15 September 2004

EUROPHYSICS LETTERS

*Europhys. Lett.*, **67** (6), pp. 941–947 (2004) DOI: 10.1209/ep1/i2004-10144-3

## Generation of nonclassical photon states using a superconducting qubit in a microcavity

YU-XI LIU<sup>1</sup>, L. F. WEI<sup>1,2</sup> and FRANCO NORI<sup>1,3</sup>

 <sup>1</sup> Frontier Research System, The Institute of Physical and Chemical Research (RIKEN) Wako-shi, Saitama, 351-0198, Japan
 <sup>2</sup> Institute of Quantum optics and Quantum information, Department of Physics Shanghai Jiaotong University - Shanghai 200030, PRC
 <sup>3</sup> Center for Theoretical Physics, Physics Department, CSCS The University of Michigan - Ann Arbor, MI 48109-1120, USA

(received 13 April 2004; accepted in final form 8 July 2004)

PACS.	42.50.Dv –	Nonclassical states of the electromagnetic field, including entangled photon
		states; quantum state engineering and measurements.
PACS.	74.50.+r -	Tunneling phenomena; point contacts, weak links, Josephson effects.

PACS. 42.50.Ct – Quantum description of interaction of light and matter; related experiments.

**Abstract.** – Based on the interaction between the radiation field and a superconductor, we propose a way to engineer quantum states using a SQUID charge qubit inside a microcavity. This device can act as a deterministic single-photon source as well as generate any Fock states and an arbitrary superposition of Fock states for the cavity field. The controllable interaction between the cavity field and the qubit can be realized by the tunable gate voltage and classical magnetic field applied to the SQUID.

The generation of quantum states of the radiation field has been a topic of growing interest in recent years. This is because of possible applications in quantum communication and information processing, such as quantum networks, secure quantum communications, and quantum cryptography [1]. Based on the interaction between the radiation field and atoms, many theoretical schemes have been proposed for the generation of Fock states [2,3] and their arbitrary superpositions [4,5]. Experiments have generated single-photon states in quantum dots [6], atoms inside a microcavity [7], and other systems [8]. A superposition of the vacuum and one-photon states can also be experimentally created by truncating an input coherent state or using cavity quantum electrodynamics [9]. However, how to generate an arbitrary photon state by virtue of the interaction between the radiation field and solid-state quantum devices seems to be unknown both theoretically and experimentally. Recent progress in superconducting quantum devices (*e.g.*, [10, 11]) makes it possible to do quantum state engineering experiments in these systems, and also there have been proposals on superconducting qubits interacting with the nonclassical electromagnetic field [12–17].

Here, we present an experimentally feasible scheme to generate quantum states of a singlemode cavity field in the microwave regime by using the photon transition between the ground and first excited states of a macroscopic two-level system formed by a superconducting quantum interference device (SQUID). This artificial two-level "atom" can be easily controlled by



Fig. 1 – (a) A charge qubit formed by a SQUID device, equivalent to a controllable macroscopic two-level system, is placed into a superconducting microwave cavity in (b). The coupling between the quantized cavity field and qubit system is realized via the magnetic flux  $\Phi_X$  through the SQUID.

an applied gate voltage  $V_{\rm g}$  and the flux  $\Phi_{\rm c}$  generated by the classical magnetic field through the SQUID (e.g., [14, 18]). The process of generating photon states in this device includes three main steps: i) The artificial atom operates at the degeneracy point by choosing appropriate values for  $V_{\rm g}$  and  $\Phi_{\rm c}$ . There is no interaction between the quantized cavity field and "atom" at this stage. ii) Afterwards, new  $V_{\rm g}$  and  $\Phi_{\rm c}$  are selected such that the cavity field interacts resonantly with the "atom" and evolves during a designated time. iii) The above two steps can be repeated until a desired state is obtained. Finally, the flux  $\Phi_{\rm c}$  can be adjusted to a special value, then the interaction is switched off, and the desired photon state appears in the cavity. This process is similar to that of a micromaser [2] and is described below.

