

Entanglement purification without controlled-NOT gates by using the natural dynamics of spin chains

Koji Maruyama^{1,2} and Franco Nori^{1,2,3}

¹*Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi 351-0198, Japan*

²*CREST, Japan Science and Technology Agency, Kawaguchi, Saitama 3320012, Japan*

³*Center for Theoretical Physics, Physics Department, Center for the Study of Complex Systems, University of Michigan, Ann Arbor, Michigan 48109-1040, USA*

(Received 22 February 2008; revised manuscript received 13 June 2008; published 7 August 2008)

We present a simple protocol to purify bipartite entanglement in spin-1/2 particles by utilizing only natural spin-spin interactions, i.e., those that can commonly be realized in realistic physical systems and S_z measurements on single spins. Even the standard isotropic Heisenberg interaction is shown to be sufficient to purify mixed state entanglement. Our protocol does not need controlled-NOT (CNOT) gates that are very hard to implement experimentally. This approach could be useful for quantum-information processing in solid-state-based systems.

DOI: [10.1103/PhysRevA.78.022312](https://doi.org/10.1103/PhysRevA.78.022312)

PACS number(s): 03.67.Bg, 03.67.Pp, 75.10.Pq

I. INTRODUCTION

Entanglement purification is one of the most important tasks in quantum-information processing [1–4]. It is a process to extract strongly entangled pairs out of initially weakly entangled ones using local operations and classical communication. By repeating the purification process, (near-) maximal entanglement can be obtained. Such a task is indispensable because maximally entangled states are an irreplaceable resource for many important protocols, such as quantum cryptography, quantum teleportation, and quantum repeaters. Unfortunately, as entanglement can only be generated by direct physical interactions, the coherent physical transfer of quantum states is necessary. However, such a transport is always fraught with difficulties due to interactions with the environment, which lowers the quality of entanglement. Therefore, a number of quantum-information processing tasks rely on the feasibility of entanglement purification, which is the key process to distill arbitrarily high entanglement out of degraded states.

Most well-known entanglement purification protocols for bipartite two dimensional systems were proposed by Bennett *et al.* [1,2] [the Bennett-Brassard-Popescu-Schumacher-Smolín-Wootters (BBPSSW) protocol] and Deutsch *et al.* [3] [the Deutsch-Ekert-Jozsa-Macchiavello-Popescu-Sanpera (DEJMPS) protocol]. Both operate on two pairs shared between two separated parties Alice and Bob to enhance entanglement in one of the pairs. An operation commonly used in these protocols is a controlled-NOT (CNOT) gate applied at both ends of the channel. A CNOT gate flips one of the spins (target spin) if the other (control spin) is in $|1\rangle$, and leaves the target spin unchanged if the control spin is in $|0\rangle$, where $|0\rangle$ and $|1\rangle$ are some orthogonal vectors. Yet, in general, carrying out a CNOT operation is one of the toughest challenges in experiments. This is why there have been only a few experiments of purification, mainly with photons [5] and one with trapped ions [6]. Throughout this paper we will consider spin-1/2 particles as a representative of two-level systems because our main interest is in the behavior of entanglement in spin chains with standard spin-spin interactions.

The main aim of this paper is to show that entanglement purification is possible only with unmodulated natural interactions between spins and simple S_z measurements on single spins, as long as the fidelity of the initial state to a Bell state is larger than 1/2. A physical intuition behind this idea is that measuring a subset of spins would project the remaining pairs onto a more strongly entangled state since there are constructive and destructive interference of magnons in a spin chain during its dynamical evolution [7–9].

