Simultaneous Cooling of an Artificial Atom and Its Neighboring Quantum System

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We propose an approach for cooling both an artificial atom (e.g., a flux qubit) and its neighboring quantum system, the latter modeled by either a quantum two-level system or a quantum resonator. The flux qubit is cooled by manipulating its states, following an inverse process of state population inversion, and then the qubit is switched on to resonantly interact with the neighboring quantum system. By repeating these steps, the two subsystems can be simultaneously cooled. Our results show that this cooling is robust and effective, irrespective of the chosen quantum systems connected to the qubit.

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Quantum devices using Josephson junctions can be used as artificial atoms (AAs) for demonstrating quantum phenomena at macroscopic scales. With states involving the two lowest energy levels, these devices are good candidates for solid-state qubits [1]. When using their three lowest levels, such a solid-state three-level system, fabricated on a microelectronic chip, can be useful for single-photon production [2] and lasing [3].

For single-photon production [2] and AA lasing [3], a state population inversion is established for the two working energy levels via a third one (i.e., transitions $|0\rangle \rightarrow |2\rangle \rightarrow |1\rangle$ in Fig. 1). Interestingly, the inverse process of state population inversion (i.e., transitions $|1\rangle \rightarrow |2\rangle \rightarrow |0\rangle$ in Fig. 1) can be used to increase the occupation probability of the ground state and thus lower the temperature of the qubit. This idea has been applied in a recent experiment [4] to cool a flux qubit. Indeed, this is analogous to the optical sideband cooling method studied earlier (see, e.g., [5,6]). The experiment [4] shows that the temperature of the flux qubit can be lowered by up to 2 orders of magnitude with respect to its surroundings. This provides an efficient approach for preparing a flux qubit in its ground state.

While the flux qubit was greatly cooled in [4], the noise sources surrounding the qubit were not. This is because of the weak coupling between the qubit and its environment in [4], where the transition rate between the ground and first excited states is small. Below we use a tunable AA (to be specific, we choose a flux qubit, but it could be another AA) to achieve a strong and switchable coupling between the AA and its neighboring quantum system, and propose an approach to simultaneously cool both of them and not just the AA. Here we consider two typical quantum systems to describe the environment surrounding the AA: (i) a quantum two-level system (TLS), which is exactly solvable, and (ii) a quantum resonator. Actually, a quantum TLS can describe the noise source like a two-level fluctuator, and the quantum resonator can model the dominant bosons of a thermal bath. In this case, the approach is to cool both the flux qubit and such noise sources. This simultaneous cooling of the flux qubit and its neighboring noise sources can significantly enhance the quantum coherence of the flux qubit because the cooled qubit is thermally activated very slowly to the first excited state, after its neighboring noise sources are also cooled. Moreover, the present approach has wide applications because the models used here can describe other quantum systems. Also, we show that different surrounding quantum systems (either a quantum TLS or a quantum resonator) give similar results, implying that the cooling is robust and effective, irrespective of the chosen neighboring quantum system.

