Universal existence of exact quantum state transmissions in interacting media

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We consider an exact state transmission, where a density matrix in one information processor A at time $t=0$ is exactly equal to that in another processor B at a later time. We demonstrate that there always exists a complete set of orthogonal states, which can be employed to perform the exact state transmission. Our result is very general in the sense that it holds for arbitrary media between the two processors and for any time interval. We illustrate our results in terms of models of spin, fermionic, and bosonic chains. This complete set can be used as a basis to study the perfect state transfer which is associated with degenerate subspaces of this set of states. Interestingly, this formalism leads to a proposal of perfect state transfer via adiabatic passage, which does not depend on the specific form of the driving Hamiltonian.

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I. INTRODUCTION

One of the central missions in quantum information theory is to transmit known or unknown quantum states from one region to another, for instance, from one information processor A to another B. The transmission may be a two quantum state swap, short-distance communications between components of a quantum device, or long-distance quantum communications through optical fibers. The simplest form of short-distance transmission, two-state swap between spin qubits, may be conveniently processed in terms of local spin couplings such as the direct Heisenberg interaction. In this case, interactions play the role of information carriers. Indeed, quantum state transmissions through interaction-linked chains [1,2], such as spin chains and bosonic lattices, have been investigated extensively (see, e.g., [3] and references therein). Early works [1] concentrated on the transmitting abilities of the naturally available interactions of spin chains, but in most cases it failed to perfectly transmit a quantum state. Afterward [2–5] interactions were proposed where perfect quantum state transfer was possible.

In spite of many works in this area, a central question remains open: given an evolution operator $U(\tau)$, governed by a time-independent or time-dependent Hamiltonian $H(t)$ linking the two processors A and B, is it possible that there exist states, during an arbitrary time interval τ, being transmitted exactly from region (or processor) A at time $t=0$ to region (processor) B at $t=\tau$? Here we first define an exact state transmission in the sense that a *pure or mixed* density matrix in region (or processor) A and at time $t=0$ is transmitted exactly to another region (processor) B (with the same internal structure as A) at the time τ. We can thus show that there always exists a complete set $\{\Psi_k(0)\}$ of orthogonal states, which can be used to perform the exact state transmission. The $\Psi_k(0)$'s are states of the entire system, which refers to two processors and also the media between them. Throughout this paper, the phrase *exact state transmission* refers specifically to the fact that the density matrices of two processors are equal because of the use of the set $\{\Psi_k(0)\}$. Our result is very general in the sense that it holds for arbitrary media between the two processors and for an arbitrary time τ (where, of course, τ is inside the light cone). We illustrate the set $\{\Psi_k(0)\}$ for models of spin, as well as spinless fermionic and bosonic chains. Indeed, the perfect state transfer (PST) in [2] occurs in a degenerate subspace of $\{\Psi_k(0)\}$. Based on this result, we propose an approach for perfect state transfer via adiabatic passage, which does not rely on the specific form of the driving Hamiltonian.

II. UNIVERSAL EXISTENCE OF EXACT STATE TRANSMISSION

We consider an M-dimensional system (M can be infinite), e.g., $M=2^K$ for K qubits, located in processor A, spanned by the bases $\{|\alpha\rangle\}$. A pure or mixed quantum state in this processor can be generically characterized by a reduced density matrix

$$\rho^A(0) = \mathrm{Tr}_B\{|\Psi(0)\rangle\langle\Psi(0)|\},$$

where the trace is taken over both processor B and media degrees of freedom. We want to transmit this state to another processor B at a given time interval τ. The exact state transmission is defined by

$$\rho^A_{\alpha\beta}(0) = \rho^B_{\alpha\beta}(\tau). \tag{1}$$

Here

$$\rho^B(\tau) = \mathrm{Tr}_A\{|\Psi(\tau)\rangle\langle\Psi(\tau)|\},$$

where the trace is taken over both processor A and media degrees of freedom, describes the quantum state of processor

B at the given time τ. Assume that the entire system (composed of processors A, B, and also the media) is initially in the state $\Psi(0)$. We show below that there exists a complete orthogonal set $\{\Psi_k(0)\}_\tau$, which depends on τ, such that an exact state transmission described by Eq. (1) occurs if the initial state is one of the states in this set.

