## Controllable Coherent Population Transfers in Superconducting Qubits for Quantum Computing

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We propose an approach to coherently transfer populations between selected quantum states in one- and two-qubit systems by using controllable Stark-chirped rapid adiabatic passages. These *evolution-time insensitive* transfers, assisted by easily implementable single-qubit phase-shift operations, could serve as elementary logic gates for quantum computing. Specifically, this proposal could be conveniently demonstrated with existing Josephson phase qubits. Our proposal can find an immediate application in the readout of these qubits. Indeed, the broken parity symmetries of the bound states in these artificial atoms provide an efficient approach to design the required adiabatic pulses.

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Introduction.—The field of quantum computing is attracting considerable experimental and theoretical attention. Usually, elementary logic gates in quantum computing networks are implemented using precisely designed resonant pulses. The various fluctuations and operational imperfections that exist in practice (e.g., the intensities of the applied pulses and decoherence of the systems), however, limit these designs. For example, the usual  $\pi$ -pulse driving for performing a single-qubit NOT gate requires both a resonance condition and also a precise value of the pulse area. Also, the difficulty of switching on/off interbit couplings (see, e.g., [1]) strongly limits the precise design of the required pulses for two-qubit gates.

Here we propose an approach to coherently transfer the populations of qubit states by using Stark-chirped rapid adiabatic passages (SCRAPs) [2]. As in the case of geometric phases [3], these population transfers are insensitive to the dynamical evolution times of the qubits, as long as they are adiabatic. Thus, here it is not necessary to design beforehand the exact durations of the applied pulses for these transfers. This is a convenient feature that could reduce the sensitivity of the gate fidelities to certain types of fluctuations. Another convenient feature of our proposal is that the phase factors related to the transfer durations (which are important for the operation of quantum gates) need only be known after the population transfer is completed, at which time they can be canceled using easily implementable single-qubit phase-shift operations. Therefore, depending on the nature of fluctuations in the system, rapid adiabatic passages (RAPs) of populations could offer an attractive approach to implementing high-fidelity single-qubit NOT operations and two-qubit SWAP gates for quantum computing. Also, the SCRAP-based quantum computation proposed here is insensitive to the geometric properties of the adiabatic passage paths. Thus, our approach for quantum computing is distinctly different from both adiabatic quantum computation (where the system is PACS numbers: 42.50.Hz, 03.67.Lx, 74.45.+c, 85.25.Cp

always kept in its ground state [4]) and holonomic quantum computating (where implementations of quantum gates are strongly related to the topological features of either adiabatic or nonadiabatic evolution paths [5]).

Although other adiabatic passage (AP) techniques, such as stimulated Raman APs (STIRAPs) [6], have already been proposed to implement quantum gates [7], the present SCRAP-based approach possesses certain advantages, such as: (i) it advantageously utilizes dynamical Stark shifts induced by the applied strong pulses (required to enforce adiabatic evolutions) to produce the required detuning chirps of the qubits, while in STIRAP these shifts are unwanted and thus must be overcome for performing robust *resonant* drivings; and (ii) it couples qubit levels directly via either one- or multiphoton transitions, while in the STIRAP approach auxiliary levels are required.

The key of SCRAP is how to produce time-dependent detunings by chirping the qubit levels. For most natural atomic or molecular systems, where each bound state possesses a definite parity, the required detuning chirps could be achieved by making use of the Stark effect (via either real, but relatively weak, two-photon excitations of the qubit levels [8] or certain virtual excitations to auxiliary bosonic modes [9]). Here we show that the breaking of parity symmetries in the bound states in currentbiased Josephson junctions (CBJJs) provides an advantage, because the desirable detuning chirps can be produced by single-photon pulses. This is because all the electric-dipole matrix elements could be nonzero in such artificial "atoms" [10]. As a consequence, the SCRAPbased quantum gates proposed here could be conveniently demonstrated with driven phase qubits [11] generated by CBJJs. In order to stress the analogy with atomic systems, we will refer to the energy shifts of the CBJJ energy levels generated by external pulses as Stark shifts.

