Model of coherent emission from disordered arrays of driven Josephson vortices

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We propose a mechanism of coherent emission from driven vortices in stacked intrinsic Josephson junctions. In contrast to super-radiance, which occurs only for *highly ordered* vortex lattices, we predict resonant radiation emission from *weakly correlated* vortex arrays. Our analytical results for the terahertz wave intensity, resonance frequencies, and the dependence of terahertz emission power on dissipation are in good agreement with the ones obtained by recent simulations.

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I. INTRODUCTION

It has been experimentally observed[1–3] and confirmed both analytically[4,5] and numerically[6,7] that moving Josephson vortices (JVs) emit sub-terahertz electromagnetic radiation. Terahertz radiation has applications in physics, astronomy, chemistry, biology, and medicine.[8] This motivates recent proposals[9] for terahertz filters,[10,11] detectors,[12] quantum devices,[13,14] and emitters[5,6] based on highly anisotropic layered superconductors (e.g., $Bi_2Sr_2CaCu_2O_{8+\delta}$), that can be modeled as coupled intrinsic Josephson junctions (IJJs).

The ultimate challenge in this field is to produce coherent terahertz radiation. It is commonly believed that this goal can be achieved by controlling super-radiance from highly ordered vortex lattices.[15,16] A vortex lattice is deemed necessary because the constructive interference of Josephson plasma waves from individual JVs is strongly suppressed by small amounts of disorder. Unfortunately, driven periodic lattices are often very unstable (especially in the presence of impurities, defects, and pinning centers), and moving JVs form either a mixture of coexisting different lattices[17] or even disordered arrays.[6] Moreover, a broad radiation spectrum by individual vortices results in a broad spectrum of the emitted radiation (e.g., Ref. 5), in contrast to a desirable resonant IJJ, where coherent radiation is characterized by sharp spectral lines.

In this context, recent interesting simulations by Tachiki and co-workers[6] show that coherent radiation may be generated by JVs moving as disordered arrays, instead of just ordered ones. This raises the question as under what conditions JVs in layered superconductors emit coherent radiation. Solving this problem is crucial to the effective design of $Bi_2Sr_2CaCu_2O_{8+\delta}$-based terahertz emitters.

The experimental demonstration[3] of terahertz radiation in zero magnetic field and various failed attempts at detecting terahertz emission in the presence of magnetic fields cast serious doubts on the initial idea that moving JVs can radiate in this frequency domain. Indeed, the now prevailing interpretation is that JVs ought to be considered as perturbing degrees of freedom, which destroy the layer coherence and thus cause the suppression of terahertz radiation. In this study, however, we reach the conclusion that, under appropriate conditions, applied magnetic fields do help amplify and tune terahertz emission. This interesting result is also consistent with the recent systematic studies in Ref. 18.

Below, we show that the *nonlocal* nature of JVs in layered superconductors is responsible for a two-scale dynamics. A longer scale, λ_{EM}, characterizes the intervortex magnetic interaction; spatial dispersion, or disorder, of vortices up to such a scale has *no* appreciable impact on the radiation mechanism. In other words, in contrast with super-radiance, which is suppressed by vortex disorder, in our approach radiation coherence is preserved even if the vortex distribution can become appreciably modulated by the radiation itself for wavelengths shorter than λ_{EM}. The shorter relevant length scale λ_G determines the cross section of the nonlinear vortex core (where the linear approximation $\sin\varphi \approx \varphi$ with the gauge-invariant phase difference across the junction having a vortex is not valid). According to this picture, for $\lambda_G \ll \lambda_{EM}$ and for a sufficiently high vortex density, radiation is emitted through a linear mechanism, as from a JV lattice whereas the vortex-radiation coupling occurs mainly in the inner JV cores. Under these conditions, the magnetic interaction among vortices is much weaker than their interaction with the emitted radiation. Our approach explains the spatial modulation of the JV density numerically found in Refs. 6 and 7. Our analytical estimates, based on a one-dimensional (1D) sine-Gordon model, prove to be in good agreement with their simulations and explain their results.

