Proceedings of
the 8th International Conference on

Quantum Communication, Measurement and Computing

28th November - 3rd December, 2006
Tsukuba International Congress Center, Tsukuba, Japan

QCMC 2006

Edited by Osamu Hirota,
Jeffrey H. Shapiro and Masahide Sasaki

NICT Press

GENERATION OF BELL STATES AND GREENBERGER-HORNE-ZEILINGER STATES IN SUPERCONDUCTING PHASE QUBITS

Shigemasa Matsuo[1], Sahel Ashhab[2], Toshiyuki Fujii[3], Franco Nori[2,4], Katsuhiko Nagai[1,3] and Noriyuki Hatakenaka[1,3,5]

[1]Graduate School of Integrated Arts and Sciences, Hiroshima University, Higashi-Hiroshima, 739-8521, Japan.
[2]Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan.
[3]AdSM, Hiroshima University, Higashi-Hiroshima, 739-8530, Japan.
[4]Center for Theoretical Physics, Physics Department, CSCS, The University of Michigan, Ann Arbor, Michigan 48109-1040, USA.
[5]MILq Project, International Project Center for Integrated Research on Quantum Information and Life Science, Higashi-Hiroshima, 739-8530, Japan.

We consider the possibility of generating macroscopic entangled states in a system of several phase qubits. We use a method that is inspired by a proposal due to Cirac and Zoller for the generation of multi-atom entangled states. We analyze the procedure that can be used to generate n-qubit entangled states, such as Bell states for the case $n = 2$ and the Greenberger-Horne-Zeilinger state for the case $n = 3$. We construct our procedures under the experimentally motivated constraint of trying to minimize the number of control lines used to manipulate the qubits in superconducting circuits.

1. INTRODUCTION

The generation of entangled states [1] is one of the important operations in quantum computing. Quantum computing has been studied in microscopic systems such as optical systems and trapped ions systems, and in macroscopic system such as superconducting devices. [2–5] The first experimental observations of entanglement were performed in optical systems involving photons. [6] Josephson-junction based quantum circuits are advantageous in terms of scalability, i.e. building a device with many qubits.

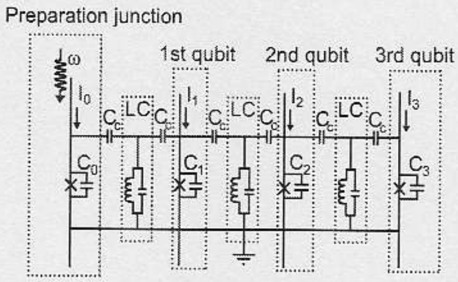

Figure 1: Superconducting circuit for generating GHZ states.

Cirac and Zoller [7] have proposed schemes of the generation of entangled states such as Bell states and the Greenberger-Horne-Zeilinger (GHZ) state [8] in two-levels atom systems by using cavity-QED. We study the possibility of generating entangled states in phase qubits by using the Cirac-Zoller scheme, with a preparation junction playing the role of the cavity. LC circuits couple the qubits as shown in Fig. 1. These LC circuits mediate the interaction between the qubits, as well as the interaction between the first qubit and the preparation junction.

2. SWAP OPERATION BETWEEN QUBITS BY LC CIRCUIT

A current-biased Josephson junction can be represented by a model of a particle in a one-dimensional tilted cosine potential. The number of energy levels and energy spacing can be controlled by the bias current. Two-level systems act as so-called phase qubits. The energy spacings can be controlled using the bias currents to generate the desired structure for a given number of qubits. For example Figure 2 shows the bias conditions needed to generate the GHZ state. The preparation junction acts a cavity in the Cirac-Zoller scheme that prepares the superposition states like $|0\rangle + |3\rangle$. The coupling between the qubits is also controlled by the bias currents. When a qubit and an LC circuit have an equal separation between their energy levels, the coupling is turned on. When they are off resonance, they evolve independently of each other. The energy levels in the LC circuit between 1st and 2nd qubit are different from in other LC circuits, in order to avoid simultaneous resonances.