*Models.*—Usually, single-qubit gates are implemented by using coherent Rabi oscillations. The Hamiltonian of

such a driven qubit reads  $H_0(t) = \omega_0 \sigma_z/2 + R(t)\sigma_x$ , with  $\omega_0$  being the eigenfrequency of the qubit and R(t) the controllable coupling between the qubit states;  $\sigma_z$  and  $\sigma_x$ are Pauli operators. If the qubit is driven resonantly, e.g.,  $R(t) = \Omega(t) \cos(\omega_0 t)$ , then the qubit undergoes a rotation  $R_x(t) = \cos[A(t)/2] - i\sigma_x \sin[A(t)/2]$ , with A(t) = $\int_{0}^{t} \Omega(t') dt'$ . For realizing a single-qubit NOT gate, the pulse area is required to be precisely designed as  $A(t) = \pi$ , since the population of the target logic state P(t) = [1 - 1] $\cos A(t)$  [/2 is very sensitive to the pulse area A(t) [in this example, we are assuming an initially empty target state]. Relaxing such a rigorous condition, we additionally chirp the qubit's eigenfrequency  $\omega_0$  by introducing a timedependent Stark shift  $\Delta(t)$ . Therefore, the qubit evolves under the time-dependent Hamiltonian  $H'_0(t) = \omega_0 \sigma_z/2 +$  $R(t)\sigma_x + \Delta(t)\sigma_z/2$ , which becomes

$$H_1(t) = \frac{1}{2} \begin{pmatrix} 0 & \Omega(t) \\ \Omega(t) & 2\Delta(t) \end{pmatrix}$$
(1)

in the interaction picture. Under the condition

$$\frac{1}{2} \left| \Omega(t) \frac{d\Delta(t)}{dt} - \Delta(t) \frac{d\Omega(t)}{dt} \right| \ll [\Delta^2(t) + \Omega^2(t)]^{3/2}, \quad (2)$$

the driven qubit adiabatically evolves along two paths the instantaneous eigenstates  $|\lambda_{-}(t)\rangle = \cos[\theta(t)]|0\rangle - \sin[\theta(t)]|1\rangle$  and  $|\lambda_{+}(t)\rangle = \sin[\theta(t)]|0\rangle + \cos[\theta(t)]|1\rangle$ , respectively. These adiabatic evolutions could produce arbitrary single-qubit gates. For example, a single detuning pulse  $\Delta(t)$  (without a Rabi pulse) is sufficient to produce a phase-shift gate:  $U_z(\alpha) = \exp(i\alpha|1\rangle\langle 1|), \alpha = -\int_{-\infty}^{+\infty} \Delta(t) dt$ . Combining the Rabi and detuning pulses for changing the mixing angle  $\theta(t) = \arctan[\Omega(t)/\Delta(t)]/2$ , from  $\theta(-\infty) = 0$  to  $\theta(+\infty) = \pi/2$ , another single-qubit gate  $U_x = \exp(i\beta_+)\sigma_+ - \exp(i\beta_-)\sigma_-$  [with  $\beta_{\pm} = -\int_{-\infty}^{+\infty} \mu_{\pm}(t) dt$ ,  $\mu_{\pm}(t) = \Delta(t) \pm \sqrt{\Delta^2(t) + \Omega^2(t)}$ ] can be adiabatically implemented:

$$U_{x}:\begin{cases} |\lambda_{-}(-\infty)\rangle = |0\rangle \xrightarrow{|\lambda_{-}(i)\rangle} |\lambda_{-}(+\infty)\rangle = -e^{i\beta_{-}}|1\rangle, \\ |\lambda_{+}(-\infty)\rangle = |1\rangle \xrightarrow{|\lambda_{+}(i)\rangle} |\lambda_{+}(+\infty)\rangle = e^{i\beta_{+}}|0\rangle. \end{cases}$$
(3)

This is a single-qubit rotation that completely inverts the populations of the qubit's logic states and thus is equivalent to the single-qubit NOT gate. Here the population transfer is insensitive to the pulse duration and other details of the pulse shape—there is no need to precisely design these beforehand. Different durations for finishing these transfers only induce different additional phases  $\beta_{\pm}$ , which can then be canceled by applying the phase-shift  $U_z(\alpha)$ .