Here, there is no need of mapping quantum information to different physical systems, e.g., photons. In addition, there is no need of artificial multiqubit gate operations, such as CNOT. With an isotropic Heisenberg interaction, the implementation of a CNOT operation has to involve at least using two two-spin operations [10], that is, the accumulation of errors is very likely to occur. From the pragmatic point of view, we naturally wish to minimize the number of artificial controls. Our proposal simplifies the necessary manipulations, compared with those in experiments [6], and also theoretical proposals for solid state circuits, such as Ref. [11]. It does not entail even single spin operations after the first round of the protocol, that is, those operations are necessary only once at the beginning. Several analyses have been done, for example, for quantum state transfer with a chain of spin-1/2 particles [7] or harmonic oscillators [12], elementary gate operations [13], and cloning transformation [14] under similar motivations, however, ours performs an invaluable, and practically very important, quantum-information processing task with minimal control.

II. ENTANGLEMENT PURIFICATION WITH SPIN CHAINS

A general scenario of entanglement purification by spin chain dynamics is depicted in Fig. 1. Alice and Bob share n pairs of weakly entangled spins forming two spin chains so that Alice's j th spin is entangled with Bob's j th spin for all j . Each chain will evolve due to the exchange interaction between nearest-neighbor spins. We will primarily consider the isotropic Heisenberg interaction, however, let us assume

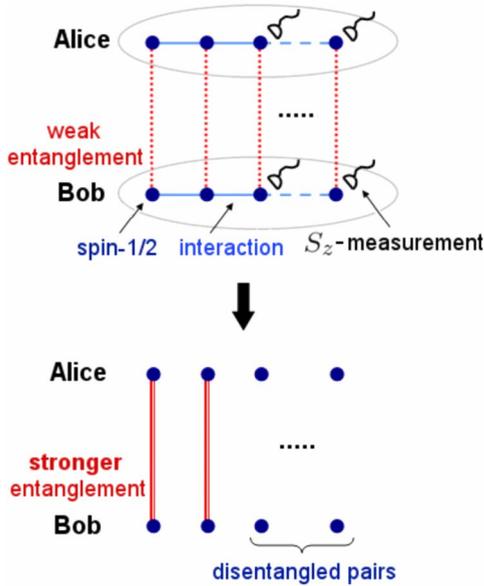


FIG. 1. (Color online) Entanglement purification using spin chains. Alice and Bob initially share n pairs of weakly entangled spins forming two spin chains. They let the spin chains evolve under a standard spin-spin interaction and perform single-spin measurements on a subset of the spins. Entanglement purification succeeds if the remaining m pairs contain higher entanglement than the initial state.

the Hamiltonian in a generic anisotropic form as

$$H = - \sum_{\langle i,j \rangle} (J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z), \quad (1)$$

where J_k and S_i^k ($k=x,y,z$) are the coupling strengths and the standard SU(2) spin operators for the i th spin, respectively. After a certain time lapse they measure $n-m$ spins each in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis, turning off the interaction, to enhance or reduce entanglement in the remaining m pairs compared with that of the initial state. All active controls we consider here are one switching-on and one switching-off of the exchange interaction Eq. (1), and single-spin measurements in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis. Single spin operations can be added to the list of available controls, however, the fewer controls, the better. In order to avoid complications due to residual correlations between unmeasured pairs, we consider cases in which only one pair is left unmeasured after a run of the protocol. We will hereafter denote vectors $|\uparrow\rangle$ and $|\downarrow\rangle$ by $|1\rangle$ and $|0\rangle$, respectively, and take $\hbar=1$.

If the initial state of the pair is a partially entangled pure state [15], then a maximally entangled pair can be attained by simply attaching only one spin to the pair locally and letting the two spins evolve under the isotropic Heisenberg interaction, that is, Eq. (1) with $J_x=J_y=J_z=J$. Suppose that the initial state of the pair is given by $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$, where $\alpha, \beta \in \mathbb{R}$, $\alpha^2 + \beta^2 = 1$ and $\alpha < \beta$ (the case of $\alpha > \beta$ can be treated in the same fashion). See Fig. 2(a) for a schematic diagram and the numbering of the spins. The three-spin state at time t can be computed as

