Cooling a micromechanical beam by coupling it to a transmission line

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We study a method to cool down the vibration mode of a micromechanical beam using a capacitively coupled superconducting transmission line. The Coulomb force between the transmission line and the beam is determined by the driving microwave on the transmission line and the displacement of the beam. When the frequency of the driving microwave is smaller than that of the transmission line resonator, the Coulomb force can oppose the velocity of the beam. Thus, the beam can be cooled. This mechanism, which may enable us to prepare the beam in its quantum ground state of vibration, is feasible under current experimental conditions.

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I. INTRODUCTION

Mechanical resonators[1,2] have important applications in high precision displacement detection,[3–5] mass detection,[6] quantum measurements,[7] and studies of quantum behavior of either mechanical motion[8–13] or phonons.[14–17] Recently, proposals[18–20] have been made for implementing qubits by using buckling nanoscale bars with quantized motion. Casimir effects on nanoscale mechanical device were also studied.[21–24] However, in previous studies (see, e.g., Refs. 18–20) of quantized mechanical resonators (and macroscopic quantum phenomena[25] in mechanical resonators; see, e.g., Refs. 8–10), it is necessary to prepare the mechanical resonators into their vibrational ground states. Therefore, one needs to cool the mechanical resonators down to ultralow temperatures to put them into their ground states. For example, a temperature below 1 mK is necessary for cooling a 20 MHz mechanical resonator to its vibrational ground state.

To reach temperatures below 1 mK, which is beyond the capability of present dilution refrigerators, alternative cooling mechanisms are now being explored. Using optomechanical couplings, the cooling of mechanical resonators was recently demonstrated experimentally.[26–31] To observe the quantized motion of a mechanical resonator, one should be able to cool the mechanical resonator down to its ground state of vibration and to detect the phonon number state. Besides optomechanical cooling, electronic cooling[32,33,35–38] was also studied. For instance, theoretical proposals for cooling a mechanical resonator were considered by coupling it either to a two-level system,[32,33,37] to an ion,[34] or to an *LC* circuit.[35] An experimental demonstration of cooling a mechanical resonator by the quantum back action of a superconducting single-electron transistor was recently reported.[36] Most of these cooling experiments (e.g., Refs. 26–29) focus on cooling mechanical resonators with a frequency lower than 1 MHz, with a mechanical quality factor higher than 10^4. It is difficult to experimentally cool mechanical resonators to their quantum ground state of vibrations because of the weak coupling between the mechanical resonators and the cooling media for optomechanical systems (see, e.g., Ref. 39).

Recently, the strong coupling between a one dimensional (1D) transmission line resonator (TLR) and a solid state qubit[40,41] was achieved,[42] and the detection of photon number states was also demonstrated.[43] Based on these experimental developments, here we consider replacing the Fabry-Pérot cavity used in previous cooling proposals[26] by a 1D TLR in order to cool a micron-scale bar.

The working mechanism of our proposal here is similar to the cooling of a tiny mirror in a Fabry-Pérot cavity.[26] This cooling mechanism can be summarized as follows. A force on the mirror is coupled to the light intensity inside the cavity. This intensity does not change instantaneously with each mirror displacement. The delayed response of the intensity to a change in the mirror displacement leads to a force that can either agree or oppose the motion of the mirror, depending on whether the laser frequency is bigger or smaller than the cavity resonant frequency.[44] By including this intensity-dependent force, in addition to a thermal force on the mirror, the mirror can be cooled.