Let us now summarize a central idea of our approach. Consider a moving JV lattice emitting radiation. This radiation will bounce back and forth the sample edges, like in a laser cavity. This radiation accumulates and creates a standing wave with a wavelength about λ_{EM}. This standing wave modulates the JV density which is now in resonance with the standing wave. This positive feedback enhances the radiation of vortices. Namely, the JV motion emits radiation, which is weaker at first. This radiation bounced inside the sample (acting as a cavity) locks the collective motion of the JVs. This collective motion produces stronger emission. The JVs then interact more strongly with the electromagnetic standing wave, compared with the now much weaker vortex-vortex interaction. Thus, the triangular vortex lattice, produced by the vortex-vortex interaction, is finally replaced by a more disordered but still modulated by the radiation and vortex structure.

II. NONLOCAL SINE-GORDON MODEL

Layered superconductors can be considered as stacks of strongly interacting IJJs. As the superconducting layers are only a few nanometers thick, i.e., the interlayer distance s is much smaller than the magnetic field penetration depth λ_{ab}, the currents flowing through different junctions are coupled. On neglecting, for the time being, external drives and internal dissipation, a system of stacked IJJs is well described by the coupled sine-Gordon equations,[19]

$$\left(1-\frac{\lambda_{ab}^2}{s^2}\Delta_n^2\right)\left(\frac{\varphi_{tt}^{(n)}}{\omega_p^2}+\sin\varphi^{(n)}\right)-\lambda^2\varphi_{xx}^{(n)}=0, \qquad (1)$$

where $\varphi^{(n)}$ is the gauge-invariant phase difference across the nth junction. Here, ω_p is the Josephson plasma frequency, λ the London penetration depth ($\lambda/\lambda_{ab}=\gamma$) along the layers, and the operator Δ_n^2 is defined by $\Delta_n^2 f=f^{(n+1)}-2f^{(n)}-f^{(n-1)}$.

A full analysis of this set of equations is a complicated problem which requires numerical simulation. However, if we restrict ourselves to the case of moderate magnetic fields, when JV cores do not overlap, we can reduce Eq. (1) to an effective 1D problem. Indeed, as was shown in Ref. 20 [see Eqs. (21) and (22) and Fig. 2 there], the phase difference φ decreases very fast away from the junction where a vortex located. Thus, a reasonable strategy could consist in neglecting the nonlinear couplings between junctions at a distance of some s from the vortex center; on solving the linearized Eq. (1) for such junctions, one would end up with a few coupled nonlinear equations for a few junctions in the vicinity of the vortex center. The case when the nonlinearity was restricted to one junction only has been considered in Ref. 5. The coupled junction system of Eq. (1) boils down to a 1D Josephson junction described by a nonlocal sine-Gordon equation. We assume below that retaining the nonlinear coupling between more junctions can lead to the same nonlocal 1D sine-Gordon equation with additional noiselike weak perturbations.

For simplicity, let us consider the pair of adjacent junctions j and $(j+1)$ locating a moving JV. We then reduce the description of the IJJ stack to a 1D problem by assuming the nonlinear coupling to be important only for the paired junctions and linearizing Eq. (1) for all other junctions (i.e., for $n\neq j,\ j+1$). It is interesting to note that the importance of the interaction between two neighboring junctions is numerically well established.[21] Moreover, Koshelev[22] recently reduced the multijunction system to two coupled junctions, and this model reproduced the simulation data in Ref. 21 and interpreted the experimental results of Ref. 3.

Following the approach in Refs. 5, 23, and 24, the equation for the averaged phase difference across a junction pair $\varphi=(\varphi^{(j+1)}+\varphi^{(j)})/2$ can be written as

$$\frac{\varphi_{tt}}{\omega_p^2}+\sin\varphi=\frac{\gamma s}{2\pi}\int dx' K_0\left(\frac{|x-x'|}{\lambda}\right)\varphi_{xx}(x')+P[\psi]\sin\varphi, \qquad (2)$$

where K_0 is the modified Bessel function and

$$P[\psi]=1-\cos\psi$$

with $\psi=(\varphi^{(j+1)}-\varphi^{(j)})/2$. The length

$$\lambda_G\equiv\frac{\gamma s}{2}=\frac{\lambda_{EM}^2}{\lambda} \qquad (3)$$

defines the size of the JV core. Again, the contribution of the next-to-neighbor junctions to the dynamics of the tagged JV weakens fast[20] with their distance from the vortex center, thus, allowing all other nonlinear Eq. (1) to be replaced by an effective nonlinear medium.