Model. – The macroscopic two-level system studied here is shown in fig. 1(a). A SQUIDtype superconducting box with  $n_c$  excess Cooper-pair charges is connected to a superconducting loop via two identical Josephson junctions with capacitors  $C_J$  and coupling energies  $E_J$ . A controllable gate voltage  $V_g$  is coupled to the box via a gate capacitor  $C_g$ . We assume that the superconducting energy gap  $\Delta$  is the largest energy. Then, at low temperatures, the quasiparticle tunneling is suppressed and no quasi-particle excitation can be found on the island. Only Cooper pairs coherently tunnel in the superconducting junctions. The above assumptions are consistent with most experiments on charge qubits. Then the standard Hamiltonian [18] is

$$H_{\rm qb} = 4E_{\rm ch}(n_{\rm c} - n_{\rm g})^2 - 2E_{\rm J}\cos\left(\frac{\pi\Phi_X}{\Phi_0}\right)\cos\Theta,\tag{1}$$

where  $\Phi_X$  is the total flux through the SQUID loop and  $\Phi_0$  the flux quantum. Thus, the superconducting loop is used to control the Josephson coupling energy by adjusting the flux through this loop. Below, we show that it can also switch on and off the qubit-field interaction. The dimensionless gate charge,  $n_{\rm g} = C_{\rm g} V_{\rm g}/2e$ , is controlled by  $V_{\rm g}$ . The single-electron charging energy is  $E_{\rm ch} = e^2/2(C_{\rm g} + 2C_{\rm J})$ .  $\Theta = (\phi_1 + \phi_2)/2$  is the quantum-mechanical conjugate of the number operator  $n_{\rm c}$  of the Cooper pairs on the box, where  $\phi_i$  (i = 1, 2) is the phase difference for each junction. The superconducting box is assumed to work in the charging regime with condition  $k_{\rm B}T \ll E_{\rm J} \ll E_{\rm ch} \ll \Delta$ , where T and  $k_{\rm B}$  are temperature and Boltzmann constant, respectively. If the gate voltages is near a degeneracy point  $n_{\rm g} = 1/2$ , the superconducting box is a charge qubit [18], which is a controllable two-level system characterized by the two lowest charge states  $|g\rangle$  (for  $n_{\rm c} = 0$ ) and  $|e\rangle$  (for  $n_{\rm c} = 1$ ). However, if the quasi-particle excitation cannot be completely suppressed, a continuum of low-lying quasi-particle states will be present, and the Hamiltonian (1) cannot be reduced to a system with two energy levels even when the gate voltage is near the degeneracy point [19].

Now we further consider that the qubit is placed in a single-mode microwave superconducting cavity, depicted in fig. 1(b), the flux  $\Phi_X$  through the SQUID can be expressed as [12–14]  $\Phi_X = \Phi_c + \Phi_q$  where the flux  $\Phi_c$  and  $\Phi_q = \eta a + \eta^* a^{\dagger}$  are generated by a classical applied magnetic field and the quantized cavity field, respectively. Here  $\eta = \int_S \boldsymbol{u}(\boldsymbol{r}) \cdot d\boldsymbol{s}$  and  $\boldsymbol{u}(\boldsymbol{r})$  is the mode function of the cavity field, with annihilation (creation) operators  $a(a^{\dagger})$ , and S is the surface defined by the contour of the SQUID. Considering the above, we obtain

$$H = \hbar \omega a^{\dagger} a + E_z \sigma_z - E_J (\sigma_+ + \sigma_-) \cos \left[ \frac{\pi}{\Phi_0} \left( \Phi_c I + \eta \, a + \eta^* \, a^{\dagger} \right) \right], \tag{2}$$