In our proposal here, the TLR, whose frequency is determined by its overall capacitance and inductance, acts as a cavity. The beam is placed near the middle of the TLR and capacitively coupled to the TLR. When the mechanical beam has a displacement, the overall capacitance of the TLR changes, thereby the resonant frequency of the TLR also changes. Now, let us consider the case where the TLR is driven by a microwave with fixed frequency. Any displacement of the beam will change, after a delay, the voltage between the TLR and the beam (and also the force between them). Recall that here we are considering two coupled oscillators: the TLR and the mechanical beam. The rf microwave drive acts directly on the TLR and indirectly on the mechanical beam. After the transients are gone, the driven damped oscillator (here, the TLR) exhibits a steady-state response which is delayed with respect to the drive. In other words, the beam displacement changes the TLR's oscillation frequency ω_a. Since the frequency ω_{rf} of the drive is fixed, this change in ω_a will affect the steady-state amplitude of the TLR oscillator, which will be reached after some delay. The displacement of the beam (i.e., the action on the TLR) causes a delayed reaction (i.e., a delayed back action) force from the

TLR to the beam. The delay is determined by the damping rate of the TLR. When the frequency of the microwave ω_{rf} is smaller than the resonant frequency ω_{a0} of the TLR, this back-action force opposes the motion the beam, thereby damping the Brownian motion of the beam.

This cooling mechanism studied here is also related to the mechanism recently employed in Refs. 35 and 46. There, cooling is produced by a capacitive force which is phase shifted relative to the cantilever motion. In their setup, when the cantilever oscillates, its motion modulates the capacitance of an *LC* circuit, therefore modulating its resonant frequency. This resonant frequency, and the potential across the capacitance, is modulated relative to the fixed frequency of the applied rf drive. The modulated force linked to this potential shifts the resonant frequency of the cantilever.[35,46] Because of the finite response time of the *LC* circuit, there is a phase lag in the force, relative to the motion. When the rf frequency is smaller than the resonant frequency, the phase lag produces a force that opposes the cantilever velocity, producing damping. When this damping is realized without introducing too much noise in the force, then the cantilever is cooled.

Our analysis, presented below, shows that it is possible to cool a 2 MHz beam, initially at ~50 mK, down to its quantum vibrational ground state at around 0.07 mK. This is a cooling factor of about 1/700. Our proposed device, which is a combination of the devices in Refs. 36 and 42, should be realizable in experiments. Moreover, because of its on-chip structure, our device has some practical advantages to be integrated in dilution refrigerators and be operated on, while optomechanical systems need an additional optical system.

II. DEVICE

Our proposed device is illustrated in Fig. 1(a). A doubly clamped microbeam is placed in the middle of a 1D superconducting TLR formed by thin coplanar striplines. The central stripline has a length l and with a capacitance C_a/l and an inductance L_a/l, per unit length. For not-very-high frequencies, the equivalent circuit of the stripline is an infinite series of inductors with each node capacitively connected to the ground, as shown in Fig. 1(b). It can be described as a series of resonators that accommodate different resonant modes.[42] Since the length of the microbeam is much smaller than that of the 1D TLR, we consider the voltage in the middle of the 1D TLR to be the voltage $V_a(t)$ on the beam. Here, we only consider the mode with the largest coupling, i.e., the lowest mode[42] coupled to the beam. The 1D TLR is coupled to both two semi-infinite TLRs to the left and right, via the capacitors C_0, and the beam via the capacitor C_b. Thus, the boundary conditions and the voltage of the 1D TLR are modified by these additional capacitors. When $C_0, C_b \ll C_a$, the circuit can be approximated by a 1D TLR with a modified frequency

$$\omega_a = \frac{1}{\sqrt{L_a C_t}}, \tag{1}$$

with $C_t = C_a + C_b + 2C_0$. Actually, due to its coupling to the environment, the 1D superconducting TLR acts as a cavity