An additional equation for ψ can be derived for a pair of JVs in two adjacent junctions so that the equations for φ and ψ form a closed set.[2,5] However, when extending Eq. (2) to describe the collective motion of N traveling JVs (randomly distributed along N_l stacked IJJs of length L), the phase φ can be regarded as a mean-field superposition of the $n=N/N_l$ individual JVs phases, $\varphi_v^{(i)}$, contained in one layer, only.

In our one-IJJ description, we assume that ψ is relatively small; this may be the case, for instance, due to the random superposition of the vortex dynamics in different junctions. Anyway, the good agreement between the analytical results reported here and earlier numerical simulations, validates *a posteriori* our assumption. Thus, the functional $P[\psi]$ can be modeled as a spatial perturbation $\delta+\epsilon P(x)$, where the real function $P(x)$ can be either periodic or random in x, depending on the operating conditions. The constant ϵ is a measure of the strength of the perturbation while the small offset δ can be conveniently eliminated by rescaling the dimensional parameters ω_p and λ_G, as appropriate. Here, we assimilate such a perturbation as an effective quenched Gaussian disorder along the IJJs; that is, $P(x)$ is modeled as a random, delta-correlated function with

$$\langle P(x)\rangle=0, \quad \langle P(x)P(x')\rangle=2\delta(x-x') \qquad (4)$$

and $\langle\cdots\rangle$ denoting the average over different disorder realizations. The constant ϵ will be taken as a perturbation parameter and only effects to leading order in ϵ will be considered. Moreover, deviations from the Gaussian statistics, implicit in the definition of $P[\psi]$, are assumed to be negligible within this approximation.

A. Josephson vortex array

We now introduce dimensionless units by expressing x and t in units of the characteristic length λ_{EM} and the reciprocal of the plasma frequency ω_p, respectively, that is,

$$x\to\tilde{x}=x/\lambda_{EM}, \quad t\to\tilde{t}=\omega_p t.$$

As a consequence, the system characteristic lengths λ and λ_G get rescaled as follows:

$$\lambda \to \tilde{\lambda} = \lambda/\lambda_{EM}, \quad \lambda_G \to \tilde{\lambda}_G = \lambda_{EM}/\lambda = 1/\tilde{\lambda},$$

and, of course, $\lambda_{EM} \to \tilde{\lambda}_{EM} = 1$. Correspondingly, $\varphi(x,t)$ is given in units of the magnetic-flux quantum ϕ_0 and all speeds in units of $\omega_p\lambda$. Hereafter, for the sake of simplicity, we shall only use dimensionless variables and, therefore, omit the "tilde" notation altogether.

The field $\varphi(x,t)$, corresponding to a dense distribution of JVs traveling with speed $V \ll 1$, can be expanded as[25,26]

$$\varphi(x,t) = p(x - Vt) - \kappa \sin[p(x - Vt)] + \cdots, \tag{5}$$

where $p = 2\pi\rho$ and $\rho = n/L$ denotes the linear JV density with number n of vortices located along the length L. The linear term in Eq. (5) corresponds to the phase difference of a uniform vortex spatial distribution whereas the periodic correction accounts for a residual phase modulation on the lattice scale $1/\rho$, with amplitude κ to be determined self-consistently. In the expansion (5), we assume high JV densities, $p \gg 1$, and small amplitudes κ. On inserting expansion (5) for $\varphi(x,t)$, the nonlocal field Eq. (2) can be approximated to an effective sine-Gordon equation, where

$$\frac{1}{\pi\lambda}\int K_0\left(\frac{|x - x'|}{\lambda}\right)\varphi_{xx}(x')dx' \to {c_p}^2\varphi_{xx},$$

$$c_p^2 = [1 + (p\lambda)^2]^{-1/2}, \tag{6}$$

and, consistently,

$$\kappa = (\gamma_V/c_p p)^2$$

with

$$\gamma_V = (1 - V^2/c_p^2)^{-1/2}.$$

In the regime considered in Refs. 6 and 7, where $\lambda \gg 1$ and $\gamma_V \simeq 1$, the parameter c_p can be further approximated to $(p\lambda)^{-1/2}$.

The validity condition for truncating the expansion (5) to its first order, $\kappa \ll 1$, or equivalently $c_p p \gg 1$, implies a direct core-core interaction; that is, $1/\lambda \gg 1/p$. We recall that here $1/\lambda$ represents the size of a vortex core in dimensionless units. Accordingly, for $\lambda \gg 1$, c_p must be regarded as the maximum velocity of a vortex array in a layered superconductor, to be compared with the maximum dimensionless velocity $1/\lambda$ of a single vortex (i.e., $\omega_p\lambda_G$ in dimensional units[2,5]).