Cooling the artificial atom and ground-state preparation.—The commonly used flux qubit [7,8] (which is an example for an AA) consists of a superconducting loop interrupted by three Josephson junctions (two equal and one smaller) and pierced by a magnetic flux Φ_e. To obtain a tunable AA, the smaller junction is here replaced by a SQUID threaded by a flux Φ_s [see Fig. 1(a)]. The Hamiltonian can be written as $H = P_p^2/2M_p + P_q^2/2M_q + U(\varphi_p, \varphi_q)$, with $P_i = -i\hbar\partial/\partial\varphi_i$ $(i = p, q)$, $M_p = 2C_J(\Phi_0/2\pi)^2$, and $M_q = M_p(1 + 4\gamma)/4$. The potential is $U(\varphi_p, \varphi_q) = 2E_J[1 - \cos\varphi_p \cos(\pi f + \frac{1}{2}\varphi_q)] + 2\gamma E_J[1 - \cos(\pi f_s)\cos\varphi_q)$, where $\varphi_p = (\varphi_1 + \varphi_2)/2$, $\varphi_q = (\varphi_3 + \varphi_4)/2$, $f_s = \Phi_s/\Phi_0$, and $f = \Phi_e/\Phi_0 + f_s/2$ (Φ_0 is the flux quantum). To drive a resonant transition between states $|E_i\rangle$ and $|E_j\rangle$, one can apply a microwave field through the circuit loop: $\Phi_w(t) = \Phi_w^{(0)} \cos(\omega_{ij}t + \theta)$, with $\omega_{ij} = (E_i - E_j)/\hbar$. When the microwave field is weak, the time-dependent perturbation Hamiltonian can be written as $V(t) = -I\Phi_w(t)$, where $I = -I_c \cos\varphi_p \sin(\pi f + \frac{1}{2}\varphi_q)$, with $I_c = 2\pi E_J/\Phi_0$. The rate of the state transition between $|E_i\rangle$ and $|E_j\rangle$ is $\Gamma_{ij} \propto |t_{ij}|^2$, where $t_{ij} = \langle E_i|I\Phi_w^{(0)}|E_j\rangle$ is the transition matrix element. When a neighboring quantum system, e.g., a noise source, is coupled to the flux qubit via a flux variation, then $\Phi_w^{(0)}$ in t_{ij} becomes the amplitude of the flux variation.

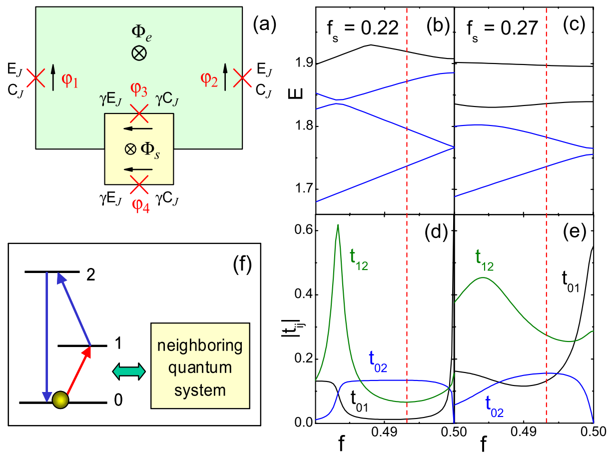

FIG. 1 (color online). (a) Schematic diagram of an artificial atom (AA) produced by a superconducting quantum circuit. A symmetric SQUID and two identical Josephson junctions with coupling energy E_J and capacitance C_J are placed in a superconducting loop pierced by a magnetic flux Φ_e (green or gray). The two junctions in the SQUID have coupling energy γE_J and capacitance γC_J, and the flux (yellow or light gray) threading through the SQUID loop is Φ_s. Here $\gamma = 0.5$, $E_J/E_c = 100$, and $E_c = e^2/2C_J$ is the single-particle charging energy of the junction. (b),(c) Energy levels of the superconducting AA as a function of the reduced magnetic flux $f = \Phi_e/\Phi_0 + f_s/2$, for $f_s \equiv \Phi_s/\Phi_0 = 0.22$ and 0.27, where only the four lowest levels are shown and the energy is in units of E_J. (d),(e) Moduli of the transition matrix elements $|t_{ij}|$ (in units of $I_c\Phi_w^{(0)}$) as a function of f, for $f_s = 0.22$ and 0.27. Note that each figure in (b)–(e) is symmetric about $f = 0.5$ and half of it is plotted. The vertical dashed lines at $f = 0.493$ are just a guide to the eye. (f) Transition diagram of the AA. At nonzero temperatures, the flux qubit is thermally activated from the ground state $|0\rangle$ to the first excited state $|1\rangle$. A resonant transition from $|1\rangle$ to the second excited state $|2\rangle$ is driven by a microwave field, so as to eliminate the unwanted thermal population of $|1\rangle$, and followed by a fast decay to $|0\rangle$. While the qubit is cooled to its ground state $|0\rangle$, the AA is then switched on, to resonantly interact with a neighboring quantum system for a period of time. Repeating these processes, both the qubit and the neighboring quantum system can be simultaneously cooled.