Similarly, the applied pulses are usually required to be exactly designed for implementing two-qubit gates. For example [12], for a typical two-qubit system described by the *XY*-type Hamiltonian  $H_{12} = \sum_{i=1,2} \omega_i \sigma_z^{(i)}/2 + K(t) \sum_{i \neq j=1,2} \sigma_+^{(i)} \sigma_-^{(j)}/2$ , with switchable real interbit-coupling coefficient K(t), the implementation of a two-qubit SWAP gate requires that the interbit interaction time *t* should be precisely set as  $\int_0^t K(t') dt' = \pi$  (when  $\omega_1 = \omega_2$ ). This difficulty could be overcome by introducing a

time-dependent dc driving to chirp the levels of one qubit. In fact, we can add a Stark-shift term  $\Delta_2(t)\sigma_z^{(2)}/2$  applied to the second qubit and evolve the system via

$$H_2(t) = \frac{1}{2} \begin{pmatrix} -\Delta_2(t) & 0 & 0 & 0\\ 0 & -\Delta_2(t) & K(t) & 0\\ 0 & K(t) & \Delta_2(t) & 0\\ 0 & 0 & 0 & \Delta_2(t) \end{pmatrix}.$$
 (4)

Three invariant subspaces;  $\text{Re}_0 = \{|00\rangle\}$ ,  $\text{Re}_1 = \{|11\rangle\}$ , and  $\text{Re}_2 = \{|01\rangle, |10\rangle\}$  exist in the above driven dynamics. Thus, the populations of states  $|00\rangle$  and  $|11\rangle$  are always unchanged, while the evolution within the subspace  $\text{Re}_2$  is determined by the reduced time-dependent Hamiltonian (1) with  $\Omega(t)$  and  $\Delta(t)$  being replaced by K(t) and  $\Delta_2(t)$ , respectively. Therefore, the APs determined by  $H_2(t)$  produce an efficient two-qubit SWAP gate; the populations of  $|00\rangle$  and  $|11\rangle$  remain unchanged, while the populations of state  $|10\rangle$  and  $|01\rangle$  are exchanged. The passages are just required to be adiabatic and again are insensitive to the exact details of the applied pulses.

Figure 1 shows schematic diagrams of two single-qubit SCRAPs. These designs could be similarly used to adiabatically implement the two-qubit SWAP gate.

Demonstrations with driven Josephson phase qubits.— In principle, the above generic proposal could be experimentally demonstrated with various physical systems [2], e.g., the gas-phase atoms and molecules, where SCRAPs are experimentally feasible. Here, we propose a convenient demonstration with solid-state Josephson junctions.

A CBJJ (see, e.g., [11]) biased by a time-independent dc current  $I_b$  is described by  $\tilde{H}_0 = p^2/2m + U(I_b, \delta)$ . Formally, such a CBJJ could be regarded as an artificial "atom," with an effective mass  $m = C_J \tilde{\Phi}_0$ , where  $\tilde{\Phi}_0 \equiv \Phi_0/2\pi$ , moving in a potential  $U(I_b, \delta) = -E_J(\cos\delta - I_b\delta/I_0)$ . Here,  $I_0$  and  $E_J = I_0 \cdot \tilde{\Phi}_0$  are, respectively, the critical current and the Josephson energy of the junction of capacitance  $C_J$ . Under proper dc bias, e.g.,  $I_b \leq I_0$ , the CBJJ has only a few bound states: the lowest two levels,  $|0\rangle$ and  $|1\rangle$ , encode the qubit of eigenfrequency  $\omega_{10} = (E_1 - E_0)/\hbar$ . During the manipulations of the qubit, the third



FIG. 1 (color online). SCRAPs for inverting the qubit's logic states by certain pulse combinations: (left) a linear detuning pulse  $\Delta(t) = v_a t$ , combined with a constant Rabi pulse  $\Omega(t) = \Omega_a$ ; and (right) a linear detuning pulse  $\Delta(t) = v_b t$ , assisted by a Gaussian-shape Rabi pulse  $\Omega(t) = \Omega_b \exp(-t^2/T_R^2)$ . Here, the solid (black) lines are the expected adiabatic passage paths, and the dashed (red) lines represent the unwanted Landau-Zener tunneling paths.