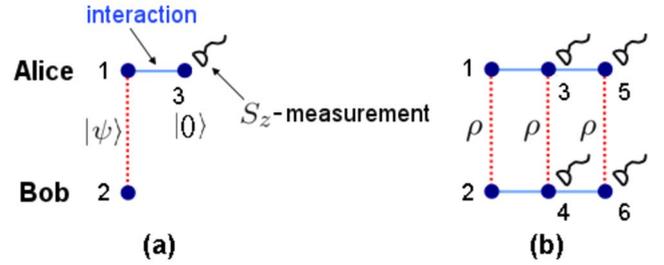


FIG. 2. (Color online) Configurations to obtain one pair with higher entanglement by spin chain dynamics. (a) If the initial state of a pair is pure, an ancillary spin suffices. (b) If the pairs are initially in a mixed state, we need three pairs, two of which are to be measured. The blue lines and red dashed lines indicate interaction and entanglement, respectively, and the measurement is in the $\{|0\rangle, |1\rangle\}$ basis.

$$|\psi(t)\rangle = \alpha e^{iJt/4} |000\rangle + \beta e^{-iJt/4} \left(i \sin \frac{Jt}{2} |011\rangle + \cos \frac{Jt}{2} |110\rangle \right), \quad (2)$$

where spins are ordered according to the numbering in Fig. 2. If an S_z measurement on the spin “3” at $Jt_{\max} = 2 \cos^{-1}(\alpha/\beta)$ gives the outcome 0, the state of the first pair (spins “1” and “2”) will be projected onto a maximally entangled state with probability $2\alpha^2$. The net result stays the same even with the (isotropic) antiferromagnetic Hamiltonian, i.e., $J < 0$.

This process can be seen as equivalent to local filtering [16], in which a spin and an ancilla evolve under some unitary operation, and then a measurement is performed on the ancilla. For specific measurement outcomes the pair becomes closer to the maximally entangled state.

Let us move on to the case in which the initial state of the pairs is mixed. In order to simplify the discussion, we shall assume that each pair is in a Werner state [17], that is,

$$\rho_W = F |\Phi^+\rangle\langle\Phi^+| + \frac{1-F}{3} (|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|), \quad (3)$$

where $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are the maximally entangled Bell states, and F is the fidelity between ρ_W and $|\Phi^+\rangle$. The reasoning for this assumption is that any mixed state can be transformed into a Werner state with random bilateral local unitary operations (twirling) without lowering its fidelity with a Bell state, thus showing the feasibility of entanglement purification for Werner states is sufficient, in principle [1]. In what follows, we shall use the BBPSSW protocol as a reference for comparisons because it is easier than others to analyze.

It turns out that higher entanglement can never be attained by starting with two pairs of spins $\rho_W^{\otimes 2}$ regardless of the combinations of values of (J_x, J_y, J_z) in the Hamiltonian. Nevertheless, entanglement in a pair can be enhanced if there are two extra pairs, i.e., three pairs in total as in Fig. 2(b). The reason why this approach fails with two pairs will be explained later.

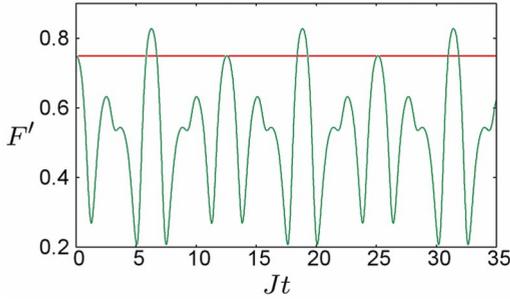


FIG. 3. (Color online) The fidelity as a function of the measurement time, with the initial fidelity equal to 0.75, indicated by the red (horizontal) line.