Proof. For the initial state $\Psi(0)$, the matrix elements of $\rho^A(0)$ in processor A at time $t=0$ are

$$\begin{aligned}\rho^A_{\alpha\beta}(0) &= \langle\alpha_A|\mathrm{Tr}_B\{|\Psi(0)\rangle\langle\Psi(0)|\}|\beta_A\rangle \\ &= \langle\alpha_A|\rho^A(0)|\beta_A\rangle \\ &= \langle\Psi(0)|\beta_A\rangle\langle\alpha_A|\Psi(0)\rangle, \end{aligned} \tag{2}$$

where $\alpha,\beta=1,2,\ldots,M$, and $|\beta_{A(B)}\rangle$ refers to a state $|\beta\rangle$ in processor A (B). At a given time $t=\tau$, the matrix elements of $\rho^B(\tau)$ in processor B are

$$\begin{aligned}\rho^B_{\alpha\beta}(\tau) &= \langle\Psi(0)|U^\dagger(\tau)|\beta_B\rangle\langle\alpha_B|U(\tau)|\Psi(0)\rangle \\ &= \langle\Psi(0)|U^\dagger(\tau)\mathcal{P}|\beta_A\rangle\langle\alpha_A|\mathcal{P}U(\tau)|\Psi(0)\rangle \\ &= \langle\Psi'(\tau)|\beta_A\rangle\langle\alpha_A|\Psi'(\tau)\rangle, \end{aligned} \tag{3}$$

where $U(\tau)$ is the evolution operator of the entire system (two processors and the media). Here we also introduce the $A\Leftrightarrow B$ exchange operator $\mathcal{P}$ satisfying

$$\mathcal{P}|\beta_A\rangle\langle\alpha_A|\mathcal{P} = |\beta_B\rangle\langle\alpha_B|$$

and $\mathcal{P}^2=1$. The exchange operator $\mathcal{P}$ swaps all the states of two processors. It can be expressed explicitly by

$$\mathcal{P} = \sum_{\alpha\beta}(|\beta_A\rangle\langle\alpha_A|)\otimes(|\alpha_B\rangle\langle\beta_B|) \tag{4}$$

and changes its form with different bases. The state $\Psi'(\tau)=G(\tau)\Psi(0)$, where we introduce the operator $G(\tau)=\mathcal{P}U(\tau)$. This operator G behaves similar (but not equal) to an evolution operator and here will be called a quasievolution operator. It is significant to note that the operator $G(\tau)$ is unitary and satisfies

$$G^\dagger(\tau)G(\tau) = U^\dagger(\tau)\mathcal{P}\mathcal{P}U(\tau) = 1. \tag{5}$$

As any unitary operator, the operator $G(\tau)$ can be diagonalized and has a complete set of orthonormal eigenvectors $\{\Psi_k(0)\}_\tau$ and exponential eigenvalues $\{\exp(i\phi_k)\}_\tau$. A vector $\Psi_k(0)$ in the set obeys the eigenequation

$$G(\tau)\Psi_k(0) = \exp(i\phi_k)\Psi_k(0). \tag{6}$$

Comparing the expressions of $\rho^A_{\alpha\beta}(0)$ in Eq. (2) with $\rho^B_{\alpha\beta}(\tau)$ in Eq. (3), it is easy to conclude that if the initial state $\Psi(0)$ is one of the $\Psi_k(0)$'s, the equality $\rho^A_{\alpha\beta}(0)=\rho^B_{\alpha\beta}(\tau)$ or Eq. (1) holds. In other words, an exact state transmission occurs. ■

The above statement or proposition could be very useful in investigating state transmissions. Generically, for an arbitrary Hamiltonian and at an arbitrary time τ, we can numerically diagonalize $G(\tau)$ to obtain its eigenstates and eigenvalues especially for small systems. In fact, studies on short-distance transmission in small systems are more significant for information exchange between the components of a quantum computing device. Below we will illustrate several eigenproblems of the operator $G(\tau)$ using analytical models. However, we emphasize that the proposition is universal and only results from the fact that a unitary operator possesses a complete set of orthogonal eigenvectors.