bound state  $|2\rangle$  of energy  $E_2$  might be involved, as the difference between  $E_2 - E_1$  and  $E_1 - E_0$  is relatively small. Because of the broken mirror symmetry of the potential  $U(\delta)$  for  $\delta \rightarrow -\delta$ , bound states of this artificial atom lose their well-defined parities. As a consequence, all the electric-dipole matrix elements  $\delta_{ij} = \langle i | \delta | j \rangle$ , i, j =0, 1, 2, could be nonzero [10]. This is essentially different from the situations in most natural atoms or molecules, where all the bound states have well-defined parities and the electric-dipole selection rule forbids transitions between states with the same parity. By making use of this property, Fig. 2 describes a SCRAP with a single CBJJ by only applying an amplitude-controlled dc pulse  $I_{dc}(t)$  (to slowly chirp the qubit's transition frequency) and a microwave pulse  $I_{ac}(t) = A_{01}(t) \cos(\omega_{01}t)$  (to couple the qubit states). Under these two pulses, the Hamiltonian of the driven CBJJ reads  $\tilde{H}_1(t) = \tilde{H}_0 - \tilde{\Phi}_0 [I_{dc}(t) + I_{ac}(t)] \delta$ . Neglecting leakage, we then get the desirable Hamiltonian (1) with  $\Delta(t) = \tilde{\Delta}(t) = -\tilde{\Phi}_0 I_{dc}(t) (\delta_{11} - \delta_{00})$  and  $\Omega(t) = \tilde{\Omega}(t) = -\tilde{\Phi}_0 A_{01}(t) \delta_{01}$ . For a natural atom or molecule with  $\delta_{ii} = 0$ , the present scheme for producing a Stark shift cannot be applied.

For typical experimental parameters [11] ( $C_J = 4.3 \text{ pF}$ ,  $I_0 = 13.3 \ \mu\text{A}$ , and  $I_b = 0.9725I_0$ ), numerical calculations show that the energy splittings of the lowest three bound states in this CBJJ  $\omega_{10} = 5.98 \text{ GHz}$  and  $\omega_{21} = 5.64 \text{ GHz}$ . The electric-dipole matrix elements between these states are  $\delta_{00} = 1.4$ ,  $\delta_{11} = 1.42$ ,  $\delta_{22} = 1.450$ ,  $\delta_{01} = 0.053$ ,  $\delta_{12} = 0.077$ , and  $\delta_{02} = -0.004$ . If the applied dc pulse is a linear function of time [i.e.,  $I_{dc}(t) = v_1 t$  with  $v_1$  constant] and the coupling Rabi amplitude  $\Omega(t) = \Omega_1$  is fixed, the above SCRAP reduces to the standard Landau-Zener problem [13].

For a typical driving with  $v_1 = 0.15$  nA/ns and  $A_{01} = 1.25$  nA, Fig. 2 shows the time evolutions of the populations in this three-level system during the designed SCRAPs. The unwanted (but practically unavoidable) near-resonant transition between the chirping levels  $|1\rangle$  and  $|2\rangle$  (due to the small difference between  $\omega_{21}$  and



FIG. 2 (color online). SCRAP-based population transfers in a phase qubit. (Left) Manipulation scheme: CBJJ levels with dashed chirped qubit energy splitting  $\Delta(t)$  are coupled (solid arrow) by a Rabi pulse  $\Omega(t)$ . The dotted red arrow shows the unwanted leakage transition between the chirping levels  $|1\rangle$  and  $|2\rangle$ . (Right) Time evolutions  $P_j(t)$  of the occupation probabilities of the lowest three levels  $|j\rangle$  (j = 0, 1, 2) in a CBJJ during the SCRAPs for inverting the desirable SCRAPs the qubit leakage is negligible.

 $\omega_{10}$ ) has been considered. Figure 2 shows that during the above passages the leakage to the third state  $|2\rangle$  is sufficiently small. Thus, the above proposal of performing the desirable SCRAPs to implement single-qubit gates should be experimentally robust.