If there are three pairs, the resulting fidelity does exceed the initial one. An example of the “time evolution of the fidelity is plotted in Fig. 3. In this plot, the isotropic nearest-neighbor interaction $H=J\sum\vec{S}_i\cdot\vec{S}_j$ is assumed and the time t indicates the measurement time. With the initial state ρ_W above, the measurement outcomes $(0,0,1,1)$ or $(1,1,0,0)$ at sites $\langle 3,4,5,6 \rangle$ or $\langle 1,2,5,6 \rangle$ lead to a successful fidelity increase (Hereafter angular brackets $\langle \rangle$ denote the spin sites). The maximum fidelity attainable can be calculated analytically as

$$F'_{\max} = \frac{16 - 53F + 118F^2}{59 - 106F + 128F^2}, \quad (4)$$

which is always larger than F if $F > 1/2$ and is achieved at times $Jt = (2n+1)2\pi$ ($n=0,1,2,\dots$). Figure 4 shows the comparison between the maximum reachable fidelity by our protocol and that by the BBPSSW protocol. The fidelity increase in the spin-chain-based protocol is approximately twice as large as that by the BBPSSW protocol, which is quite significant despite the simplicity of the protocol. There are other combinations of outcomes, $(0,0,0,0)$ and $(1,1,1,1)$ which give a fidelity increase, but we exclude this possibility

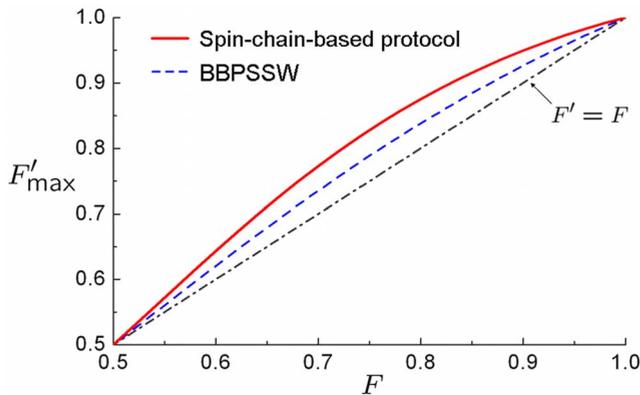


FIG. 4. (Color online) Maximum attainable fidelity F'_{\max} between the resulting state and a Bell state. The top (red) solid line represents the maximum fidelity after a run of our protocol with three pairs. The dashed (blue) line shows the fidelity change by the BBPSSW protocol. The bottom (black) dash-dotted line represents $F' = F$.

because the increase is small (roughly half of the BBPSSW protocol) while the probability is slightly higher than the above case.

What about the necessary time precision for the protocol? The duration (FWHM) Δt to have a fidelity increase larger than $(F'_{\max} - F)/2$ depends on F , but it can be computed to be $J\Delta t < 0.5$ for a wide range of F , i.e., $0.61 < F < 0.94$. With $J \sim 1 \mu\text{eV}$ in, e.g. Ref. [18], this condition gives $\Delta t < 300$ ps, which is much longer than the experimentally feasible accuracy of ~ 10 ps.

Another nice feature of this protocol is that the purified pair stays in the Werner state with the new fidelity F'_{\max} , i.e., $\rho' = F'_{\max}|\Phi^+\rangle\langle\Phi^+| + (1 - F'_{\max})/3(|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|)$. This means that further twirling operations are not necessary when iterating the protocol with three new pairs in ρ' , unlike the BBPSSW protocol.

III. DISCUSSION

Let us now discuss the case where there are only two pairs of spins for our scheme. As we have mentioned above, two pairs of a Werner state cannot be purified if there are no extra single spin operations. We sketch its reasoning and show that the protocol can be as efficient as the BBPSSW protocol when single spin operations are available in addition to a single switching on and off of the two-spin interaction of the form of Eq. (1).