III. FULLY CONTROLLABLE MODELS

We first consider a one-dimensional lattice (a chain) with N local units (particles); each has the same (could be infinite) dimensional eigenspace spanned by the basis $\{|s\rangle\}$, e.g., $s=0,1$ for a spin chain. Assume that we are able to control the interactions between nearest-neighbor sites $(j-1)$ and j. We then turn on/off the permutation operators $E_{j,j-1}$ in chronological order such that

$$U_f(\tau) = E_{N,N-1},\ldots,E_{32}E_{21}, \tag{7}$$

where

$$E_{jj-1} = \sum_{s,r}(|s_j\rangle\langle r_j|)\otimes(|r_{j-1}\rangle\langle s_{j-1}|), \tag{8}$$

which, e.g., in a spin chain, can be represented by the XY or Heisenberg interactions [6]. An arbitrary state $|\phi\rangle_1|R\rangle$, at the first site as processor A, is an eigenstate of $G_f(\tau)$, where $|R\rangle$ is an arbitrary *symmetric* state for the rest of the chain $(j=2,\ldots,N)$.

Let us now consider the simple, but general, problem of a system with symmetry. If $[U(\tau),\mathcal{P}]=0$, e.g., in a time-independent system Hamiltonian with $U(\tau)=e^{-iH\tau}$ leading to $[H,\mathcal{P}]=0$, the eigenproblem of $G(\tau)$ becomes that of $U(\tau)$ and $\mathcal{P}$. Specially, the common eigenstates of the time-independent Hamiltonian H and $\mathcal{P}$ may be those of $G(\tau)$ as well.

IV. ADIABATIC EIGENSTATES

The adiabatic approximation is usually applied to describe systems under slowly varying time-dependent Hamiltonians. The adiabatic quantum state transfer for spin chains was studied in [7,8]. We now consider a time-dependent Hubbard-type Hamiltonian [9],

$$H(t) = -J(\Delta)T_h + \omega(\Delta)h_s + H_U, \tag{9}$$

where

$$T_h = \sum(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) \tag{10}$$

is the hopping term. $n_j=a_j^\dagger a_j$ is the number operator at site j. The creation operator $a_j^\dagger$ satisfies the standard commutation relations for bosons $a_j=b_j$ and anticommutation relations for spinless fermions $a_j=c_j$. This model is equivalent to a general XY model in spin chains through the Jordan-Wigner transformation [6]. The parameter $\Delta=\frac{\tau}{2}-t$; and $H_U=U\Sigma_j n_j(n_j-1)$ is the on-site repulsion ($H_U\equiv 0$ for fermions). The single-particle energy $h_s=\Sigma_j\epsilon_j n_j$ is designed such that $\epsilon_j<\epsilon_{j+1}$ for all j's and $\omega(\frac{\tau}{2})>0$. We also require that $J(\Delta)[\omega(\Delta)]$ is an even [odd] function of Δ and $J(\pm\tau/2)=0$ when $t=0(+\tau/2)$ and $\tau(-\tau/2)$. Also,

$$H(0) = \omega\left(\frac{\tau}{2}\right)h_s + H_U$$

and

$$H(\tau) = -\omega\left(\frac{\tau}{2}\right)h_s + H_U.$$

In this case processor A (B) is at the first (last) site. An initial state $a_1^\dagger|\mathbf{0}\rangle$ is the lowest eigenstate of $H(0)$ when the total particle number is 1, where $|\mathbf{0}\rangle$ is the bosonic or fermionic vacuum state. If we control the evolution of $H(t)$ adiabatically from time 0 to τ, the final state will be $e^{-i\phi(\tau)}a_N^\dagger|\mathbf{0}\rangle$, where $\phi(\tau)$ is the sum of the dynamic phase angle and geometric phase angle. The state $a_1^\dagger|\mathbf{0}\rangle$ satisfies