The adiabatic manipulations proposed above could also be utilized to read out the qubits. In the usual readout approach [11], the potential barrier is lowered fast to enhance the tunneling and subsequent detection of the logic state  $|1\rangle$ . Recently [14], a  $\pi$  pulse resonant with the  $|1\rangle \leftrightarrow$  $|2\rangle$  transition was added to the readout sequence for improved fidelity. The tunneling rate of the state  $|2\rangle$  is significantly higher than those of the qubit levels, and thus could easily be detected. The readout scheme used in [14] can be improved further by utilizing the above SCRAP by combining the applied microwave pulse and the biascurrent ramp. The population of state  $|1\rangle$  is then transferred to state  $|2\rangle$  with very high fidelity. In contrast to the above APs for quantum logic operations, here the population transfer for readout is not bidirectional, as the population of the target state  $|2\rangle$  is initially empty. The fidelity of such a readout could be very high, as long as the relevant AP is sufficiently fast compared to the qubit decoherence time.

SCRAPs could also be used to implement two-qubit gates in Josephson phase qubits. With no loss of generality, we consider a superconducting circuit [11] produced by capacitively coupling two identical CBJJs. The SWAP gate is typically performed by requiring that the two CBJJs be biased identically (yielding the same level structures) and the static interbit coupling between them reaches the maximal value  $K_0$ . If one waits precisely for an interaction time  $\tau = \pi/2K_0$ , then a two-qubit SWAP gate is produced [15]. In order to relax such exact constraints for the coupling procedure, we propose adding a controllable dc current,  $I_{dc}^{(2)}(t) = v_2 t$ , applied to the second CBJJ. Thus one can drive the circuit under Hamiltonian  $\bar{H}_{12}(t) = \sum_{k=1,2} H_{0k} +$  $\tilde{\Phi}_0^{-2} p_1 p_2 / \bar{C}_m - \tilde{\Phi}_0 I_{dc}^{(2)}(t) \delta_2$ . Here, the last term is the driving of the circuit, and the first term  $H_{0k} = \tilde{\Phi}_0^{-2} p_k^2 / (2\bar{C}_J) -$  $E_I \cos \delta_k - \tilde{\Phi}_0 I_b \delta_k$  is the Hamiltonian of the kth CBJJ with a renormalized junction capacitance  $\bar{C}_J = C_J(1 + C_J)$  $\zeta$ ), with  $\zeta = C_m/(C_J + C_m)$ . The coupling between these two CBJJs is described by the second term with  $\bar{C}_m^{-1} =$  $\zeta/[C_I(1+\zeta)]$  being the effective coupling capacitance. Suppose that the applied driving is not too strong, such that the dynamics of each CBJJ is still safely limited within the subspace  $\mathscr{O}_k = \{|0_k\rangle, |1_k\rangle, |2_k\rangle\}$ :  $\sum_{l=0}^2 |l_k\rangle \langle l_k| = 1$ . The circuit consequently evolves within the total Hilbert space  $\emptyset = \emptyset_1 \otimes \emptyset_2$ . Using the interaction picture defined by the unitary operator  $U_0 = \prod_{k=1,2} \exp(-it \sum_{l=0}^2 |l_k\rangle \langle l_k|),$ we can easily check that, for the dynamics of the present circuit, three invariant subspaces (relating to the computational basis) exist: (i)  $Im_1 = \{|00\rangle\}$  corresponding to the sub-Hamiltonian  $\bar{H}_1 = E_{00}(t)|00\rangle\langle 00|$  with  $E_{00}(t) =$  $-\tilde{\Phi}_0 I_{dc}^{(2)}(t) \delta_{00} + \tilde{\Phi}_0^{-2} p_{00}^2 / \bar{C}_m, \ p_{ll'} = \langle l_k | p_k | l'_k \rangle, \ \text{and} \ \delta_{ll'} = \langle l_k | \delta_k | l'_k \rangle; \ \text{(ii)} \ \text{Im}_2 = \{ |01\rangle, |10\rangle \} \ \text{corresponding to the}$ 