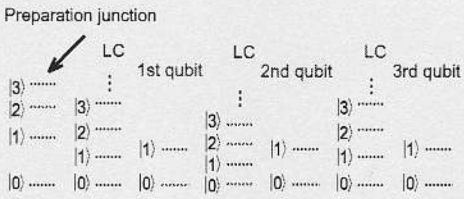

Figure 2: Energy levels for superconducting circuit in Fig. 1.

Resonance occurs under an appropriate bias I_i, and in such a case quantum states is swapped between the resonant qubits. Using the rotating-wave approximation, the effective Hamiltonian [9] for the $i-th$ qubit and LC circuit can be written as

$$\hat{H}_{SWAP} = \frac{\epsilon(I_i)}{2}\sigma_z^{(i)} + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \bar{V}_{i,LC}\left(\sigma_-^{(i)} \otimes a^\dagger + H.c.\right) \tag{1}$$

Resonance occurs when $\epsilon(I_i) = \hbar\omega$ (with no other resonance conditions). If the system is allowed to evolve under the effect of this effective Hamiltonian for a period of time given by

$$\Delta t_{i,LC,n}^{\mathrm{SWAP}} = \frac{\pi}{2}\frac{\hbar}{|\bar{V}_{i,LC}|\sqrt{n+1}}, \tag{2}$$

an i-SWAP operation is performed between the $i-th$ qubit and the LC circuit as shown in Fig. 3.

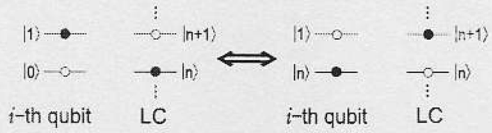

Figure 3: Schematic diagram of the on-resonance dynamics of $i-th$ qubit and LC circuit.

We consider swapping quantum states between the $i - th$ qubit and the $(i+1) - th$ qubit through the LC circuit. For example, we consider the time evolution of state $|0\rangle_i|1\rangle_{LC}|0\rangle_{i+1}$. When the $i-th$ qubit and the LC circuit are on resonance, and $(i+1)-th$ qubit is independent, the state evolve to $|0\rangle_i|1\rangle_{LC}|0\rangle_{i+1}$. Next, when $(i+1) - th$ qubit and the LC circuit are on resonance, and $i - th$ qubit is independent, the state evolve to $|0\rangle_i|0\rangle_{LC}|1\rangle_{i+1}$ from $|0\rangle_i|1\rangle_{LC}|0\rangle_{i+1}$. Then, the state of $i - th$ and $(i+1) - th$ qubit become $|0\rangle_i|1\rangle_{i+1}$ from $|1\rangle_i|0\rangle_{i+1}$. The state $|0\rangle_i|0\rangle_{i+1}$ do not evolve with time, apart from the evolution of their phase factors.

3. GENERATION OF ENTANGLED STATES

For clarity, we focus on the case of generating the GHZ state. This state can be prepared as follows:

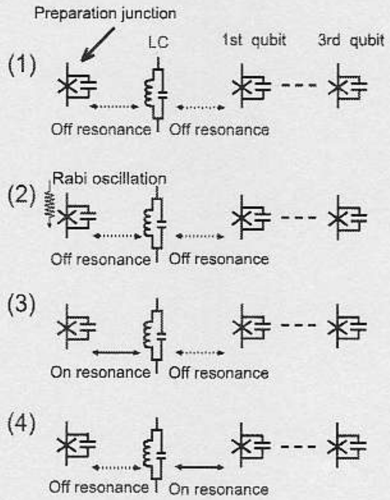

Figure 4: A part of procedure for generating the GHZ state.