$$G_{\mathrm{ad}}(\tau)a_1^\dagger|\mathbf{0}\rangle = e^{-i\phi(\tau)}a_1^\dagger|\mathbf{0}\rangle, \tag{11}$$

which is an eigenstate of the quasievolution operator $G_{\mathrm{ad}}(\tau)$. Another trivial eigenstate of $G_{\mathrm{ad}}(\tau)$ is the vacuum state $|\mathbf{0}\rangle$. For spin or spinless fermionic chains and total particle number larger than 1, the n-particle product states of $c_1^\dagger c_2^\dagger \ldots c_n^\dagger|\mathbf{0}\rangle$ are also eigenstates of $G_{\mathrm{ad}}(\tau)$. However, if the total boson number is larger than 1, there are two situations due to the relative strength of $\omega(\frac{\tau}{2})$ and U. In the limit when $H_U=0$, the eigenstates of $G_{\mathrm{ad}}(\tau)$ will be condensed to $c_1^{\dagger n}|\mathbf{0}\rangle$. When the on-site repulsion is strong, the eigenstates have the same form as those of fermionic chains if the total boson number is not large.

It is important to note that the hopping term T_h does not affect the above formalism, as long as it drives the system within the adiabatic regime. In other words, this adiabatic protocol is insensitive to the driving Hamiltonian, which makes it a promising candidate in state transmission. This approach is applicable to higher dimensional systems.

V. LINEAR MODEL

We now consider a linear Hamiltonian with the same notations for operators as in Eq. (9),

$$H = \sum_j J_j(e^{i\vartheta}a_j^\dagger a_{j+1} + e^{-i\vartheta}a_{j+1}^\dagger a_j) + \sum_j \epsilon_j n_j, \tag{12}$$

where ϑ is a phase angle. It may be a linearly coupled bosonic Hamiltonian which is equivalent to that of the on-chip coupled cavities (e.g., in Refs. [10–12]) or a generalized Bose-Hubbard model without on-site repulsion [9]. The desired parameters ϵ_j and J_j can be realized by experimental methods such as tunable transmission line resonators, superconducting quantum interference device couplers [13], and external magnetic traps [9]. The Hamiltonian may also represent a general XY model of a spin chain.

A recent work [14] discussed the mapping between a rank l irreducible spherical tensor bosonic operator $A_{lm}^\dagger$ and the creation operator $a_k^\dagger$ at site $k=m+\frac{N+1}{2}$. Those results can be used directly to the spinless fermionic and spin cases modeled by Eq. (12). The three components of the angular momentum vector $\mathbf{L}$ may be expressed by creation and annihilation operators of fermions or bosons,

$$L_x = \sum_j D_j(a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j),$$

$$L_y = i\sum_j D_j(a_j^\dagger a_{j+1} - a_{j+1}^\dagger a_j),$$

$$L_z = \sum_j \left(j - \frac{N+1}{2}\right)n_j,$$

where

$$D_j = \frac{\sqrt{j(N-j)}}{2}.$$

If we select J_j and ϵ_j in Hamiltonian (12) such that $J_j=JD_j$ [2] and $\epsilon_j=0$ [the case $\epsilon_j=\epsilon(j-\frac{N+1}{2})$ will be considered in the next section], the time-evolution operator of the Hamiltonian becomes

$$U(\tau) = \exp(-iJ\tau L_x). \tag{13}$$

The evolution operator $U(\tau)$ corresponds to a rotation operator $R(\Omega)$. The irreducible tensor operator $A_{lm}^\dagger$ in the Heisenberg representation evolves as

$$A_{lm}^\dagger(\tau) = \sum_{m'} (-)^{i(\pi/2)(m'-m)}d_{m'm}^l(-J\tau)A_{lm'}^\dagger(0), \tag{14}$$