FIG. 3 (color online). SCRAPs within the invariant subspace Im<sub>3</sub> = { $|02\rangle$ ,  $|11\rangle$ ,  $|20\rangle$ } for the dynamics of two identical threelevel capacitively coupled CBJJs driven by an amplitudecontrollable dc pulse. (Left) Adiabatic energies and the desirable AP path (the middle solid line with arrows):  $A \rightarrow C_1 \rightarrow C_2 \rightarrow$  $C_3 \rightarrow B$ . (Right) Time evolutions of populations  $P_{\alpha}(t)$ ,  $\alpha = 20$ , 11, 02, within the invariant subspace Im<sub>3</sub> during the designed SCRAPs for inverting the populations of  $|10\rangle$  and  $|01\rangle$ . It is shown that the initial population of the  $|11\rangle$  state (corresponding to the *A* regime in the left figure) is adiabatically partly transferred to the two states  $|20\rangle$  and  $|02\rangle$  in the  $C_1$ ,  $C_2$ , and  $C_3$ regimes, respectively. Note that the population of the state  $|11\rangle$ vanishes at t = 0 and completely returns after the passages.

sub-Hamiltonian  $\bar{H}_2(t)$  taking the form of Eq. (1) with  $\Omega(t) = \bar{\Omega} = 2\tilde{\Phi}_0^{-2}p_{10}^2/\bar{C}_m$  and  $\Delta(t) = \bar{\Delta}(t) = \tilde{\Phi}_0 I_{dc}^{(2)}(t) \times (\delta_{11} - \delta_{00})$ ; and (iii) Im<sub>3</sub> = { $|02\rangle = |a\rangle$ ,  $|11\rangle = |b\rangle$ ,  $|20\rangle = |c\rangle$ } corresponding to

$$\bar{H}_{3}(t) = \begin{pmatrix} E_{a}(t) & \Omega_{ab}e^{-it\vartheta} & \Omega_{ac} \\ \Omega_{ba}e^{it\vartheta} & E_{b}(t) & \Omega_{bc}e^{-it\vartheta} \\ \Omega_{ca} & \Omega_{cb}e^{it\vartheta} & E_{c}(t) \end{pmatrix},$$

with  $E_a(t) = -\tilde{\Phi}_0 I_{dc}^{(2)}(t) \delta_{22} + \tilde{\Phi}_0^{-2} p_{00} p_{22}/\bar{C}_m$ ,  $E_b(t) = -\tilde{\Phi}_0 I_{dc}^{(2)}(t) \delta_{11} + \tilde{\Phi}_0^{-2} p_{11}^2/\bar{C}_m$ ,  $E_c(t) = -\tilde{\Phi}_0 I_{dc}^{(2)}(t) \delta_{00} + \tilde{\Phi}_0^{-2} p_{22} p_{00}/\bar{C}_m$ ;  $\Omega_{ab} = \Omega_{ba} = \Omega_{bc} = \Omega_{cb} = \tilde{\Phi}_0^{-2} p_{01} p_{12}/\bar{C}_m$ ,  $\Omega_{ac} = \Omega_{ca} = \tilde{\Phi}_0^{-2} p_{02}^2/\bar{C}_m$ , and  $\vartheta = \omega_{10} - \omega_{21}$ . Under the APs for exchanging the populations of the states  $|10\rangle$  and  $|01\rangle$ , we can easily see that the population of  $|00\rangle$  remains unchanged. Also, after the desired APs, the population of the state  $|11\rangle$  should also be unchanged. Indeed, this is verified numerically in Fig. 3 for the typical parameters  $\zeta = 0.05$  and  $v_2 = 3.0$  nA/ns. Therefore, the desirable two-qubit SWAP gate could also be effectively produced by utilizing the proposed SCRAPs.

Discussions and conclusions.—By using SCRAPs, we have shown that populations could be controllably transferred between selected quantum states, insensitive to the details of the applied adiabatic pulses. Assisted by readily implementable single-qubit phase-shift operations, these adiabatic population transfers could be used to generate universal logic gates for quantum computing. Experimentally existing superconducting circuits were treated as a specific example to demonstrate the proposed approach.

Like other RAPs, the adiabatic nature of the present SCRAPs requires that the passages should be sufficiently slow (compared to the usual Rabi oscillations) and fast (compared to the decoherence times of the qubits). Satisfying both conditions simultaneously does not pose any serious difficulty with typical experimental parameters. Indeed, as shown above, experimentally feasible APs could be applied within tens of nanoseconds. This time interval is significantly longer than the typical period of an experimental Rabi oscillation, which usually does not exceed a few nanoseconds, and could be obviously shorter than the typical decoherence times of existing qubits, which might reach hundreds of nanoseconds, e.g., for the phase qubits reported in [11].

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