In the opposite limit, $p/\lambda \ll 1$, vortices only weakly interact on the magnetic length scale λ_{EM} (rescaled here to 1); the limiting velocity c_p grows larger than $1/\lambda$ and the JV array becomes unstable.

B. Radiation mechanism

The emission of radiation by fast-moving JVs also takes place on the magnetic length scale λ_{EM}. The effective phase difference φ associated with an array of JVs moving along an IJJ, thus, obeys the perturbed local sine-Gordon equation,

$$\varphi_{tt} - {c_p}^2\varphi_{xx} + \sin\varphi = -\beta\varphi_t - f + \epsilon P(x)\sin\varphi. \tag{7}$$

Note that, in leading order, $\varphi = p\ (x - Vt)$, as can be seen from Eq. (5). Here, for completeness, we have restored the viscous term $-\beta\varphi_t$ and the current-induced drive f, that allow us to control the net JV speed V (see, e.g., Ref. 24).

Like in the more conventional single sine-Gordon-soliton perturbation schemes,[27,28] we consider the Ansatz,

$$\varphi(x,t) \to \varphi(x,t) + \chi(x,t), \tag{8}$$

which, inserted in Eq. (7), yields[26]

$$\chi_{tt} - {c_p}^2\chi_{xx} + (\cos\varphi)\chi = -\beta\chi_t + \epsilon P(x)\sin\varphi, \tag{9}$$

where $\varphi = 0$ is the ground state and only terms $\mathcal{O}(\epsilon)$ have been kept. The wave number q and the angular frequency ω of the unperturbed plasmon modes (i.e., for $\beta = \epsilon = 0$) form a continuum spectrum,[27] with

$$\omega^2 = 1 + {c_p}^2q^2. \tag{10}$$

However, as for the field [Eq. (5)] with $p \gg 1$, the radiation-vortex coupling $(\cos\varphi)\chi$ becomes negligible, and the plasma-wave dispersion relation can be approximated to $\omega = c_p|q|$. On introducing the spatial Fourier components of $P(x)$ and $\chi(x,t)$, defined by

$$P(x) = \frac{2}{\pi}\int_{-\infty}^{\infty} P(k)e^{ikx}dk, \quad \chi(x,t) = \frac{2}{\pi}\int_{-\infty}^{\infty}\chi_q(t)e^{iqx}dq,$$

Eq. (9) can be rewritten as[29]

$$\frac{d}{dt}B(q) - \frac{\beta}{2}B(q) = \frac{\epsilon}{2i}[e^{i(\omega - pV)t}P(q - p) - (p \to -p)],$$

where

$$B(q) \equiv (\dot{\chi}_q - i|q|\chi_q)e^{i\omega t}$$

is directly related to the spectral density of the array emission power,

$$W(q) = \frac{4}{\pi}\frac{d}{dt}|B(q)|^2,$$

that is,

$$W(q) = \frac{2\epsilon^2}{\pi}\left[\frac{|P(q - p)|^2(\beta/2)}{(\beta/2)^2 + (\omega - pV)^2} + (p \to -p)\right]. \tag{11}$$

Here we use that $P(x)$ is a real function so that $P^*(k) = P(-k)$. The notation "$(p \to -p)$" denotes the symmetric term obtained by replacing $p \to -p$ in the first term inside the square brackets. This means that two waves propagate in opposite directions with the same frequency ω; for $|P(q - p)| = |P(q + p)|$, they generate standing plasma oscillations, like those reported in Refs. 6 and 7. To simplify our notation, hereafter we restrict ourselves to JVs driven in one assigned direction, say $V > 0$.