where $A_{lm'}^\dagger(0)=A_{lm}^\dagger$. When $\tau=\pi/J$, the expression is reduced to a simple form

$$U^\dagger(t_0)a_i^\dagger U(t_0) = ra_{N-i+1}^\dagger, \tag{15}$$

where the factor

$$r = \exp\left(i\pi\frac{N-1}{2}\right),$$

and its interesting effect has been discussed in [14]. It is easy to show that, in this case, we can use

$$\mathcal{P} = r^* \exp(i\pi L_x)$$

as an exchange operator, which is equivalent to the exchange operator within the two processors and mirror exchanging the site indices. The quasievolution operator $G(\tau)$ now becomes

$$G_l(\tau) = r^* \exp[-i(J\tau - \pi)L_x]. \tag{16}$$

An eigenstate $\Psi_k(0)$ may be expressed by the product of the operators

$$\tilde{A}_{lm}^\dagger = \exp\left(-i\frac{\pi}{2}L_y\right)A_{lm}^\dagger \exp\left(i\frac{\pi}{2}L_y\right) \tag{17}$$

acting on the vacuum state $|\mathbf{0}\rangle$. At the time $\tau=\pi/J$, the operator $G_l(\pi/J)=r^*$ is a constant. All product states of $a_i^\dagger$ acting on the vacuum state are the eigenstates of $G_l(\pi/J)$.

VI. GENERALIZATION USING THE DRESSING TRANSFORMATION

The time-independent dressing transformations W preserve the commutation relation [14] among the angular momentum components but introduce new effects. The whole family of Hamiltonians generated [15] by dressing transformations W can behave as the linear cases studied in the last section. We obtain the quasievolution operators and their eigenstates of two models via individual dressing transfor-

mations. As an example, under the transformation

$$W_j = \exp\left[-\vartheta\left(j-\frac{N+1}{2}\right)n_j\right], \tag{18}$$

the Hamiltonian $H_l = JL_x$ in Ref. [2] becomes

$$H_l' = J(\cos\ \vartheta L_x + \sin\ \vartheta L_y). \tag{19}$$

The eigenstates of H_l' and the set $\{\Psi_k(0)\}$ can be expressed by a product of the tensor operators

$$A_{lm}^{\dagger}(\vartheta) = \exp(i\vartheta L_z)A_{lm}^{\dagger}\exp(-i\vartheta L_z) \tag{20}$$

acting on the vacuum state. The quasievolution operator is

$$G_r(\tau) = r^*\exp[-i(J\tau-\pi)(\cos\ \vartheta L_x + \sin\ \vartheta L_y)]. \tag{21}$$

At the time $\tau=\pi/J$, the quasievolution operator $G_r(\pi/J) = r^*$ is a constant. Again, all product states of $A_{lm}^{\dagger}(\vartheta)$ acting on the vacuum state are the eigenstates of $G_r(\pi/J)$.

Another example is the one-mode squeezing transformation

$$W_j = \exp\left[\frac{\xi}{2}(b_j^2 - b_j^{\dagger 2})\right], \tag{22}$$

where the transformed Hamiltonian reads as

$$H_l' = J(\cosh\ \xi L_x + \sinh\ \xi L_x'), \tag{23}$$

with

$$L_x' = \sum_j D_j(b_j^{\dagger}b_{j+1}^{\dagger} + b_{j+1}b_j). \tag{24}$$

In this case, the quasievolution operator is

$$G_s(\tau) = r^*\exp[-i(J\tau-\pi)(\cosh\ \xi L_x + \sinh\ \xi L_x')]. \tag{25}$$

At the time $\tau=\pi/J$, again $G_s(\pi/J)=r^*$ is a constant.

VII. PERFECT STATE TRANSFER

Perfect state transfer was found [2–5] by designing specific strengths of the coupling constants in spin chains. Interesting studies of generic properties of perfect state transfer have been carried out in the last few years (see, e.g., [16–19]). Recently, it has also drawn experimentalist's attention [20]. Below we will look into PST in terms of the complete orthogonal set $\{\Psi_k(0)\}$.

