1}]\rho_{z_1,z_2} - i\sum_{j=1}^N\Omega_j(\rho_{g_j(z_1),z_2} - \rho_{z_1,g_j(z_2)}) - [\Gamma_{z_1}^{1/2} - \Gamma_{z_2}^{1/2}]^2\rho_{z_1,z_2}, \tag{9}$$

where $z_1, z_2 = (1111), (1110), \ldots, (0000)$ for four qubits (256 equations) and $z_1, z_2 = (11), (10), (01), (00)$ for two qubits (16 equations). K_{z_1} is the energy of the z_1 state and depends on ϵ_i and J_{ij} in Eq. (1). For example, for two qubits, $K_{(11)} = \epsilon_1 + \epsilon_2 + J_{12}$ while $K_{(10)} = \epsilon_1 - \epsilon_2 - J_{12}$.

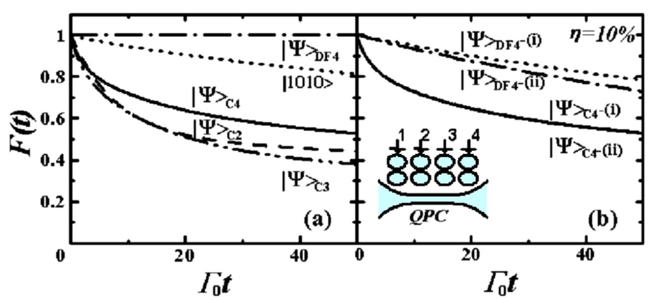

FIG. 1 (color online). Time-dependent fidelities $F(t)$ of cluster states $|\Psi\rangle_{C2}$, $|\Psi\rangle_{C3}$, $|\Psi\rangle_{C4}$, a product state $|1010\rangle$ and four-qubit DF states $|\Psi\rangle_{\rm DF4} = (|1100\rangle - |1001\rangle - |0110\rangle + |0011\rangle)/2$, for $\Gamma_0 = J$ and $\Delta\Gamma = 0.6J$. $\Omega = 4J$ and $n_E = 2$ in Eq. (3); thus, $\epsilon_2 = \epsilon_3 \sim 13.4$ and $\epsilon_1 = \epsilon_4 \sim 14.4$. (a) Comparison of the cluster states, the DF state, and $|1010\rangle$. (b) The case when nonuniformity in the qubit parameters is introduced as $\Omega_i = 4J(1 - \eta_i)$, $\epsilon_i = \epsilon + \eta_i J$, and $\Gamma_i^{(\pm)} = (1 - \eta_i)\Gamma^{(\pm)}$, with i indicating the ith qubit. Here $\eta_i = 0$ for all qubits besides (i) $\eta_3 = 0.1$ and (ii) $\eta_4 = 0.1$. The fidelities of $|\Psi\rangle_{C4}$ for (i) and (ii) mostly overlap. (Inset) Four qubits that use double dot charged states are capacitively coupled to a QPC detector. We consider a similar detection setup for two and three qubits. These calculations are carried out by the $H_{\rm cq}$, that is, they include $H_{\rm uw}$ and higher-order terms.

The $g_j(z_i)$s are introduced for notational convenience and are determined by the relative positions between qubit states. We assume that the tunneling rate Γ of the QPC detector in the presence of N qubits satisfies $\Gamma^{-1} = \sum_i \Gamma_i^{-1}$, where the tunneling rate Γ_i is determined by the state $\sigma_{iz} = \pm 1$ of the ith qubit. The strength of the measurement can be parametrized by $\Delta\Gamma_i$ as $\Gamma_i^{(\pm)} = \Gamma_{i0} \pm \Delta\Gamma_i$, where Γ_{i0} is the tunneling rate of the QPC in the absence of the qubits. The time-dependent fidelity $F(t) \equiv \mathrm{Tr}[\hat{\rho}(0)\hat{\rho}(t)]$ can be calculated by tracing over the elements of the reduced DM obtained from Eq. (9). $F(t)$ can be expanded in time as $F(t) = 1 - \sum_{n=1}(1/n!) \times (t/\tau^{(n)})^n$, where the lifetime is $1/\tau^{(n)} = \{-\mathrm{Tr}[\hat{\rho}(0)d^n\hat{\rho}(0)/dt^n]\}^{1/n}$.