where the first two terms represent the free Hamiltonians of the cavity field with frequency  $\omega =$  $4E_{\rm ch}/\hbar$  and the qubit with the energy  $E_z = -2E_{\rm ch}(1-2n_{\rm g})$ , I is the identity operator. The third term is the nonlinear photon-qubit interaction which is switchable by the flux  $\Phi_c$ . The charge excited state  $|e\rangle$  and ground state  $|g\rangle$  correspond to the eigenstates  $|\downarrow\rangle$  and  $|\uparrow\rangle$  of the spin operator  $\sigma_z$ , respectively. The cosine in eq. (2) can be further decomposed into classical and quantized parts, and the quantized parts  $\sin[\pi(\eta a + \text{H.c.})/\Phi_0]$  and  $\cos[\pi(\eta a + \text{H.c.})/\Phi_0]$ can be further expanded as a power series in  $a(a^{\dagger})$ . Here, the single-photon transition between the states  $|e,n\rangle$  and  $|g,n+1\rangle$  satisfies the condition  $(\pi|\eta|/\Phi_0)\sqrt{n+1} \ll 1$ , where n is the number of photons; therefore all higher orders of  $\pi |\eta| / \Phi_0$  can be neglected and only a singlephoton transition is kept in the expansion of eq. (2). Using the notation for trapped ion systems (e.q., [20]), the first red (blue) sideband excitations  $\beta a \sigma_+$  + H.c. ( $\beta a \sigma_-$  + H.c.) for interactions of the cavity field and the qubit [13], with photon-qubit coupling constant  $\beta =$  $(\pi \eta E_{\rm J}/\Phi_0)\sin(\pi \Phi_{\rm c}/\Phi_0)$ , can be obtained by adjusting the gate voltages  $V_{\rm g}$  and the flux  $\Phi_{\rm c}$ . They correspond to  $2E_z = \hbar\omega (2E_z = -\hbar\omega)$  and dimensionless gate charge  $n_g = 1$   $(n_g = 0)$ . Also  $\xi(\sigma_+ + \sigma_-)$  with  $\xi = E_J \cos(\pi \Phi_c/\Phi_0)$  is called the carrier [13], which corresponds to  $n_{\rm g} = 1/2$ . The Hamiltonian (2), with the above assumptions, is our model.

Preparation process. – We choose  $|0, g\rangle$  as our initial state, where the cavity field is in the vacuum state  $|0\rangle$  and the qubit is in the ground state  $|g\rangle$ . The goal is to prepare an arbitrary pure state of the cavity field,

$$|\psi\rangle = \sum_{n=0}^{N} c_n |n,g\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle,$$
(3)

where  $|n\rangle$  denotes the Fock states of the cavity field with excitation number  $n = 0, 1, 2, \cdots$ . A Fock state  $|m\rangle$  with m photons is a special case of eq. (3) with conditions  $c_n = 0$  for all  $n \neq m$  with  $0 < m \leq N$ .

Thermal photons in the cavity have to be suppressed in order to obtain the vacuum state  $|0\rangle$ . In the microwave region  $0.1 \sim 15$  cm, the mean number of thermal photons  $\langle n_{\rm th} \rangle$  satisfies  $3.0 \times 10^{-208} \leq \langle n_{\rm th} \rangle \leq 0.043$  at T = 30 mK, and  $1.7 \times 10^{-104} \leq \langle n_{\rm th} \rangle \leq 0.26$  at T = 60 mK. These temperatures can be obtained experimentally (*e.g.*, in [11,21]).

After the system is initialized, two different processes are required to engineer the state of the cavity field. The first process involves rotating the qubit state, but keeping the cavity field state unchanged. This stage can be experimentally realized by tuning the gate voltage and classical magnetic field such that  $n_{\rm g} = 1/2$  and  $\Phi_{\rm c} = 0$ ; then the time evolution operator  $U_{\rm C}(t)$  of the qubit in the interaction picture is

$$U_{\rm C}(t) = \cos(\Omega_1 t)I + i\sin(\Omega_1 t)(|g\rangle\langle e| + |e\rangle\langle g|), \tag{4}$$

where  $\Omega_1 = E_J/\hbar$ . The subscript "C" in  $U_C(t)$  denotes the carrier process, which can superpose two levels of the qubit, and it can also flip the ground state  $|g\rangle$  or the excited state  $|e\rangle$  to each other, after a time  $t = \pi (2p-1)/2\Omega_1$ , with positive integer p.