The spectral emission power [Eq. (11)] is key to our analysis of a resonant IJJ. The spectrum $W(q)$ can be easily specialized for any choice of $P(x)$. In the case of quenched Gaussian disorder, see Eq. (4),

$$\langle|P(k)|^2\rangle = \frac{1}{8} \tag{12}$$

so that on disorder-averaging Eq. (11), we obtain the IJJ spectral emission power per unit of length,

$$w(\omega) = \frac{\epsilon^2}{4\pi}\left[\frac{\beta/2}{(\beta/2)^2 + (\omega - pV)^2} + (\omega \to -\omega)\right]. \quad (13)$$

This spectrum holds for $\beta \ll \omega$ or, equivalently, for $V \gg \beta/p$, and has a sharp resonance maximum,

$$w^{\max} = \frac{\epsilon^2}{2\pi\beta} \quad (14)$$

for

$$\omega_r = pV. \quad (15)$$

C. Vortex dynamics

Subject to a drive f produced by an externally applied electrical current, the vortices in an IJJ flow with an average speed V and, simultaneously, their cores interact with the electromagnetic waves they radiate. For the relatively weak vortex-core repulsion, $1/\lambda \gtrsim 1/p$, simulated in Refs. 6 and 7, we expect that the vortex array can be modulated, both in space and time, by the resonant plasma modes.

To express the average speed V of a JV array with $p/\lambda \gg 1$ as a function of the drive f, from Eq. (7) we derive the energy balance equation per unit of length of radiating IJJ,[15]

$$w(V) + \beta(pV)^2 = pVf. \quad (16)$$

Equation (16) tells us that the rate at which the drive pumps energy into the system (right-hand side), must be equilibrated by the radiative, $w(V)$, and the viscous loss, $\beta(pV)^2$, of the soliton array $\varphi(x,t)$ (left-hand side).

For $V \gg \beta/p$, the total emission power of the radiating sine-Gordon solitons,

$$w(V) = \frac{\epsilon^2}{2} \quad (17)$$

is computed by integrating the spectral emission power [Eq. (11)]; solving the ensuing Eq. (16) with respect to V, we obtain

$$V(f) = \frac{f \pm (f^2 - 2\beta\epsilon^2)^{1/2}}{2\beta p}, \quad (18)$$

where only the rising branch with the + sign is stable.[29] Therefore, the observable velocity-drive characteristic $V(f)$ is expected to show a step at

$$f_{\mathrm{th}} = (2\beta)^{1/2}\epsilon \quad (19)$$

and to grow linearly with f for $f \gg f_{\mathrm{th}}$, when the radiation loss becomes negligible, namely,

$$V = \frac{f}{\beta p}. \quad (20)$$

Note that, at variance with an emitting JV lattice,[29,30] no multiple hysteretic steps in the $V(f)$ are predicted. Indeed, the condition $f > f_{\mathrm{th}}$ simply implies that the effective phase φ is not pinned by disorder;[31] for $f \lesssim f_{\mathrm{th}}$, instead, the JV array can move only by creeping, namely, through the nucleation and the subsequent migration of array defects.[32,33] Creeping is likely responsible for the smooth low-current J-$\mathcal{V}$ characteristics shown in Fig. 4 of Ref. 6. Moreover, in the linear regime [Eq. (20)], the wavelengths λ_r of the emitted radiation are expected to be much shorter than the length L of the IJJ (see below) so that corrections due to the appropriate standing-wave periodic boundary conditions are on the order of λ_r/L.

A vortex is sensitive to the radiation field only when the wavelengths λ_r excited in the IJJ are larger than its size. For the parameter choice of Refs. 6 and 7, this can only occur on the JV core scale λ_G because $\lambda_G \lesssim \lambda_r$.

The interaction between the radiation standing wave, say,

$$\chi(x,t) = \chi_0 \cos(qx)\cos(\omega t + \phi) \quad (21)$$

and a single JV solution of the nonlocal sine-Gordon Eq. (2),

$$\varphi(x) = \pi + 2\arctan(\lambda x) \quad (22)$$

(both in dimensionless units) is well described by the nonrelativistic quasiparticle approach of Ref. 15. The JV center of mass with coordinate $X(t)$ is subject to an oscillating sinusoidal trap,

$$\ddot{X} = -\beta\dot{X} + \chi_0 \frac{q^2}{\lambda} e^{-q/\lambda} \cos(qX)\cos(\omega t + \phi) \quad (23)$$

with an amplitude which is exponentially suppressed at short wavelengths, i.e., for $q/\lambda \gg 1$. However, for sufficiently large trap amplitudes, the vortices in each layer get spatially distributed with wave vector q.