The Hamiltonian (1) has its invariant subspaces spanned by the following combinations of Bell-product states, namely, $(\Phi^+\Phi^-, \Phi^-\Phi^+)$, $(\Phi^+\Psi^\pm, \Psi^\pm\Phi^+)$, $(\Psi^+\Psi^-, \Psi^-\Psi^+)$, $(\Psi^\pm\Phi^-, \Phi^-\Psi^\pm)$, and $(\Phi^+\Phi^+, \Phi^-\Phi^-, \Psi^+\Psi^+, \Psi^-\Psi^-)$. Note that these subspaces are independent of the values of the coupling strengths J_k ($k=x,y,z$), because all terms in Eq. (1) commute with each other, thus simultaneously diagonalizable regardless of J_k 's. In the following, the dominant component in the initial Werner state is assumed to be Φ^+ and we will look at the resulting state of one of the pairs when the outcomes of the S_z measurement on the other pair are the same. We do not lose any generality by such assumptions thanks to the symmetry of the Hamiltonian and the Werner state.

The initial state $\rho_W^{\otimes 2}$ is diagonal in the Bell-product basis and there are three values of the weight for those Bell-product components. Let us write those weights as $w_L = F^2$, $w_M = F(1-F)/3$, and $w_S = (1-F)^2/9$. Since the components in the same rank-2 subspace above have an equal weight, $\rho_W^{\otimes 2}$ is equivalent to an identity operator in each rank-2 subspace. Thus the time evolution of entanglement is determined by that of components in the rank-4 subspace. If we evaluate the fidelity with respect to Φ^+ , its maximum is obtained when the weight of $\Phi^+\Phi^+$ comes back to w_L . The (unnormalized) state of spins $\langle 1,2 \rangle$ after measuring $(0,0)$ or $(1,1)$ at sites $\langle 3,4 \rangle$ then becomes $w_L|\Phi^+\rangle\langle\Phi^+| + w_S(|\Phi^-\rangle\langle\Phi^-| + |\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|) + w_S(|\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-|)$. The fidelity between this state and Φ^+ is $(w_L + w_M)/(w_L + 4w_M + 3w_S) = F$, hence there is no fidelity increase. The same logic can be applied straightforwardly to cases with different expected outcomes and a different reference Bell state for fidelity evaluation.

If we allow anisotropy for the interaction, entanglement purification can be achieved with as big fidelity increase as in the BBPSSW protocol with the help of extra single spin operations. This is simply because a CNOT gate can be decomposed into a sequence of single spin operations and one-time activation of the XY- or Ising-type interaction [19]. But with the isotropic Heisenberg interaction, such a decomposition is known to be impossible [20].

The effect of the Heisenberg Hamiltonian is quite different when there are three pairs and its invariant subspaces can no longer be described neatly by the Bell states. Physically, this difference can be attributed to the behavior of spinless fermions that appear after the Jordan-Wigner transformation. When there are only two spins in a chain the fermion propagates (or hops) freely between sites, hence the simple dynamics described above. On the contrary, in a three-spin chain, the fermions interact with each other and there can be some nontrivial interferences between different modes that result in the concentration of entanglement.

Despite such an intuitive picture, the application of the Jordan-Wigner transformation to our problem makes the whole calculation quickly untractable. Instead, here we attempt to characterize the three-pair Hamiltonian in terms of the ability of making correlations between Bell pairs. Such an analysis is useful because entanglement purification can be seen as a process to extract information on a given pair by correlating other pairs to it. The BBPSSW protocol makes use of the correlating power of the bilateral CNOT, which correlates two Bell states as

$$|\Psi_{kl}\rangle|\Psi_{mn}\rangle \rightarrow |\Psi_{k,l+n}\rangle|\Psi_{k+m,n}\rangle, \quad (5)$$

where $\Psi_{00}, \Psi_{01}, \Psi_{10}, \Psi_{11}$ are in our notation $\Phi^+, \Phi^-, \Psi^+, \Psi^-$, respectively, and additions in the indices are in modulo 2. If a bilateral CNOT is applied on a randomly chosen Bell pair and another pair in a known Bell state, the mutual information between these two pairs is increased from 0 bits to 1 bit. Similarly, the time evolution operator $U(Jt=2\pi) = \exp[-2\pi i \sum_i \vec{S}_i \cdot \vec{S}_j]$ for three pairs increases the mutual information (in the Bell basis), between a given unknown pair and the other two, from 0 to 1.48 bits, while $U = \exp(-iHt)$ generates no correlation between pairs in the two-pair case. The correlation generated between pairs then gives (partial) information on the unmeasured pair, enabling us to single out pairs containing higher entanglement.