The reparation junction (0th junction) has four energy levels. (1) We start from the ground state $|0\rangle_0|0\rangle_1|0\rangle_2|0\rangle_3$ for junctions and the ground state $|0\rangle_{LC}$ for all LC circuits. All neighboring junctions and LC circuits are kept off resonance as shown in Fig. 2. (2) the quantum state $(|0\rangle_0+|3\rangle_0)/\sqrt{2}$ is generated in the 0th junction by driving Rabi oscillations. All neighboring junctions and LC circuits are kept off resonance with each other. We now have as the superposition state $(|0\rangle_0 + |3\rangle_0)|0\rangle_1|0\rangle_2|0\rangle_3/\sqrt{2}$. (3) Using a sequence of the basic gates introduced in Sec. 2 with neighboring preparation junction and LC circuit, we have the state $(|0\rangle_0|0\rangle_{LC} + |2\rangle_0|1\rangle_{LC})|0\rangle_1/\sqrt{2}$ from the state $(|0\rangle_0 + |3\rangle_0)|0\rangle_{LC}|0\rangle_1/\sqrt{2}$. (4) Performing the basic gates between $1st$ qubit and LC circuit, we obtain the state $(|0\rangle_0|0\rangle_{LC}|0\rangle_1 + |2\rangle_0|0\rangle_{LC}|1\rangle_1)/\sqrt{2}$ and then we have the state $(|0\rangle_0|0\rangle_1 + |2\rangle_0|1\rangle_1)|0\rangle_2|0\rangle_3$ for junctions. The above procedure is also shown schematically in Fig.4. Using a sequence of the basic gates introduced in Sec. 2, the state on $1st$ qubit is moved to $3rd$ qubit. We have the state $(|0\rangle_0|0\rangle_3 + |2\rangle_0)|1\rangle_3)|0\rangle_1|0\rangle_2/\sqrt{2}$. Similarly we perform a sequence of basic gates to reach the state $(|0\rangle_1|0\rangle_2|0\rangle_3 + |1\rangle_1)|1\rangle_2|1\rangle_3)|0\rangle_0/\sqrt{2}$. As a result, we now have a GHZ state involving the 1st, 2nd and 3rd qubits. The generalization of the above procedure to n-qubit entangled states, such as Bell states for the case $n = 2$ is straightforward.

ACKNOWLEDGMENTS

We would like to thank L.-F. Wei for useful discussions. This work was supported in part by a Grand-in-Aid for Scientific Research (18540352) and for Scientific Research on Priority Areas (17071009) from the Ministry of Education Culture, Sports, Science and Technology of Japan, the National Security Agency (NSA), the Laboratory for Physical Sciences (LPS) and the Army Research Office (ARO); and also by the National Science Foundation (NSF) grant No. EIA-0130383. One of us (S. A.) was supported by the Japan Society for the Promotion of Science (JSPS).

REFERENCES

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935); N. Bohr, Phys. Rev. **48**, 696 (1935).

[2] C.-P. Yang, and S. Han, Phys. Rev. A **70**, 062323 (2004).

[3] L.-F. Wei, Y.-X. Lui and F. Nori, Phys. Rev. Lett. **96**, 246803 (2006).

[4] L.-F. Wei, Y.-X. Liu, and F. Nori, Phys. Rev. B **72**, 104516 (2005).

[5] F. W. Strauch, P. R. Johnson, A. J. Dragt, C. J. Lobb, J. R. Anderson, and F.C.Wellstood, Phys. Rev. Lett. **91**, 167005 (2003).

[6] See, e.g., A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981); and references therein for previous experiments.

[7] J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994).

[8] D. M. Greenberger, M. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed. M. Kafatos (Kluwer Academic, Dordrecht, The Netherlands, 1989), pp.73-76; D. M. Greenberger, M. Horne, A. Shimony and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990).

[9] G. Johansson, L. Tornberg, V. S. Shumeiko and G. Wendin, J. Phys.: Condens. Matter **18**, S901 (2006).