The second process is the first red (blue) sideband excitation, which can be realized by tuning the gate voltage and classical magnetic field such that  $n_{\rm g} = 1$  ( $n_{\rm g} = 0$ ) and  $\Phi_{\rm c} = \Phi_0/2$ . Thus, in the interaction picture, the time evolution operators  $U_{\rm R}(t)$  for the red ( $U_{\rm B}(t)$ )

for the blue) of the cavity field and qubit can be expressed [22] as

$$U_{\rm R}(t) = R_{ee}(t)|e\rangle\langle e| + R_{gg}(t)|g\rangle\langle g| - iR_{ge}(t)|g\rangle\langle e| - iR_{eg}(t)|e\rangle\langle g|$$
(5)

or

$$U_{\rm B}(t) = R_{gg}(t)|e\rangle\langle e| + R_{ee}(t)|g\rangle\langle g| - iR_{ge}(t)|e\rangle\langle g| - iR_{eg}(t)|g\rangle\langle e|$$
(6)

with  $R_{eg}(t) = [e^{i\theta} \sin(|\Omega_2|t\sqrt{aa^{\dagger}})/\sqrt{aa^{\dagger}}]a$ ,  $R_{ge}(t) = B_{eg}^{\dagger}(t)$ ,  $R_{ee}(t) = \cos(|\Omega_2|t\sqrt{aa^{\dagger}})$ , and  $R_{gg}(t) = \cos(|\Omega_2|t\sqrt{a^{\dagger}a})$ , where we have assumed that  $\Omega_2 = \pi \eta E_J/\hbar \Phi_0 = |\Omega_2|e^{i\theta}$ , in which the phase  $\theta$  depends on the mode function of the cavity field  $u(\mathbf{r})$ . The red sideband excitation described by operator  $U_{\rm R}(t)$  can entangle  $|g, n+1\rangle$  with  $|e, n\rangle$ , or flip  $|g, n+1\rangle$  to  $|e, n\rangle$  and vice versa, by choosing the duration of the interaction between the cavity field and the qubit. From eq. (5), it is easy to verify that the emission probability  $P_g$  of the upper level for the qubit is  $P_g = \sin^2(|\Omega_2|t\sqrt{n+1})$ . We find that  $P_g = 1$  when  $|\Omega_2|t\sqrt{n+1} = \pi(2k-1)/2$ , with positive integer k. So when  $t = \pi(2k-1)/(2|\Omega_2|\sqrt{n+1})$ , there are n+1 photons in the cavity and the qubit is in its ground state. The first blue sideband excitation, denoted by  $U_{\rm B}(t)$ , can entangle state  $|e, n+1\rangle$  with state  $|g, n\rangle$ , or flip  $|e, n+1\rangle$  to  $|g, n\rangle$  and vice versa. Below we use the carrier and the first red sideband excitation, represented by  $U_{\rm C}(t)$  and  $U_{\rm R}(t)$ , as an example showing the generation of an arbitrary quantum state of the cavity field.

Using the quantum operations  $U_{\rm C}(t)$  and  $U_{\rm R}(t)$  in eqs. (4) and (5), the single photon state  $|1\rangle$  can be generated from the initial vacuum state  $|0\rangle$ . That is, we can first flip the ground state of the qubit to the excited state when the condition  $\Omega_1 t_1 = \pi/2$  is satisfied for the carrier  $U_{\rm C}(t_1)$ , then we turn on the first red sideband excitation  $U_{\rm R}(t_2)$  and let the photon-qubit system evolve a time  $t_2$  satisfying the condition  $|\Omega_2|t_2 = \pi/2$ . Finally, we adjust the classical magnetic field such that  $\Phi_{\rm c} = 0$ ; thus the interaction between the cavity field and qubit vanishes, and a single-photon state exists in the cavity, that is

$$|1\rangle \otimes |g\rangle = U_{\rm R}(t_2|) U_{\rm C}(t_1) |0\rangle \otimes |g\rangle.$$
(7)

