Critical currents in quasiperiodic pinning arrays: One-dimensional chains and Penrose lattices

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Summary

We have studied the critical depinning current J_c versus the applied magnetic flux Φ, for quasiperiodic (QP) one-dimensional (1D) chains and 2D arrays of pinning centers placed on the nodes of a five-fold Penrose lattice. In 1D QP chains, the peaks in $J_c(\Phi)$ are determined by a sequence of harmonics of the long and short segments of the chain. The critical current $J_c(\Phi)$ has a remarkable self-similarity. In 2D QP pinning arrays, we predict analytically and numerically the main features of $J_c(\Phi)$, and demonstrate that the Penrose lattice of pinning sites (which has many built-in periods) provides an enormous enhancement of $J_c(\Phi)$, even compared to triangular and random pinning site arrays. This huge increase in $J_c(\Phi)$ could be useful for applications.

Model

We perform simulated annealing simulations of

$$\eta\,\mathbf{v}_i = \mathbf{f}_i = \mathbf{f}_i^{vv} + \mathbf{f}_i^{vp} + \mathbf{f}_i^{T} + \mathbf{f}_i^{d}.$$

- The force due to the vortex-vortex interaction is

$$\mathbf{f}_i^{vv} = \sum_j^{Nv} f_0\, K_1(|\,\mathbf{r}_i - \mathbf{r}_j\,| / \lambda)\, \mathbf{r}_{ij},$$

  N_v is the number of vortices, K_1 is a modified Bessel function, λ is the penetration depth, $\mathbf{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j) / |\,\mathbf{r}_i - \mathbf{r}_j\,|$, $f_0 = \Phi_0^2/8\pi^2\lambda^3$, and $\Phi_0 = hc/2e$.
- The pinning force is

$$\mathbf{f}_i^{vp} = \sum_k^{Np} f_p \cdot (|\,\mathbf{r}_i - \mathbf{r}_k^{(p)}\,| / r_p)\, \Theta[(r_p - |\,\mathbf{r}_i - \mathbf{r}_k^{(p)}\,|) / \lambda]\, \mathbf{r}_{ik}^{(p)},$$

  N_p is the number of pinning sites, f_p (expressed in f_0) is the maximum pinning force of each short-range parabolic potential well located at $\mathbf{r}_k^{(p)}$, r_p is the range of the pinning potential, Θ is the Heaviside step function.
- All the lengths (fields) are expressed in units of λ (Φ_0/λ^2).
- In the equation of motion, $\mathbf{f}_i^T$ is the thermal stochastic force, and $\mathbf{f}_i^d$ is the driving force; η is the viscosity.

1D Quasicrystal

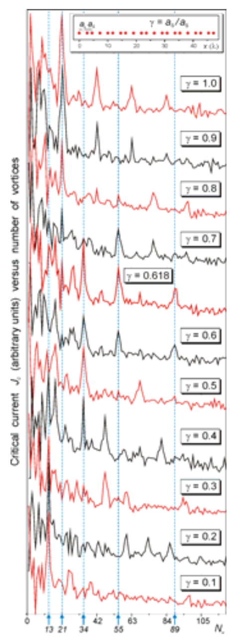
Construction of a 1D QP chain (1D quasicrystal): Iteratively apply the Fibonacci rule *L→LS, S→L*

LSLLSLSLLSLLSLSLLSLS …

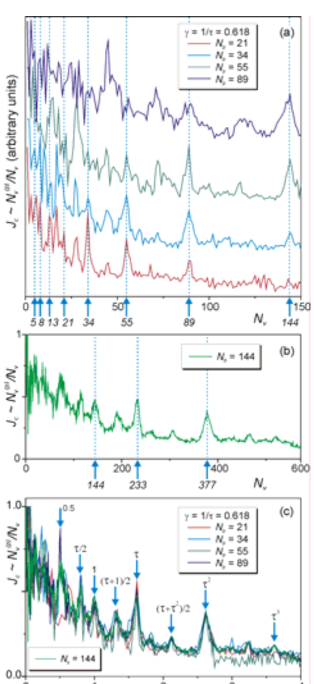
For an infinite QP sequence, the ratio of the numbers of *L* to *S* elements is the *golden mean* $\tau = (1 + \sqrt{5})/2$.
The *n*th point where a new segment (e.g., for *S* = 1 and *L* = τ) begins is:

$$x_n = n + [n/\tau] / \tau,$$

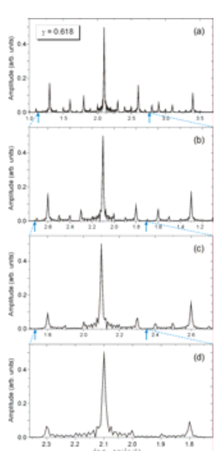
where $[x]$ denotes the integer part of x. This sequence exhibits self-similarity and has a hierarchy of built-in periods.