In Figs. 1(b)–1(e), we show the energy levels of the AA and the transition matrix elements $|t_{ij}|$ for two values of f_s. The SQUID gives an effective Josephson coupling energy αE_J with $\alpha = 0.77$ and 0.66, respectively. For any nonzero temperature, the system will be thermally activated from the ground state $|0\rangle \equiv |E_0\rangle$ to the first excited state $|1\rangle \equiv |E_1\rangle$. Here we consider the case in Fig. 1(b), with the system working at, e.g., $f = 0.493$. As shown in Fig. 1(d), at this f, the corresponding transition matrix elements are $|t_{01}| \approx 0.01$, $|t_{12}| \approx 0.07$, and $|t_{02}| \approx 0.13$. When a microwave field is applied to drive a resonant transition $|1\rangle \rightarrow |2\rangle \equiv |E_2\rangle$, because $\Gamma_{20} > \Gamma_{21} \gg \Gamma_{10}$ at $f \sim 0.493$, the system can be pumped from $|1\rangle$ to $|2\rangle$ and then quickly decays to the ground state $|0\rangle$, while the process for thermally activating the system from $|0\rangle$ to $|1\rangle$, via coupling to the environment, will be very slow. Note that the coupling strength of the states $|2\rangle$ and $|0\rangle$ to the flux noise source is also proportional to the transition matrix element $|t_{20}|$, so the decay rate from $|2\rangle$ to $|0\rangle$ is proportional to Γ_{20} ($\propto |t_{20}|^2$) according to the Fermi golden rule [9]. Therefore, the flux qubit is "cooled" because the population probability for the ground state $|0\rangle$ can be greatly increased, with respect to any unwanted excited state $|1\rangle$. Interestingly, this cooling mechanism corresponds to an "inverse process" of the usual state population inversion. For simplicity, here we use a weak microwave field. The driving field would need to be stronger to achieve cooling when the relevant transition matrix elements are small. This puts some constraints on the specific amplitudes used to achieve the desired result [10]. Indeed, a recent experiment [4] has successfully realized the microwave-induced cooling, lowering the temperature of a flux qubit relative to its surroundings. Thus, this microwave-induced cooling provides an efficient method for preparing the flux qubit in its ground state. Below we use this prepared ground state to further cool a quantum system connected to the qubit.

Cooling a quantum two-level system.—In the subspace spanned by $|0\rangle$ and $|1\rangle$, the flux qubit (our AA) is modeled by $H_q = \frac{1}{2}\hbar\omega_{10}\sigma_z$. Here we consider a qubit-TLS system described by $H_t = H_q + H_{\rm TLS} + V + H_{\rm env}$, where $H_{\rm TLS} = \frac{1}{2}\hbar\Omega\sigma_z'$ is the Hamiltonian of a quantum TLS and $H_{\rm env}$ describes all the degrees of freedom in the environment and their coupling to the TLS. Hereafter, the Pauli operators with primes refer to the neighboring TLS. The interaction Hamiltonian between the qubit and the TLS is $V = \hbar g(\sigma_+\sigma_-' + {\rm H.c.})$, with $g = |t_{01}|/\hbar$. In the experimental case [4], corresponding to Fig. 1(d), because $|t_{01}|$ is small at $f \sim 0.493$, the coupling between the qubit and its environment is weak. To cool the TLS effectively, after the qubit with $f_s = 0.22$ is cooled to the ground state, we change the reduced magnetic flux f_s to $f_s = 0.27$, which corresponds to Fig. 1(e). For the qubit parameters used here, it is shown [2] that at $f \sim 0.493$ (i.e., in between the level-crossing points), the adiabatic condition $|\hbar\langle E_i|(d/dt)|E_j\rangle/(E_i - E_j)| \ll 1$ can still be fulfilled for the three lowest levels by changing the applied flux as fast as $0.1\Phi_0$ ns^{-1}. This means that around this f the quantum states can be well preserved even when changing the flux very fast. More importantly, in the case of Fig. 1(e), because $|t_{01}|$ is much increased, then the qubit-TLS interaction $\hbar g$ is strengthened by 1 order of magnitude. Here we assume that the quantum TLS is resonant to the qubit with $f_s = 0.27$. Since the level spacing $\hbar\omega_{01}$ of the qubit with $f_s = 0.22$ is different from that with $f_s = 0.27$, thus at $f \sim 0.493$ the qubit with $f_s = 0.22$ is off-resonant to the TLS. This gives an even smaller effective qubit-TLS coupling.