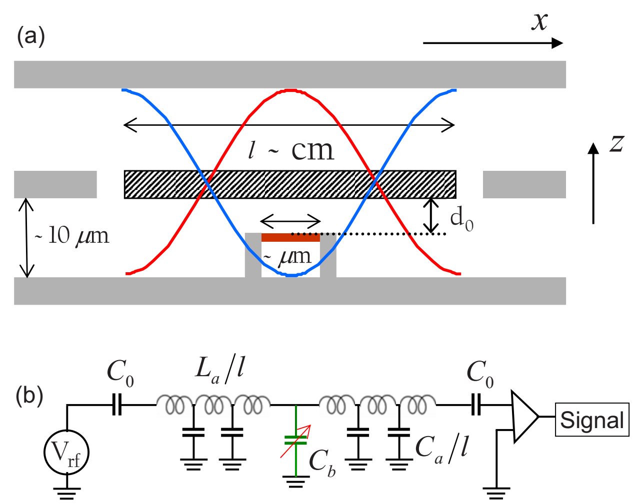

FIG. 1. (Color online) (a) Schematic layout for the proposed device and (b) its equivalent circuit. A superconducting coplanar stripline, forming a 1D TLR, provides a cavity. A doubly clamped beam (central small dark red rectangle) is placed between two horizontal superconducting lines. This red beam is capacitively coupled to the central (hatched) superconducting line at a maximum of the voltage standing wave in the 1D TLR. The capacitances C_0 allow the input and output signals to be coupled to the central (hatched) stripline. This allows us to measure the amplitude and phase of the 1D TLR and apply dc and rf pulses to the 1D TLR.

with finite quality factor $Q_a = \omega_a / 2\gamma$, where 2γ is the damping rate of the 1D TLR.

The fundamental vibration mode of the doubly clamped beam can be approximated by a mechanical resonator with frequency ω_b and effective mass m. The beam is coupled to a conductor (the 1D TLR) via a capacitor, and its equivalent circuit is illustrated in Fig. 2. The beam is exposed to a Coulomb force from the 1D TLR. Please note that for the case we studied in this paper, the amplitudes of the oscillates are small and thus the beam is essentially in the linear regime. For a review of nonlinear oscillators, see, e.g., Ref. 45.

III. COULOMB FORCE ON THE BEAM

This force gives rise to a cooling mechanism which is similar to the cavity cooling of the vibrating mirror in Ref. 26. As shown in Fig. 2(a), it is assumed that the beam vi-

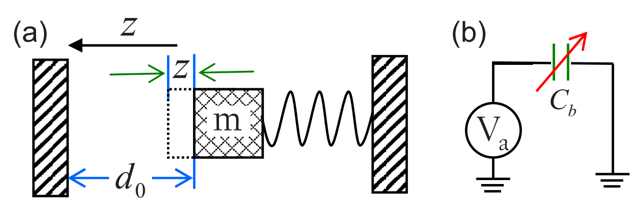

FIG. 2. (Color online) (a) Schematics of the beam (mass and spring) capacitively coupled to a conductor and (b) its equivalent circuit. The charging energy of the beam is determined by the voltage V_a and variable capacitor C_b between the beam and the 1D transmission line.

brates around its equilibrium position with an amplitude $z(t) \equiv z$, which is much smaller than the distance d_0 between its equilibrium position and the TLR, i.e., $z \ll d_0$. The averaged Coulomb force on the beam can be written as[35]

$$F_C(z) = \frac{d_0}{4(d_0 - z)^2} C_{b0} V_a^2(z), \tag{2}$$

when $\omega_a \gg \omega_b$. Here, C_{b0} is the capacitance between the beam and TLR for $z=0$. Assuming that an external driving source $V_d = V_{\mathrm{rf}} \cos(\omega_{\mathrm{rf}} t)$ acts on the central TLR via the capacitor C_0, $V_a^2(z)$ will reach a steady amplitude after a time delay $\tau_d \sim 1/\gamma$. To first order in z, Eq. (2) can be rewritten as

$$F_C(z) \cong F_0 + K' z, \tag{3}$$

where the effective elastic constant K' of the Coulomb force on the beam by the TLR is

$$K' = K_E D(\alpha)\left[1 - \alpha^2 \frac{C_{b0}}{C_{t0}} D(\alpha)\left(\alpha^2 - 1 + \frac{1}{Q_a^2}\right)\right]. \tag{4}$$

The term F_0, which is independent of the displacement of the beam, will change the equilibrium position of the beam. F_0 does not contribute to the cooling of the beam and can be canceled by applying an appropriate dc voltage between the TLR and the beam. Therefore, hereafter it will be omitted. The term

$$K_E = \frac{C_{b0} V_{\mathrm{rf}}^2}{2 d_0^2} \tag{5}$$

describes the coupling strength between the beam and the TLR. C_{t0} is the total capacitance of the TLR for $z=0$.