Also any Fock state  $|m\rangle$  can be easily created from the vacuum state  $|0\rangle$  by alternatively turning on and off the quantum operations in eqs. (4)-(5) to excite the qubit and emit photons during the time interval T. The latter is divided by 2m subintervals  $\tau_1, \tau_2, \dots, \tau_{2l-1}, \tau_{2l}, \dots, \tau_{2m-1}, \tau_{2m}$  which satisfy conditions  $|\Omega_1|\tau_{2l-1} = \pi/2$  and  $|\Omega_2|\tau_{2l}\sqrt{l+1} = \pi/2$ , where  $l = 1, \dots, m$ . This process can be described as

$$|m\rangle \otimes |g\rangle = U_{\mathcal{R}}(\tau_{2m})U_{\mathcal{C}}(\tau_{2m-1})\cdots U_{\mathcal{R}}(\tau_{2})U_{\mathcal{C}}(\tau_{1})|0\rangle \otimes |g\rangle.$$
(8)

Finally, the classical magnetic field is changed such that  $\Phi_c = \Phi_0$ , and an *n*-photon state is provided in the cavity.

Our next goal is to prepare superpositions of different Fock states  $(e.g., \alpha_1|0\rangle + \alpha_2|1\rangle)$  for the vacuum  $|0\rangle$  and single-photon  $|1\rangle$  states. This very important state can be deterministically generated by two steps,  $U_{\rm C}(t'_1)$  and  $U_{\rm R}(t'_2)$ , with  $t'_2 = \pi/2|\Omega_2|$ ; that is

$$(\alpha_1|0\rangle + \alpha_2|1\rangle) \otimes |g\rangle = U_{\rm R}(t_2')U_{\rm C}(t_1')|0\rangle \otimes |g\rangle, \tag{9}$$

where the operation time  $t'_1$  determines the weights of the coefficients of the superposition  $\alpha_1 = \cos(\Omega_1 t'_1)$  and  $\alpha_2 = e^{-i\theta} \sin(\Omega_1 t'_1)$ . If the condition  $t'_1 = \pi/4\Omega_1$  is satisfied, then we have a superposition  $(|0\rangle + e^{-i\theta}|1\rangle)/\sqrt{2}$  with equal probabilities for each component and the relative phase between them can be further specified by the phase of the mode of the cavity field.

An arbitrary target state (3) can be generated from the initial state by alternatively switching on and off the carrier and first red sideband excitation during the time T', which can

be divided into 2n subintervals  $\tau'_1, \dots, \tau'_{2n}$ . That is, the target state can be deterministically generated as follows:

$$|\psi\rangle = \sum_{n=0}^{N} c_n |n,g\rangle = U(T')|0,g\rangle, \qquad (10)$$

where U(T') is determined by a sequence of time evolution operators associated with chosen time subintervals as  $U(T') = U_{\rm R}(\tau'_{2n})U_{\rm C}(\tau'_{2n-1})\cdots U_{\rm R}(\tau'_{2})U_{\rm C}(\tau'_{1})$ . Therefore, the coefficients  $c_n$  are

$$c_n = \langle g, n | U_{\mathcal{R}}(\tau'_{2n}) U_{\mathcal{C}}(\tau'_{2n-1}) \cdots U_{\mathcal{R}}(\tau'_2) U_{\mathcal{C}}(\tau'_1) | 0, g \rangle.$$

$$\tag{11}$$

Reference [4] has explicitly discussed how to adjust the rescaled times to obtain the expected state by solving the inverse evolution of eq. (10). Ideally, any state of the cavity field can be created according to our proposal by adjusting the gate voltage, classical magnetic field, and duration of the photon-qubit interaction. It is very easy to check that the state (3) can also be created by the carrier and blue sideband excitation whose time evolutions  $U_{\rm C}(t)$  and  $U_{\rm B}(t)$ are described by eqs. (4) and (6).