A price we have to pay for the simplicity and the larger fidelity increase is the probability of success, which is about 2/5 of that of the BBPSSW protocol. Let us make a rough estimation of the number l of initial pairs to obtain a final pair achieving a desired fidelity increase the for BBPSSW protocol (l_B) and for our spin-chain-based protocol (l_{SC}). These can be written as

$$l_B = \left(\frac{2}{p_B}\right)^{r_B} \quad \text{and} \quad l_{SC} = \left(\frac{3}{p_{SC}}\right)^{r_{SC}}, \quad (6)$$

where p_B (p_{SC}) and r_B (r_{SC}) are the success probability per run and the number of rounds (“rounds” as in a sport

tournament), respectively. We approximate p_{SC} by its average over the range $1/2 < F < 1$ for simplicity as it does not vary much. Also, by expanding the form of F' with respect to F around 1, we see that

$$r_B \approx \left(\ln \frac{2}{3}\right)^{-1} \ln \left(\frac{1-F_f}{1-F_i}\right) \approx \left(\ln \frac{2}{3}\right)^{-1} \ln \left(\frac{11}{27}\right) r_{SC} = 2.21 r_{SC} \quad (7)$$

for a fidelity change from F_i to F_f . With these approximations, we find that

$$\frac{l_{SC}}{l_B} \approx 1.16^{r_{SC}} \quad (8)$$

holds for any fidelity increase. This might make our protocol appear less attractive than the BBPSSW. However, more important is the number of multispin operations the resulting pair experiences. In our scheme this number is r_{SC} , which is only 23% of $2r_B$, the operations needed for the standard CNOT-based BBPSSW protocol. In addition, the difference in l is rather small. Even if $r_{SC}=5$, which corresponds to a fidelity increase from 0.75 to 0.990, l_{SC}/l_B is only 2.07. We therefore wish to state that the efficiency of our protocol is still notable, considering that only natural evolutions and an initial one-time twirling are involved.

Before summarizing, let us briefly note the effect of anisotropy. Our protocol does not work with the XY or Ising interaction. Yet, a small amount of anisotropy is useful to purify mixed state entanglement. For example, suppose that there is an antisymmetric exchange interaction, also known as the Dzyaloshinskii-Moriya interaction, in addition to the XY interaction as

$$H = -J \sum (S_i^x S_j^x + S_i^y S_j^y) + \vec{d} \cdot \vec{S}_i \times \vec{S}_j, \quad (9)$$

where \vec{d} is the coupling vector that reflects the anisotropy. If a set of three pairs, each of which is in the Werner state with fidelity $F=0.75$, evolves under the Hamiltonian with $J=1$ and $\vec{d}=\{0.1, 0, 0\}$, the fidelity can be as high as 0.81 when two pairs are measured at $t \approx 352$.

IV. SUMMARY AND OUTLOOK

We have shown that entanglement purification is possible with only the free dynamics of spin chains and simple S_z measurements on single spins. Arbitrarily high entanglement can be obtained by repeating the process. The amount of operations is minimal, in the sense that single spin operations (twirling) are needed only once for the entire process and only one activation of the interspin interaction is required for each run. Hence the error accumulations due to artificial controls should be much less likely compared with conventional schemes. More detailed and quantitative comparisons of errors under realistic conditions would thus be one of the important future research topics.

The proposed scheme might also be experimentally realizable in the near future as the number of spins in this

protocol is not large. Candidate physical systems would be, for example, semiconductor quantum dots [10,18,22], superconducting qubits [21], atoms trapped in optical lattices [23], etc., where the interqubit couplings are often described by Heisenberg-type interactions.