$$D(\alpha) = \left[(\alpha^2 - 1)^2 + \frac{\alpha^2}{Q_a^2}\right]^{-1} \tag{6}$$

is a dimensionless parameter determined by the ratio

$$\alpha = \frac{\omega_{\mathrm{rf}}}{\omega_a}. \tag{7}$$

$D(\alpha)$ takes its maximum value on resonance $\omega_{\mathrm{rf}}/\omega_a = 1$. The typical behavior of K', versus the detuning

$$\Delta = \omega_{\mathrm{rf}} - \omega_{a0}, \tag{8}$$

is plotted in Fig. 3. Here, ω_{a0} is the frequency of the TLR for $z=0$. There is an optimal detuning point for the driving microwave where K' takes its maximum value. As shown in Fig. 3, the sign of the effective elastic constant K' of the Coulomb force is determined by the detuning between the frequency of the driving microwave (ω_{rf}) and that of the TLR (ω_{a0}). When $\omega_{\mathrm{rf}} < \omega_{a0}$, additional damping is induced by the Coulomb force, cooling the beam because of its delayed response to the displacement of the beam.

IV. COOLING MECHANISM

We define the effective temperature T_{eff} of the fundamental vibration mode of the beam according to the equipartition law

$$T_{\mathrm{eff}} = K_{\mathrm{eff}} \frac{\langle z^2 \rangle}{k_B}, \tag{9}$$

where k_B is the Boltzmann constant and K_{eff} a modified elastic constant of the beam after considering the existence of the TLR. In some papers (e.g., in Refs. 26 and 31), the effective temperature is defined and/or estimated from the *original* elastic constant K_0 of the beam, instead of the *effective* elastic constant K_{eff} of the beam. For the case when the effective elastic constant K' of the force F_C is much smaller than the original elastic constant K_0 of the beam, the definition in Eq. (9) and the one in Refs. 26 and 31 give almost the same result of the effective temperature. However, please note that for a large effective elastic constant K' of the force F_C, one should not neglect the modification of the elastic constant of the beam.

The Coulomb force from the TLR has two effects on the mean kinetic energy of the beam. First, because of its delayed response to the displacement of the beam, the Coulomb force introduces additional damping in the beam motion, thereby increases or reduces the mean kinetic energy of the beam. Below, it is shown that the damping rate of the beam increases when $\omega_{\mathrm{rf}} < \omega_{a0}$. Second, fluctuations in the Coulomb force from the TLR also introduce additional noise in the motion of the beam, thereby increasing the mean kinetic energy of the beam. The balance of these two competing effects gives the theoretical lower limit of the attainable effective temperature by this cooling mechanism.

To evaluate the cooling effect of the Coulomb force from TLR, we use the following equation of motion for the beam:[26]

$$m\frac{d^2 z}{dt^2} + m\Gamma \frac{dz}{dt} + K_0 z = F_{\mathrm{th}} + \int_0^t \frac{dF_C[z(t')]}{dt'} h(t-t') dt', \tag{10}$$

where $\Gamma = \omega_b / Q_b$ describes the coupling strength between the beam and its thermal environment. Here, Q_b is the quality factor of the beam, $K_0 = m\omega_b^2$ the elastic constant of the beam, and F_{th} the thermal noise force on the beam, with a spectral density[47]