For simplicity, we now assume that the flux qubit is ideally cooled to the ground state $|0\rangle$ and then begins to resonantly interact with the quantum TLS at time t_i. When $H_{\rm env}$ is not included, the time evolution of the density operator of the TLS is governed by $\rho(t_i + \tau) = M(\tau)\rho(t_i)$

and the gain operator is defined by $M(\tau)\rho = \mathrm{Tr}[\exp(-iV\tau/\hbar)\rho \otimes |0\rangle\langle 0| \exp(iV\tau/\hbar)]$, where Tr denotes the trace over the qubit states and τ is the interaction time between the TLS and the flux qubit.

When $H_{\rm env}$ is considered, the dynamics of the density operator is described by [11]

$$\frac{d\rho}{dt} = r_a \ln[M(\tau)]\rho + L\rho, \qquad (1)$$

where r_a is the rate for "switching on" the AA to resonantly interact with the TLS (each cycle includes the time required to cool the qubit) and L describes the dissipation of the TLS due to $H_{\rm env}$. We model the environment in $H_{\rm env}$ by a thermal bath. The operator L can be written as [12] $L\rho = -\frac{1}{2}\kappa(n_{\rm th}+1)(\sigma'_+\sigma'_-\rho - \sigma'_-\rho\sigma'_+) - \frac{1}{2}\kappa n_{\rm th}(\sigma'_-\sigma'_+\rho - \sigma'_+\rho\sigma'_-) + \mathrm{H.c.}$, where κ is the decay rate of the TLS and $n_{\rm th}$ is the average number of bosons in the thermal bath (particularly, $n_{\rm th} = 0$ at zero temperature). Here we assume $g > \kappa$, ensuring coherence between the qubit and its ancillary circuitry.

For the neighboring quantum TLS, Eq. (1) can be exactly solved. The solution for $p_e \equiv \langle e|\rho|e\rangle$ is

$$p_e(t) = \left[p_e(0) - \frac{n_{\rm th}}{\Lambda}\right]\exp(-\Lambda\kappa t) + \frac{n_{\rm th}}{\Lambda}, \qquad (2)$$

and $p_g \equiv \langle g|\rho|g\rangle = 1 - p_e$, where $\Lambda = (2n_{\rm th}+1) - N_t \ln[\cos^2(g\tau)]$, with $N_t = r_a/\kappa$ denoting the number of cycles for switching on the AA during the lifetime ($\equiv 1/\kappa$) of the TLS. Because of the coupling to the qubit, the decay rate is now scaled by a factor Λ. Clearly, $p_e = n_{\rm th}/\Lambda$ and $p_g = 1 - n_{\rm th}/\Lambda$ at steady state.

Because Λ is a periodic function of $g\tau$, both p_e and p_g are also periodic; e.g., at $g\tau = (2n-1)\pi/2$, with $n = 1, 2, \ldots$, $\Lambda \to +\infty$ and p_g abruptly changes to $p_g = 1$; at $g\tau = n\pi$, with $n = 0, 1, \ldots$, $\Lambda = 2n_{\rm th}+1$ and p_g slowly approaches $p_g = 1 - n_{\rm th}/(2n_{\rm th}+1)$. These features are clearly shown in Fig. 2(a) for p_g with $N_t = 150$. To implement an efficient cooling, a smaller τ is desirable, so we can only focus on the region $g\tau \in [0, \pi/2]$. Figure 2(b) shows the time evolution of p_g as a function of N_t for $g\tau = 0.2\pi$. Though $g\tau$ is away from $g\tau = \pi/2$, one can still drastically cool the TLS by evolving $p_g(t)$ to $p_g \sim 1$ with a large N_t.