Although exact state transmissions exist universally, not all of them are significant for PST. For instance, the vacuum state $|\mathbf{0}\rangle$ is an eigenstate of $G(\tau)$ in the above example, such that $\rho_{\alpha\beta}^A(0)=\rho_{\alpha\beta}^B(\tau)$, but there is no actual information transmitted in the process since the two processors share the same information on this state. A significant exact state transmission for the PST requires that at least some of the eigenstates of $G(\tau)$ in the set $\{\Psi_k(0)\}$ are biased to occupy processors A and B. Ideally, if an eigenstate of $G(\tau)$ is localized at processor A then that state can be perfectly transferred. Here, localization means that the targeted density matrix $\rho^A(0)$ is a state that does not entangle other states outside processor A.

In quantum information theory, quantum state transfer often refers to transferring an unknown state. Since the set $\{\Psi_k(0)\}$ is a complete orthogonal set, a known or unknown initial state can be expanded as $\Psi(0)=\Sigma_{k=1}^M C_k\Psi_k(0)$, which is usually not an eigenstate of $G(\tau)$ because

$$G(\tau)\Psi(0) = \sum_{k=1}^{M} C_k e^{i\phi_k}\Psi_k(0). \tag{26}$$

At a specific time τ^*, $\Psi(\tau^*)$ may again become an eigenstate of $G(\tau^*)$ such that

$$\Psi(\tau^*) = e^{i\phi(\tau^*)}\Psi(0). \tag{27}$$

The condition to satisfy this equation is

$$C_k\{\exp(i\phi_k) - \exp[i\phi(\tau^*)]\} = 0. \tag{28}$$

Therefore, for $C_k\neq 0$, one obtains the condition $\phi(\tau^*)=\phi_k + 2\pi K_k$, where K_k are arbitrary integers. This is a very restrictive condition if there are many coefficients $C_k\neq 0$, which may only happen for particular systems with symmetry. As examples, in the above linear system [when $\tau^*=\pi/J$, because $G_l(\pi/J)$, $G_s(\pi/J)$, and $G_r(\pi/J)$ are constants] any state in processor A is an eigenstate of $G(\tau^*)$. However, if there are few nonzero C_k's, the perfect state transfer still happens even without symmetry. In the cases of two nonzero C_k's, the perfect state transfer will *always* happen. This interesting result appears directly from the theoretical framework presented above.

The adiabatic process is an example. Assume that we are initially in a superposition state $\Psi(0)=C_1|\mathbf{0}\rangle + C_2a_1^{\dagger}|\mathbf{0}\rangle$, then

$$G(\tau)\Psi(0) = C_1|\mathbf{0}\rangle + C_2e^{-i\phi(\tau)}a_1^{\dagger}|\mathbf{0}\rangle$$

is not an eigenstate of $G(\tau)$ but becomes an eigenstate at a given time τ^* when $\phi(\tau^*)=2\pi$. Thus, $\Psi(\tau^*)$ is an eigenstate of $G(\tau^*)$. The state evolves from time 0 to τ^* such that

$$C_1|\mathbf{0}\rangle + C_2a_1^{\dagger}|\mathbf{0}\rangle \rightarrow C_1|\mathbf{0}\rangle + C_2a_N^{\dagger}|\mathbf{0}\rangle.$$

This is a perfect adiabatic state transfer. When the on-site repulsion is small, an arbitrary function $f(b_1^{\dagger})$ can also be perfectly transferred. As a specific example, here we set $T_h = L_x$, $h_s=L_z$, and $H_U=0$ in Eq. (9), so that the Hamiltonian

$$H(t) = G\exp[i\phi(\Delta)L_y]L_z\exp[-i\phi(\Delta)L_y], \tag{29}$$

where

$$G = \sqrt{J(\Delta)^2 + \omega(\Delta)^2} \tag{30}$$

and

$$\phi(\Delta) = \arctan\left(\frac{J(\Delta)}{\omega(\Delta)}\right). \tag{31}$$

If G is a constant, the ground state becomes $\exp[i\phi(\Delta)L_y]a_1^{\dagger}|\mathbf{0}\rangle$, with a time-independent eigenvalue $-G(N-1)/2$. Thus, one can easily obtain the adiabatic condition for the ground state

$$\left|\frac{1}{2}\frac{d\phi}{dt}\sqrt{N-1}\right| \ll 1, \tag{32}$$

which can be satisfied by controlling the ratio $J(\Delta)/\omega(\Delta)$.