Environmental effects. – We now discuss the environmental effects on the prepared states, which are actually limited by the following time scales: the relaxation time  $T_1$ , the preparation time  $\tau_e$  of the excited state, and the dephasing time  $T_2$  of the qubit, the lifetime  $\tau_p$  of the photon and an effective interaction time  $\tau_c^{(n)}$  which corresponds to the transition from  $|n, e\rangle$  and  $|n + 1, g\rangle$ . If  $T_1, \tau_p \gg \tau_e, \tau_c^{(n)}$ , then the Fock states can be prepared. If the condition  $T_1, T_2, \tau_p \gg \tau_e, \tau_c^{(n)}$  is satisfied, then the superposition can also be obtained.

Now let us estimate the photon number of the obtainable Fock state in a full-wave cavity. In microwave experiments, it is possible to obtain very high-Q superconducting cavities, with Q values around  $3 \times 10^8$  to  $5 \times 10^{10}$  [2,23], which correspond to the lifetimes of the microwave region from  $0.001 \le \tau_p \le 0.15$  s to  $0.167 \le \tau_p \le 25$  s. The parameters of the charge qubit [24] without the cavity are  $2E_{\rm J}/h = 13.0$  GHz (so the operation time corresponding to a completely excited qubit is about  $\tau_e \approx 3.8 \times 10^{-11}$  s). The lifetime of the excited state for the qubit  $T_1 = 1.3 \times 10^{-6}$  s, *i.e.*  $\tau_e \ll T_1$ . For an estimate of the interaction coupling between the cavity field and the qubit, we assume that the cavity mode function is taken as a standing-wave form such as  $B_x = -i\sqrt{\hbar\omega}/\varepsilon_0 V c^2(a-a^{\dagger})\cos(kz)$ , with polarization along the normal direction of the surface area of the SQUID, located at an antinode of the standing-wave mode; then the interaction between the cavity field and the qubit reaches its maximum and the interaction strength can be expressed as  $|\beta| = \pi |\eta| E_J / \Phi_0 = (\pi S E_J / c \Phi_0) \sqrt{\hbar \omega / \varepsilon_0 V}$ . For example, if the wavelength of the cavity mode is taken as  $\lambda_1 = 0.1 \,\mathrm{cm}$ , then  $\pi |\eta| / \Phi_0 \approx 7.38 \times 10^{-5} \ll 1$ , where the dimension of the SQUID is taken as 10  $\mu$ m and the mode function u(r) is assumed to be independent of the integral area because the dimension of the SQUID,  $10 \,\mu m$ , is much less than 0.1 cm, for the wavelength of the cavity mode. In this case,  $\tau_c^{(0)} \approx 5.0 \times 10^{-7} \,\mathrm{s}$ , which is less than one order of magnitude of the excited lifetime  $T_1$ . This means that the qubit in its excited state can emit a photon before it relaxes to its ground state. But if we take the dimension of the SQUID as  $1 \,\mu m$ , the coupling between the cavity field and the qubit is two orders of magnitude smaller than for the  $10 \,\mu m$  SQUID, and then the interaction time is  $5.0 \times 10^{-5} > T_1$ . Therefore, in this case, the qubit relaxes to the ground state before the photon can be emitted from the qubit, and thus it is difficult to obtain a photon state. In fig. 2, we plot the ratio  $\tau_p^{(n)}/\tau_c^{(n)}$  between the lifetime [25]  $\tau_p^{(n)} = \tau_p/n$  (here,  $\tau_p^{(1)} = \tau_p$ ) of the Fock state  $|n\rangle$  for the zero-temperature environment and the effective operation time  $\tau_p^{(n)}$  of transitions from state  $|n, e\rangle$  to  $|n + 1, g\rangle$  for different values of Q and for  $\lambda = 0.1$  cm. Figure 2 shows that the photon number of the prepared Fock states can reach  $10^2$  in the above-mentioned high-Q cavity. But if the Q values are less than  $10^7$ , it might be difficult to