If we look at our protocol from a more general perspective, it can be understood as an execution of a quantum-information processing task by a natural time evolution and simple measurements. While we have considered only one specific task in this paper, it would be very intriguing if more complicated tasks could be performed with simple multiparticle interactions.

ACKNOWLEDGMENTS

K.M. is grateful to Martin Plenio for a careful reading of the manuscript. Some calculations were performed with the RSCC system of RIKEN. This work was supported in part by the National Security Agency (NSA), the Army Research Office (ARO), the Laboratory for Physical Sciences (LPS), the National Science Foundation (NSF) Grant No. EIA-0130383, the CREST program of the Japan Science and Technology Agency (JST), the Japan Society for the Promotion of Science Core-to-Core (JSPS-CTC) program, and the JSPS-RFBR Grant No. 06-02-91200.

-
- [1] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, *Phys. Rev. Lett.* **76**, 722 (1996).
- [2] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, *Phys. Rev. A* **54**, 3824 (1996).
- [3] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, *Phys. Rev. Lett.* **77**, 2818 (1996).
- [4] For recent reviews, see, W. Dür and H.-J. Briegel, *Rep. Prog. Phys.* **70**, 1381 (2007); R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, e-print arXiv:quant-ph/0702225.
- [5] J.-W. Pan, C. Simon, C. Brukner, and A. Zeilinger, *Nature (London)* **410**, 1067 (2001); P. G. Kwiat, S. Barraza-Lopez, A. Stefanov, and N. Gisin, *ibid.* **409**, 1014 (2001); T. Yamamoto, M. Koashi, S. K. Ozdemir, and N. Imoto, *ibid.* **421**, 343 (2003).
- [6] R. Reichle, D. Leibfried, E. Knill, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland, *Nature (London)* **443**, 838 (2006).
- [7] S. Bose, *Phys. Rev. Lett.* **91**, 207901 (2003).
- [8] K. Maruyama, T. Iitaka, and F. Nori, *Phys. Rev. A* **75**, 012325 (2007).
- [9] For reviews, see, D. Burgath, *Eur. Phys. J. Spec. Top.* **151**, 147 (2007); S. Bose, *Contemp. Phys.* **48**, 13 (2007).
- [10] D. Loss and D. P. DiVincenzo, *Phys. Rev. A* **57**, 120 (1998).
- [11] J. M. Taylor, W. Dür, P. Zoller, A. Yacoby, C. M. Marcus, and M. D. Lukin, *Phys. Rev. Lett.* **94**, 236803 (2005).
- [12] M. B. Plenio, J. Hartley, and J. Eisert, *New J. Phys.* **6**, 36 (2004).
- [13] M.-H. Yung, D. W. Leung, and S. Bose, *Quantum Inf. Comput.* **4**, 174 (2004).
- [14] G. De Chiara, R. Fazio, C. Macchiavello, S. Montangero, and G. M. Palma, *Phys. Rev. A* **72**, 012328 (2005).
- [15] In this case, “entanglement concentration” is a more appropriate term than “purification.”
- [16] N. Gisin, *Phys. Lett. A* **210**, 151 (1996).
- [17] R. F. Werner, *Phys. Rev. A* **40**, 4277 (1989).
- [18] J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, *Science* **309**, 2180 (2005).
- [19] N. Schuch and J. Siewert, *Phys. Rev. A* **67**, 032301 (2003).
- [20] G. Burkard, D. Loss, D. P. DiVincenzo, and J. A. Smolin, *Phys. Rev. B* **60**, 11404 (1999).
- [21] J. Q. You and F. Nori, *Phys. Today* **58**, 42 (2005).
- [22] S. Das Sarma, R. de Sousa, X. Hu, and B. Koiller, *Solid State Commun.* **133**, 737 (2005).
- [23] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hansch, and I. Bloch, *Nature (London)* **425**, 937 (2003).