VIII. PERFECT MIXED-STATE TRANSFER

If the eigenstate $\Psi(\tau^*)$ is not completely localized at processor A, a reduced density matrix at processor A with matrix elements,

$$\rho^A_{\alpha\beta} = \langle\Psi(\tau^*)|\beta_A\rangle\langle\alpha_A|\Psi(\tau^*)\rangle, \tag{33}$$

can be perfectly transferred from processor A to B. As an example, we will consider a *collective* dressing transformation. Equation (13) with $\epsilon \neq 0$ can be regarded as an evolution operator via a dressing transformation $W=\exp(i\theta L_y)$, where $\theta=\arctan(\epsilon/J)$, for which we have

$$H = JL_x + \epsilon L_z = \sqrt{J^2+\epsilon^2}WL_xW^\dagger. \tag{34}$$

At the time

$$\tau^* = \frac{\pi}{\sqrt{J^2+\epsilon^2}}, \tag{35}$$

a dressed tensor is

$$A^\dagger_{lm}(\theta) = WA^\dagger_{lm}W^\dagger = \sum_{m'} d^l_{mm'}(\theta)A^\dagger_{lm'}. \tag{36}$$

Products of tensors $A^\dagger_{lm}(\vartheta)$ acting on the vacuum state, for example, $A^\dagger_{lm}(\vartheta)|\mathbf{0}\rangle$, are eigenstates of $G(\tau^*)$. A state

$$\Psi(0) = C_1|\mathbf{0}\rangle + C_2A^\dagger_{l-l}(\theta)|\mathbf{0}\rangle \tag{37}$$

cannot be prepared at processor A located at the first site. The reduced density matrix at site 1 now becomes

$$\rho^A = \begin{bmatrix} |C_1|^2 & C_1C_2^*\left(\cos\frac{\theta}{2}\right)^{N-1} \\ C_2C_1^*\left(\cos\frac{\theta}{2}\right)^{N-1} & |C_2|^2\left(\cos\frac{\theta}{2}\right)^{2N-2} \end{bmatrix}, \tag{38}$$

where we have used

$$d^l_{-l-l}(\theta) = \left(\cos\frac{\theta}{2}\right)^{N-1}.$$

However, this mixed state can still be perfectly transferred from site 1 to site N during the time interval τ^*. Density matrix (38) becomes a pure state when N goes to infinity or θ is small (i.e., when ϵ is small, as in experiments [21]). In that case, it becomes a perfect pure-state transfer. We emphasize here that although we transfer a mixed state, our transfer is still perfect. Our result is different from previous ones where the fidelity is less than 100% (see, e.g., Ref. [22] and references therein).

IX. CONCLUSION

We have shown the general existence of a set of initial states such that exact state transmissions can take place. The result is universal in the sense that it holds for arbitrary interactions, through *any* media between the two processors and at *any* given evolution time. The existence of such a set is essentially based on the properties of a unitary operator. We have shown a unitary operator, called the quasievolution operator, whose complete orthogonal set of eigenstates can perform the exact state transmission. We illustrate the "broad applicability" of this set of eigenstates through analytical models. Generally the quasievolution operator can be numerically diagonalized especially for small systems. The ability to transfer unknown states is essential to quantum information processing. The set can be used as a basis to perform perfect state transfer in its degenerate subspaces. In addition, the present formalism leads us to propose an adiabatic perfect state transfer protocol, which is insensitive to the Hamiltonian driving the transfer.

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