III. COMPARISON WITH NUMERICAL RESULTS

The results in Refs. 6 and 7 can be easily analyzed within the above theoretical framework. To make contact with their numerical data, one must express all lengths in units of λ, with $\lambda = 200$ μm; the velocities in units of the light speed in the dielectric $c = c_0/\sqrt{\epsilon_c}$, where c_0 is the speed of light *in vacuo* and $\epsilon_c = 10$ is the simulated dielectric constant; the forces in units of J/J_c, where J_c is the critical JJ current and J is the superconducting current across the IJJs; and the angular frequencies in units of the plasma gap frequency $\nu_p = \omega_p/2\pi = c/\lambda = 0.47 \times 10^{12}$ Hz. Moreover, the actual layer JV density is $\rho = n/L \simeq 0.6$ $\mu\mathrm{m}^{-1}$ with $L = 100$ μm, the layer thickness is $s = 15$ Å, and the penetration length ratio $\gamma = 500$. For this choice of numerical parameters, the length scales we introduced in the previous section read, respectively, $\lambda_G = 0.38$ μm, $\lambda_{EM} = 8.7$ μm, and $1/p = 0.27$ μm.

First, we note that the simulations of Refs. 6 and 7 correspond to the physical condition where $\lambda_G \lesssim \lambda_r$. As for the resonant modes $\chi_0 \propto \epsilon$, see Eq. (14), the amplitude of the driving force in Eq. (23) turns out to scale like $\epsilon(\lambda/\lambda_G)^{1/2}$, which is strong enough to drag a JV against the disorder field [Eq. (4)] and the array of restoring forces. This explains the disordered spatial distribution of the emitting JVs, which, far from forming any ordered lattice, seem rather to get trapped by the plasma standing waves. In spite of the coherent nature of the plasma radiation, the vortex distributions in each IJJ can differ from one another because of the intrinsic disorder brought about by the layer-layer coupling.

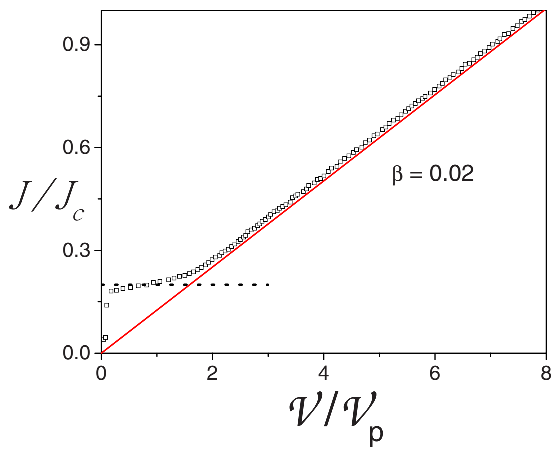

FIG. 1. (Color online) Current-voltage characteristics (in dimensionless units) for a damping constant $\beta=0.02$. The square symbols are the simulation data extracted from Fig. 4 of Ref. 6. The red straight line is our prediction from Eq. (24) for the linear Ohmic branch. The horizontal black dotted line is an estimate, from the numerics, of the depinning threshold in Eq. (19).

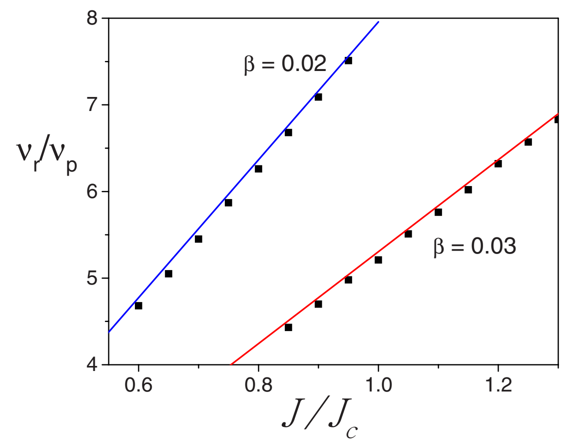

FIG. 2. (Color online) Resonance frequency ν_r versus current intensity J (both in dimensionless units) for two values of the viscous constant β. The black dots are simulation data extracted from Fig. 5 of Ref. 6. The two colored straight lines represent our theoretical predictions based on Eq. (28) and using two different values of the damping parameter β.