Fig. 2 – Ratio  $\tau_p^{(n)}/\tau_c^{(n)}$ , vs. photon number  $n \ge 1$ , of the lifetime  $\tau_p^{(n)}$  of the photon number state  $|n\rangle$  and the effective operation time  $\tau_c^{(n)}$ . The latter corresponds to the transition from  $|n, e\rangle$  to  $|n+1, g\rangle$  for a 10  $\mu$ m × 10  $\mu$ m SQUID in the full-wave cavity.

prepare a photon state with our estimated coupling. We also find that a longer microwave in the full-wave cavity corresponds to a longer  $\tau_c^{(n)}$  for a fixed Q, which means that it is easy to create photon states for shorter microwaves. For example, if the wavelength is taken as  $\lambda = 1$  cm, then the coupling between the qubit and the photon in the full-wave cavity might not be strong enough for generating Fock states within the currently known experimental data for  $T_1$ . So for longer microwaves, we can make a smaller cavity and place the qubit where the qubit-photon interaction is maximum.

If we want to prepare a superposition of different Fock states of the cavity field, we need to consider  $T_2$ , which is of the order of a few ns (*e.g.*, 5 ns in [11]). Then the survival time of the entangled state between the cavity field and qubit, which is required for the preparation of the superposition of the Fock states, may be very short. With the improvement of read-out techniques, a longer dephasing time can make our proposal far more realizable.

*Discussions.* – We propose a scheme for deterministically generating nonclassical photon states via the interaction of photons and a charge qubit. Indeed, the Fock state can be prepared with current technology. The superposition would be easier to obtain by increasing the dephasing time  $T_2$  and the qubit-photon coupling strength. Our discussions above are based on experimental values for  $T_1$  and  $T_2$  without the cavity; the decoherence may become shorter when the SQUID is placed inside the cavity. Further, in order to obtain a stronger coupling, the following steps would help to increase the qubit-field coupling strength: i) decrease the volume V of the cavity; ii) increase the area S of the SQUID; iii) increase the Josephson coupling energy  $E_{\rm J}$  under the condition  $E_{\rm J} \ll E_{\rm ch}$ . We can also put a high permeability  $\mu$ material inside the SQUID loop [14], then the qubit-field coupling strength can increase to  $\mu|\beta|$ , because the relative permeability in ferromagnetic materials can be  $10^2$  to  $10^6$ , and might partly compensate some of the decoherence effects due to the  $\mu$  material itself. Increasing the SQUID dimension and decreasing the cavity volume will reduce the maximum allowed number of SQUIDs inside the cavity making it unadvantageous for quantum computing. However, one qubit is enough for the generation of nonclassical photon states, our goal here. We note that Girvin et al. [17] proposed a different system in which the coupling of the photon-qubit can reach 10<sup>8</sup> Hz, corresponding to  $\tau_c^{(0)} \sim 10^{-9}$  s. We are considering how to generate nonclassical photon states by using such a system. This scheme might not be easy to generalize in a straightforward manner to the flux qubit case. This because the interaction between the flux-qubit and the cavity field cannot be switched on and off in the same way for the charge qubit. However, in some modified manner, it should be possible to generalize this scheme.

947

\* \* \*

We thank X. HU helpful comments. This work was supported in part by the US NSA and ARDA under AFOSR contract No. F49620-02-1-0334, and by the NSF grant No. EIA-0130383.