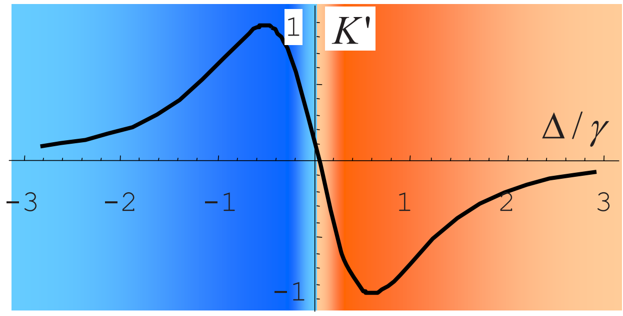

FIG. 3. (Color online) The effective elastic constant K' (arbitrary units) versus the detuning $\Delta = \omega_{\mathrm{rf}} - \omega_{a0}$ (scaled by γ). When $\omega_{\mathrm{rf}} < \omega_{a0}$, the beam can be cooled. Here, the region $\Delta > 0$ ($\Delta < 0$) is shaded by the red (blue) color, representing the heating (cooling) of the beam, respectively.

$$S_{\mathrm{th}}=4k_BT_0m\Gamma. \tag{11}$$

T_0 is the temperature of the environment. $F_C(z)$ is the Coulomb force on the beam, acting on the beam via a delay function:

$$h(t)=1-e^{-\gamma t}, \tag{12}$$

for $t>0$. Using the Laplace transform, we obtain the mean-squared motion of the beam:

$$\langle z^2\rangle=\frac{k_BT_0\omega_{a0}^2\Gamma}{\pi K_0}\int_{-\infty}^{+\infty}d\omega\left[\left(\omega^2-\frac{K_{\mathrm{eff}}}{K_0}\omega_{a0}^2\right)^2+\omega^2\Gamma_{\mathrm{eff}}^2\right]^{-1}. \tag{13}$$

There are three measurable effects on the vibration mode ω_b of the beam from $F_C(z)$: a modified effective elastic constant K_{eff},

$$K_{\mathrm{eff}}=K_0\left(1-\frac{1}{1+\beta^2}\frac{K'}{K_0}\right), \tag{14}$$

a modified damping rate Γ_{eff},

$$\Gamma_{\mathrm{eff}}=\Gamma\left(1+Q_b\frac{\beta}{1+\beta^2}\frac{K'}{K_0}\right), \tag{15}$$

with $\beta=\omega_b/\gamma$, and additional noise in the motion of the beam generated by the fluctuation of $F_C(z)$.

For the case $|K'|/K_0\ll 1$, neglecting fluctuations of $F_C(z)$ in Eq. (10), the steady value of the mean-squared motion of the beam is given by

$$\langle z^2\rangle\approx k_BT_0\frac{\Gamma}{\Gamma_{\mathrm{eff}}}, \tag{16}$$

which defines an effective temperature of the beam. For the case when $|K'|\approx K_0$ or $|K'|>K_0$, the frequency of the beam will be greatly shifted away from the original one, resulting in weak cooling of the beam.[37]

V. EFFECTS OF FLUCTUATIONS OF $F_C(z)$

In the above discussions, we did not consider the effect of fluctuations of $F_C(z)$ on the effective temperature. Equation (15) shows that the damping rate of the beam is modified because of the existence of the TLR. Thus, the effective temperature is changed [see Eqs. (9) and (16)]; however, according to the fluctuation and dissipation theorem, dampings are always accompanied with noises. We now study noises on the beam introduced by the force F_C of the TLR. Actually, there are several noise sources affecting $F_C(z)$, such as fluctuations in the driving microwave, back action due to measurements on the TLR, and thermal noise in the TLR. Among these noise sources, the thermal noise provides an intrinsic limit of the fluctuations of $F_C(z)$. Therefore, a lower limit S_{TLR} of the spectral density of $F_C(z)$ can be obtained by considering the voltage fluctuation S_V from the thermal noise in the TLR, which is given by $S_V=4k_BT_0R$, with $R=2\gamma L_a$ the effective resistance in the TLR. The voltage fluctuation S_V gives rise to a fluctuation of the charge on the capacitor C_b, giving rise to fluctuations of $F_C(z)$ on the beam through the capacitor C_b. Since $C_b\approx C_{b0}$ for small vibration amplitudes of the beam, we find that the thermal noise in the TLR gives a fluctuation of $F_C(z)$ on the beam,