From Eq. (9), we obtain the first-order lifetime for two-, three-, and four-qubit cluster states $|\Psi\rangle_{C_2}$, $|\Psi\rangle_{C_3} = (|+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3)/\sqrt{2}$ and $|\Psi\rangle_{C_4} = (|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4 + |+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4 + |-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4 + |-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4)/2$, respectively, as follows:

$$1/\tau_{C2}^{(1)} = \sum_{z_1,z_2=(11),\ldots,(00)} \Gamma_d(z_1, z_2)/8, \tag{10}$$

$$1/\tau_{C3}^{(1)} = \sum_{z_1,z_2=(111),\ldots,(000)} \Gamma_d(z_1, z_2)/32, \tag{11}$$

$$1/\tau_{C4}^{(1)} = \sum_{z_1,z_2=(1111),\ldots,(0000)} \Gamma_d(z_1, z_2)/128, \tag{12}$$

where the dephasing rate is defined as $\Gamma_d(z_1, z_2) \equiv [\Gamma_{z_1}^{1/2} - \Gamma_{z_2}^{1/2}]^2$. Note that the lifetime of the cluster states is an average over all of the dephasing rates between different product states. This is in contrast with other entangled states. For example, the lifetime of two-qubit Bell states $|c\rangle = (|10\rangle + |01\rangle)/\sqrt{2}$ and $|d\rangle = (|10\rangle - |01\rangle)/\sqrt{2}$ takes the form $1/\tau_c^{(1)} = 1/\tau_d^{(1)} = (1/2)\Gamma_d(10, 01)$. It is well known that the singlet state $|d\rangle$ is the most robust two-qubit state [20] when there is a symmetry between qubits. However, solid-state qubits generally decohere due to various kinds of local causes, which often break the symmetry of the qubit state. Our results in Eqs. (10)–(12) indicate that cluster states might be robust against nonuniformity or local defects because of the averages.

In Fig. 1, we compare the fidelity of cluster states with a product state and a four-qubit decoherence-free (DF) state [20]. Figure 1(a) shows the time-dependent fidelities of two-qubit and four-qubit cluster states and the product state $|1010\rangle$, when $\Gamma_0 = J$ and $\Delta\Gamma = 0.6J$. Our results show that the strongly entangled cluster states are more fragile than a product state such as $|1010\rangle$. We can also see that the robustness of the cluster state depends on the number of qubits in the cluster state. Figure 1(b) shows the time-dependent fidelities of both cluster states and DF states, for nonuniform qubits. Here the parameters Ω_i, ϵ_i, and Γ_i for the third or fourth qubit deviate from those of other qubits by 10%. Note that the fidelities of $|\Psi\rangle_{C4}$ show almost the same behavior irrespective of the distribution of the nonuniformity. Furthermore, a comparison between Figs. 1(a) and 1(b) shows that the nonuniformity has almost no effect on the fidelity of the $|\Psi\rangle_{C4}$ cluster state. These results vividly illustrate our analysis of the lifetime (that it is an average over all the product states). In contrast, in Ref. [19], we showed that the robustness of the DF states strongly depends on the nonuniformity. Thus, even though cluster states are generally more fragile than DF states, they are more robust against nonuniformities among qubits than DF states.

In conclusion, we describe how to efficiently generate cluster states in solid-state qubits. By manipulating the gate bias voltage, we explicitly show how to generate "one-touch" entanglement via cluster states in charge qubits. We also investigate the robustness of cluster states and find that one-way quantum computing could be viable for solid-state qubits.

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