## REFERENCES

- NIELSEN M. N. and CHUANG I. L., Quantum Computation and Quantum Information (Cambridge University Press) 2000.
- [2] KRAUSE J. et al., Phys. Rev. A, 36 (1987) 4547; 39 (1989) 1915.
- [3] LEONSKI W. et al., Phys. Rev. A, 49 (1994) R20; LEONSKI W., Phys. Rev. A, 54 (1996) 3369;
   BRUNE M. et al., Phys. Rev. Lett., 65 (1990) 976; HOLLAND M. J. et al., Phys. Rev. Lett., 67 (1991) 1716.
- [4] LAW C. K. et al., Phys. Rev. Lett., **76** (1996) 1055.
- [5] VOGEL K. et al., Phys. Rev. Lett., 71 (1993) 1816; PARKINS A. S. et al., Phys. Rev. Lett., 71 (1993) 3095; Phys. Rev. A, 51 (1995) 1578; PEGG D. T. et al., Phys. Rev. Lett., 81 (1998) 1604.
- [6] MICHLER P. et al., Science, 290 (2000) 2282; KIM J. et al., Nature, 397 (1999) 500; MICHLER P. et al., Nature, 406 (2000) 968.
- [7] HOOD C. J. et al., Science, 287 (2000) 1447; VARCOE B. T. H. et al., Nature, 403 (2000) 743;
   PINKSE P. W. H. et al., Nature, 404 (2000) 365; BERTET P. et al., Phys. Rev. Lett., 88 (2002) 143601; KUHN A. et al., Phys. Rev. Lett., 89 (2002) 067901.
- [8] LOUNIS B. et al., Nature, 407 (2000) 491; HONG C. K. et al., Phys. Rev. Lett., 56 (1986) 58;
   BRUNEL C. et al., Phys. Rev. Lett., 83 (1999) 2722; KURTSIEFER C. et al., Phys. Rev. Lett., 85 (2000) 290; SANTORI C. et al., Phys. Rev. Lett., 86 (2001) 1502.
- [9] LVOVSKY A. I. et al., Phys. Rev. Lett., 88 (2002) 250401; MAITRE X., Phys. Rev. Lett., 79 (1997) 769.
- [10] MOOIJ J. E. et al., Science, 285 (1999) 1036; VAN DER WAL C. H. et al., Science, 290 (2000) 773; HAN S. et al., Science, 293 (2001) 1457; VION D. et al., Science, 296 (2002) 886; YU Y. et al., Science, 296 (2002) 889; CHIORESCU I. et al., Science, 299 (2003) 1869; BERKLEY A. J. et al., Science, 300 (2003) 1548; FRIEDMAN J. R. et al., Nature, 406 (2000) 43.
- [11] NAKAMURA Y. et al., Nature, 398 (1999) 786; PASHKIN Y. A. et al., Science, 421 (2003) 823.
- [12] AL-SAIDI W. A. et al., Phys. Rev. B, 65 (2002) 224512.
- [13] ZHU S. L. et al., Phys. Rev. A, 68 (2003) 034303.
- [14] YOU J. Q. and NORI F., Phys. Rev. B, 68 (2003) 064509.
- [15] YOU J. Q. et al., Phys. Rev. B, 68 (2003) 024510.
- [16] YANG C. P. et al., Phys. Rev. A, 67 (2003) 042311.
- [17] GIRVIN S. M. et al., cond-mat/0310670.
- [18] MAKHLIN Y. et al., Rev. Mod. Phys., 73 (2001) 357.
- [19] AVERIN D. V. and NAZAROV Y. V., Phys. Rev. Lett., 69 (1992) 1993; TINKHAM M., Introduction to Superconductivity (McGraw-Hill, New York) 1996.
- [20] MEEKHOF D. M. et al., Phys. Rev. Lett., **76** (1996) 1796; BEN-KISH A. et al., Phys. Rev. Lett., **90** (2003) 037902; KNEER B. et al., Phys. Rev. A, **57** (1998) 2096; WEI L. F. et al., quant-ph/0308079.
- [21] NAKAMURA Y. et al., Phys. Rev. Lett., 87 (2001) 246601; 88 (2002) 047901.
- [22] Stenholm S., Phys. Rep., 6 (1973) 1.
- [23] BRATTKE S. et al., Phys. Rev. Lett., 86 (2001) 3534; RAIMOND J. M. et al., Rev. Mod. Phys., 73 (2001) 565.
- [24] LEHNERT K. W. et al., Phys. Rev. Lett., 90 (2003) 027002.
- [25] PERINOVA V. and LUKS A., Phys. Rev. A, 41 (1990) 414.