$$S_{\mathrm{TLR}}=2k_BT_0D(\alpha)RC_{b0}K_E. \tag{17}$$

Assuming an Ohmic friction for the beam, the temperature T' and the damping rate Γ' of the beam and the spectral density S of the noise on the beam have the following relation:[48]

$$S\propto\Gamma'\omega\coth\frac{\hbar\omega}{2k_BT'}. \tag{18}$$

For not-very-low temperature near the beam's resonant frequency, we find that an effective temperature of the beam could be related to the spectral density S and the damping rate of the beam: $T'\propto S/\Gamma'$. Therefore, after considering the fluctuation and dissipation theorem, we further modify the effective temperature of the beam to

$$T_{\mathrm{eff}}=T_0\left(\frac{\Gamma}{\Gamma_{\mathrm{eff}}}\right)\frac{S_{\mathrm{TLR}}+S_{\mathrm{th}}}{S_{\mathrm{th}}}, \tag{19}$$

where we only take into account the thermal noise in the TLR. Indeed, the attainable lowest effective temperature of the beam would be higher than the above limit, since there are also other fluctuations acting on the beam, from both the driving microwave and the back action of the measurement on the beam. These fluctuations would add more noises, which depend on the special parameters of the circuit for the driving microwave and the circuit for the measurement, e.g., the noise from amplifiers,[46] in the numerator of Eq. (19). We do not address them here.

VI. COOLING ABILITY

Now let us estimate the cooling effect T_{eff}/T_0. Using experimentally feasible parameters,[36,42] we take $\omega_{a0}=10$ GHz, $\gamma=200$ kHz, $d_0=0.1$ μm, $C_{b0}=400$ aF, $C_{t0}=10^3$ C_{b0}, and $K_0=10$ N/m. When the driving power of the microwave is set at $V_{\mathrm{rf}}=0.05$ mV, the effective spring constant K' of the Coulomb force can be as large as 0.55 N/m for optimal detuning of the driving microwave. It is possible to obtain stronger coupling between the beam and the TLR by increasing the driving power of the microwave, as long as the voltage V_a between the beam and the TLR is kept below the breakdown voltage. Using the parameters listed above and assuming $Q_b=10^5$, the cooling effect T_{eff}/T_0 depends on the oscillating frequency ω_b of the beam and the detuning Δ, as shown in Fig. 4. In Fig. 4, we assume $|K'|\ll K_0$, and then take the effective spring constant $K_{\mathrm{eff}}\approx K_0$ in Eq. (14). Otherwise, the optimal value of ω_b/γ to reach the lowest T_{eff} will be slightly drifting away from unity.[37] The best cooling effect on a 200 kHz beam is estimated to be $T_{\mathrm{eff}}/T_0\approx 3.6\times 10^{-4}$ for the parameters given above. Therefore, if this beam is precooled by the dilution refrigerator to a temperature of 1 K, it can be further cooled down to 0.36 mK using the TLR.

For a 2 MHz beam, we use a stronger microwave $V_{\mathrm{rf}}=0.5$ mV. The best cooling effect is about $T_{\mathrm{eff}}/T_0\approx 1.4$

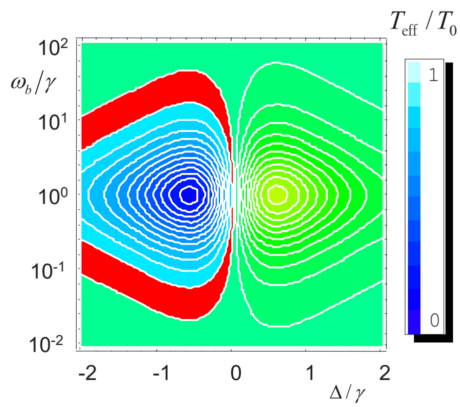

FIG. 4. (Color online) Cooling effect T_{eff}/T_0 versus the oscillating frequency of the beam ω_b and the detuning $\Delta=\omega_{\mathrm{rf}}-\omega_{a0}$, both rescaled by γ, for the typical parameters listed in the text. There is a cooling window for ω_b and Δ. The cooling effect is normalized to unity. The vertical bar at the right refers to cooling effects (in blue) and not heating (in red). The darker blue corresponds to a greater cooling effect T_{eff}/T_0. The beam is heated in the red region and becomes unstable in the green region.