Cooling a quantum resonator.—When the system connected to the flux qubit is a quantum resonator, the total Hamiltonian becomes $H_t = H_q + H_{\rm res} + V + H_{\rm env}$, where $H_{\rm res} = \hbar\omega a^\dagger a$ describes the quantum resonator, $V = -\hbar g(\sigma_+ a + \mathrm{H.c.})$ is the interaction between them, and $H_{\rm env}$ describes all the degrees of freedom in the environment and their coupling to the quantum resonator. Also, we assume that when cooling the quantum resonator the flux qubit is tuned in resonance to it.

For the quantum resonator coupled to the flux qubit as well as to a thermal bath, the dynamics of the density operator of the quantum resonator is also described by Eq. (1). The operator L describes the dissipation of the quantum resonator induced by the thermal bath [12]: $L\rho = -\frac{1}{2}\kappa(n_{\rm th}+1)(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger) - \frac{1}{2}\kappa n_{\rm th}(aa^\dagger\rho + \rho aa^\dagger - 2a^\dagger\rho a)$, where κ is the damping rate of the quantum resonator and $n_{\rm th}$ is the average number of bosons in the thermal bath coupled to the quantum resonator. In the present case, Eq. (1) can only be solved approximately. Here we use $\ln[M(\tau)] \approx (M-1) - \frac{1}{2}(M-1)^2$, which corresponds to neglecting terms of order $O(\sin^6(g\tau\sqrt{n}))$. The equation of motion for the boson number distribution $p_n = \langle n|\rho|n\rangle$ of the quantum resonator becomes

$$\frac{dp_n}{dt} = a_{n+1}p_{n+1} - b_{n+1}p_{n+2} - c_{n+1}p_n - a_n p_n + b_n p_{n+1} + c_n p_{n-1}, \qquad (3)$$

with $a_n = r_a S(n)[1 + \frac{1}{2}S(n)] + \kappa(n_{\rm th}+1)n$, $b_n = \frac{1}{2}S(n) \times S(n+1)$, and $c_n = \kappa n_{\rm th} n$, where $S(n) = \sin^2(g\tau\sqrt{n})$.

At steady state, $dp_n/dt = 0$, which leads to a recursion relation for the steady boson number distribution p_n:

$$p_{n-1} = p_n\left\{\frac{n_{\rm th}+1}{n_{\rm th}} + \frac{N_t S(n)[2+S(n)]}{2nn_{\rm th}}\right\} - p_{n+1}\frac{N_t S(n)S(n+1)}{2nn_{\rm th}}, \qquad (4)$$

where $N_t = r_a/\kappa$ represents the number of cycles for switching on the AA during the lifetime of the quantum resonator. For $N \gg 1$, $p_N(n_{\rm th}+1) \gg p_{N+1}N_t S(N)S(N+1)/2N$, so we approximately have $p_{N-1} = \{(n_{\rm th}+1)/n_{\rm th} + N_t S(N)[2+S(N)]/2Nn_{\rm th}\}p_N$. This is the initial condition for Eq. (4) and p_N is determined by $\sum_{n=0}^{N} p_n = 1$.

Figure 3 displays the time evolution of both vacuum-state probability p_0 and average boson number $\langle n\rangle$ as a

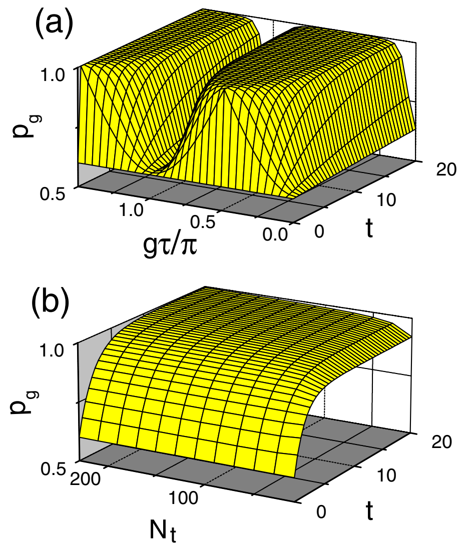

FIG. 2 (color online). (a) Ground-state probability p_g versus time t (in units of $1/r_a$) and $g\tau$ for $N_t \equiv r_a/\kappa = 150$. (b) Probability p_g versus t and N_t for $g\tau = 0.2\pi$. Here $n_{\rm th} = 0.5$ and p_e is chosen to be 0.4 at the initial time $t = 0$; g (τ) is the interaction strength (time).