In Fig. 1, we compare the current-voltage characteristics from simulation, reported in Fig. 4 of Ref. 6, with the force-velocity (f-V) curve of Eq. (20). In the units of Ref. 6,

$$\frac{J}{J_c} = 2\pi\beta\left(\rho\lambda\frac{V}{c}\right) = 2\pi\beta\frac{\mathcal{V}}{\mathcal{V}_p}, \tag{24}$$

where $\mathcal{V}=\rho\lambda(V/c)$ is the flux-flow voltage across a IJJ layer and $\mathcal{V}_p \equiv \nu_p\Phi_0$ with $\Phi_0=h/2e$ denoting the flux quantum. The agreement is quite good in the linear regime whereas the depinning threshold [Eq. (19)] is clearly visible for $J/J_c \simeq 0.2$, which, in our units, corresponds to setting $\epsilon=1$.

The resonant plasma radiation is investigated in Refs. 6 and 7 on the linear branch of the J-$\mathcal{V}$ characteristics. Three plots of the plasma standing waves, two in Ref. 6 and one in Ref. 7, are shown for different J/J_c; from there we read out the corresponding resonance wavelengths λ_r. Furthermore, the resonance frequencies $\nu_r=2\pi\omega_r$ are either given explicitly in the text or shown in the figure (see, e.g., Fig. 5 of Ref. 6). The product $\lambda_r\nu_r$ appears to define a Swihart velocity, denoted here by c_S, independent of the simulation parameters J/J_c and β, that is,

$$\frac{c_S}{c} \simeq 0.04. \tag{25}$$

In view of our radiation mechanism [Eq. (9)], the ratio c_S/c can be identified with c_p in Eq. (6); accordingly, for $\lambda \gg 1$ and $\gamma_V \simeq 1$,

$$\frac{c_S}{c} \simeq \frac{1}{\sqrt{\lambda p}} \tag{26}$$

is predicted to be on the order of 0.036, which is reasonably close to the result in Refs. 6 and 7, given the accuracy of the data available.

The average JV speed in a resonant IJJ structure is proportional to the resonance frequency, that is, from Eq. (15),

$$V = \frac{\nu_r}{\rho}. \tag{27}$$

This equation holds for *all* different choices of the simulation parameters presented in Refs. 6 and 7. Note that the measured JV speeds are relatively small, $V \ll c$, as assumed in our nonrelativistic treatment of Eq. (9), where $\gamma_V \simeq 1$. Moreover, when combined with Eq. (24), this equation yields the dependence of ν_r on the simulation control parameters J/J_c and β. The ensuing law

$$\frac{\nu_r}{\nu_p} = \frac{1}{2\pi\beta}\frac{J}{J_c} \tag{28}$$

closely matches all spectral resonance peaks reported in Ref. 6, as shown in Fig. 2. Note that combining Eqs. (24) and (28) yields the simple β-independent relation

$$\frac{\nu_r}{\nu_p} = \frac{\mathcal{V}}{\mathcal{V}_p}. \tag{29}$$

Finally, we notice from Eqs. (14) and (20) that $w^{\max}$ is proportional to ϵ^2/β and V is proportional to f/β; as a consequence, one would expect that on decreasing β the IJJ spectral emission band shifts to lower J/J_c while growing in intensity, both inversely proportional to β. This is exactly the dependence displayed in Fig. 6 of Ref. 6.

IV. CONCLUSIONS

We propose a mechanism of coherent radiation from the moving Josephson vortices in layered superconductors. We show that due to the two-scale structure of Josephson vortices, they radiate terahertz radiation on a characteristic scale λ_{EM}, which is much longer than the Josephson vortex-core size $\lambda_G \sim \gamma s$. Among all emitted waves, only standing modes in the sample (working as a cavity) survive. These standing modes produce modulation of the density of JVs. This, in turn, makes vortices mainly radiate with wavelengths corresponding to standing waves. Such positive feedback can result in relatively strong radiation with well-pronounced maxima in the spectra. All our analytical estimates are in a good agreement with numerical data.[6,7]

The experimental demonstration[3] of terahertz radiation in zero magnetic field and various failed attempts at detecting terahertz emission in the presence of magnetic fields cast serious doubts on the initial idea that moving JVs can radiate in this frequency domain. Indeed, the now prevailing interpretation is that JVs ought to be considered as perturbing degrees of freedom, which destroy the layer coherence and thus cause the suppression of terahertz radiation. In this study, however, we reach the conclusion that, under appropriate conditions, applied magnetic fields do help amplify and tune terahertz emission. This interesting result is also consistent with the recent systematic studies in Ref. 18.

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