Critical current J_c for 1D QP chains
Self-similarity of J_c in real space

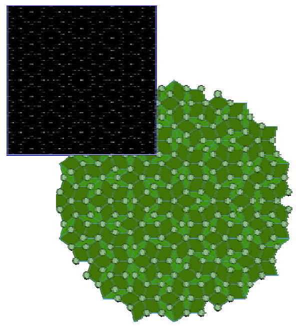


Self-similarity of J_c in *k*-space



2D Quasicrystal: Penrose lattice

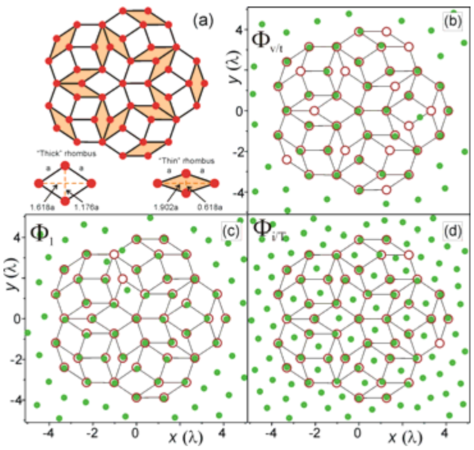
Five-fold Penrose lattice



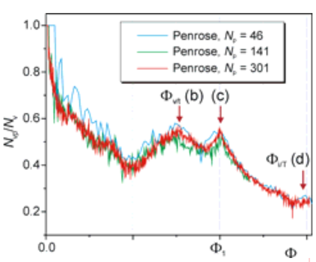
A *Penrose lattice* is a 2D quasiperiodic (QP) structure, or 2D quasicrystal.

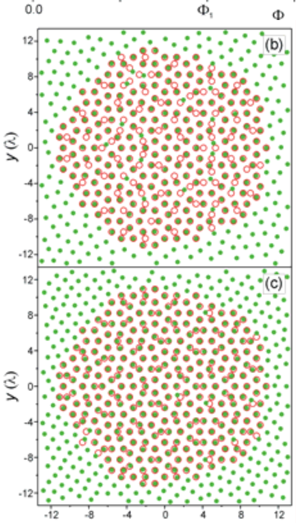
- Has a local rotational (five- or ten-fold) symmetry, but does not have translational long-range order.
- Is constructed using certain simple shapes combined according to specific local rules, and can extend to infinity without any defects.
- Self-similar diffraction patterns of a Penrose lattice exhibit a dense set of "Bragg" peaks.

Critical current J_c in a Penrose-lattice array of pinning sites



J_c for increasing sample size



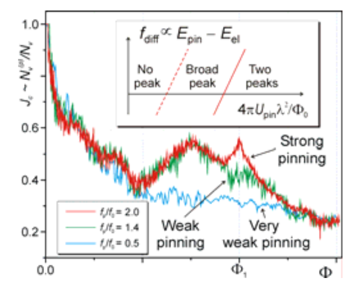


Stability of the maxima of J_c

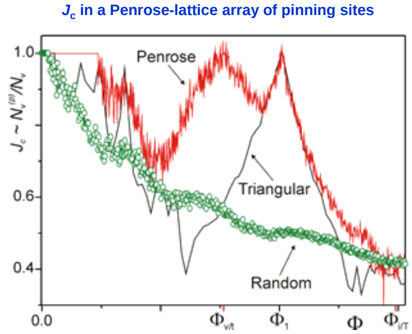
The dimensionless difference of the pinning and elastic energies is $E_{pin} - E_{el} = \beta\, f_{diff}\, n_{pin}\, \Phi_0^2 / (4\pi\lambda^2)$, where

$$f_{diff} = 4\pi\lambda^2 U_{pin} / \Phi_0^2 - \beta \cdot [1 - b\,(3^{1/2}\,\beta\, n_{pin} / 2)^{1/2}]^2.$$

Here $E_{pin} = U_{pin}\,\beta\, n_{pin}$; $E_{el} = C_{11}\,[(a_{eq} - b) / a_{eq}]^2$; n_{pin} is the density of pinning centers, $\beta(H \le H_1) = H/H_1 = B/(\Phi_0\, n_{pin})$, and $\beta(H > H_1) = 1$ is the fraction of occupied pinning sites, $a_{eq} = [2/(3^{1/2})\,\beta\, n_{pin}]^{1/2}$ and b are the inter-vortex distances in triangular and distorted lattices ($b = a/\tau$ for $H = H_1$ and $b = a$ for $H = H_{v/t}$), and $C_{11} = B^2 / [4\pi(1 + \lambda^2 k^2)]$ is the compressibility modulus for local deformations. Near matching fields, J_c has a peak only if $f_{diff} > 0$.



Strong enhancement of the critical current J_c in a Penrose-lattice array of pinning sites



Quasiperiodic (Penrose) lattice provides an ***unusually broad*** critical current $J_c(\Phi)$, that could be useful for practical applications demanding *high J_c's over a wide range of fields*

References: V.R. Misko, S. Savel'ev, and F. Nori, Phys. Rev. Lett. **95**, 177007 (2005); Phys. Rev. B **74**, 024522 (2006); cond-mat/0502480.