$\times 10^{-3}$ with $\gamma=2$ MHz. If the beam is precooled by the dilution refrigerator to a temperature of 50 mK, it could be further cooled down to 0.07 mK by the TLR. This implies that the thermal phonons in the 2 MHz beam will be less than 0.24, where a quantum description is expected.[49,50] It should be noticed that when the beam reaches a quantum regime, a quantum theory is expected to give the cooling efficiency in the quantum regime.

Above, we do not consider matching the impedance of the TLR to that of the conventional microwave components.[42] To obtain an optimal impedance, e.g., 50 Ω, of TLR, one needs to carefully design the geometry of the TLR and the beam. If one simply considers the TLR as a straight coplanar transmission line, then C_{t0} might be ~ 1 pF for a 10 GHz TLR. Thus, a larger C_{b0} is necessary for this larger C_{t0} to maintain the cooling effects described above.

VII. DISCUSSIONS

The fabrication of superconducting TLR now is good enough to provide a 1D TLR with an electrical quality factor as high as 10^6. The reduced effective temperature T_{eff} of the beam can be inferred from the power spectrum of the 1D TLR around the oscillating frequency ω_b, whose integral is proportional to the effective temperature. Detection of microwave photon was achieved in recent experiments, capable of resolving a single microwave photon number.[43] Therefore, in principle, the information of the beam could be inferred by detecting the field in the TLR.

The working principle of our proposal is similar to that in the optomechanical cooling by a Fabry-Pérot cavity.[26] The cooling or heating is determined by the detuning between the driving laser and cavity, which is determined by the detuning between the driving microwave and the TLR in our case. In both cases, a high mechanical quality factor is needed, since it measures heating effects on the beam by its thermal environments.

Our proposal is also similar to the one in Ref. 35, which deals with a cantilever and a coupled *LC* circuit. Here, we consider a doubly clamped beam coupled to a coplanar TLR. Besides considering different physical systems, in Ref. 35, they analyze only *two* special detunings between the frequency of the driven microwave and the resonant frequency of the *LC* circuit. Here, we present a more general result for the effective elastic constant versus the detuning, valid for all values of the detuning between the frequency of the driven microwave and the resonant frequency of the TLR. We also explain how the damping rate of the beam and the noise on the beam are changed by the TLR. Our studies enable us to optimize the setup of experimental parameters for achieving a lower effective temperature of the beam, as shown in Fig. 4.

Since the best cooling is obtained when $\omega_b/\gamma\approx 1$, the cooling efficiency of optical-cavity cooling would be efficient for beams with tens of megahertz, or even higher frequency, considering current experimental parameters. For a typical optical cavity, with a resonant frequency of $\sim 10^{14}$ Hz, the damping rate is about 10^8 Hz, for an optical quality factor 10^6, making it favorable for cooling a 100 MHz beam. However, to cool a beam with an ~ 1 MHz vibration frequency, the optimal damping rate of the optical cavity would also be ~ 1 MHz. This requires an optical quality factor $\sim 10^8$ for a tiny mirror. A high mechanical quality factor is also required at the same time, which is a great challenge for the fabrication of the tiny mirror. However, in our case, the damping rate of the TLR can be as small as 200 kHz. The damping rate of the TLR can be easily increased to match the frequency of the beam by attaching an additional circuit to the TLR, while it is very difficult to decrease the damping rate of an optical cavity to match the lower-frequency beam. Thus, a megahertz beam could be cooled down to its quantum ground state and also reach the regime $\gamma \ll \omega_b$, where the cavity linewidth is much smaller than the mechanical frequency and the corresponding cavity detuning. Then, the photon sidebands could be resolved when the beam is cooled down to the quantum regime.[50] A recent interesting study of the lower limit for resonator-based side-band cooling can be found in Ref. 51.

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