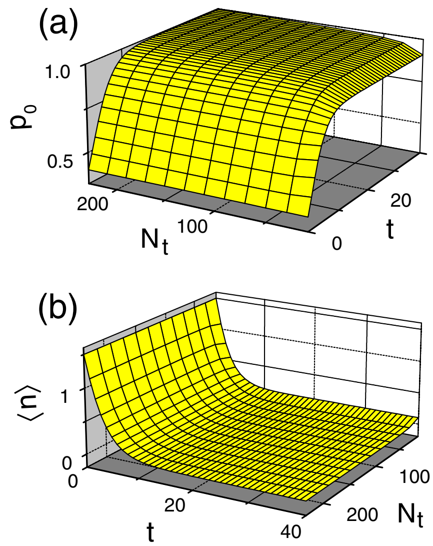

FIG. 3 (color online). (a) Vacuum-state probability p_0 versus time t (in units of $1/r_a$) and $N_t \equiv r_a/\kappa$. (b) Average boson number $\langle n \rangle$ versus t and N_t. Here $n_{\rm th} = 0.5$, $g\tau = 0.2\pi$, and $\langle n \rangle$ is chosen to be 1.5 at the initial time $t = 0$.

function of N_t for the quantum resonator, where $g\tau = 0.2\pi$. As shown in Fig. 3(a), one can evolve $p_0(t)$ to $p_0 \sim 1$ using a large N_t. Figure 3(b) shows that $\langle n \rangle \sim 0$ when $p_0 \sim 1$, revealing that the quantum resonator can also be effectively cooled. More interestingly, Figs. 2(b) and 3(a) give quite similar results, although very different models are used for the quantum systems connected to the qubit. This reveals that the cooling is robust and effective, irrespective of the chosen neighboring quantum systems.

Discussion and conclusion.—The cooling approach studied here has potentially wide applications. For instance, the environmental noise is sometimes explained as mainly due to two-level fluctuators, in which one or a few fluctuators play a dominant role. Also, the environment is often modeled by a boson bath, in which the bosons in resonance to the qubit play a dominant role. Here the quantum TLS can be used to model a two-level fluctuator and the quantum resonator can be used to model the dominant bosons of the environment in resonance to the qubit. Actually, the TLS defect that is most strongly coupled to the qubit may be off resonant to the qubit. If the off resonance is large, the effect of the TLS on the qubit is not important. Otherwise, to cool the TLS defect, one can vary the reduced flux f to tune the qubit to be in resonance with the defect. Also, when the environmental bosons in resonance with the qubit are cooled, one can tune the qubit by changing f to further cool the off-resonant bosons. After cooling the dominant noise sources of the qubit, the quantum coherence of the cooled flux qubit will be enhanced. The quantum TLS can also model a solid-state qubit and the approach can be used to describe cooling two coupled qubits. Naturally, the quantum resonator can model a mechanical resonator at the nanometer scale. The cooling of mechanical resonators is currently a popular topic and its study provides opportunities to observe the transition between classical and quantum behaviors of a mechanical resonator [13]. In our proposal, the quantum states can be manipulated quickly, due to the advantages of the proposed solid-state three-level system. Moreover, the cooling of both the flux qubit and the mechanical resonator can simultaneously enhance the quantum behaviors of the two subsystems. This will help observe the transition between classical and quantum behaviors of the mechanical resonator via measuring the quantum states of the qubit.

In conclusion, we have proposed an approach to simultaneously cool a flux qubit and its neighboring quantum system. In each cycle of cooling, the flux qubit is first prepared to the ground state, following an inverse process of the state population inversion, and then switched on to resonantly interact with the neighboring quantum system. As typical examples, we model the quantum system connected to the qubit by either a TLS or a resonator. Our results show that the cooling is robust and effective, irrespective of the chosen quantum systems.

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