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Nonlinear Josephson plasma waves in slabs of layered superconductors

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Abstract

For layered superconductors, we study the specific nonlinear Josephson plasma waves (NJPWs) propagating along thin superconducting slabs and damping away from them. Two cases are considered, when the superconductor is surrounded by either vacuum or metals. The magnetic field of the NJPW is distributed symmetrically with respect to the middle of the sample and can change its sign inside the slab. The impedance ratio of the tangential electric and magnetic field amplitudes for NJPWs can be of the order of unity. For the case of a superconductor surrounded by the vacuum, this results in a non-monotonic dispersion relation, $\omega(k)$, strongly sensitive to the NJPW amplitudes. Therefore, the "stopping light" phenomenon can be observed at frequencies where $\mathrm{d}\omega(k)/\mathrm{d}k = 0$. Resonance excitations of the NJPWs should produce anomalies in the amplitude dependence of the reflectivity and transmissivity of the incident THz waves, which could be useful for THz devices. Animations illustrating the results presented here, are available online at dml.riken.go.jp/nonlinear.

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A layered superconductor is a medium that allows the propagation of both nonlinear [1] and surface [2] electromagnetic waves in the, important for applications, THz frequency range. Surface waves and nonlinear effects are both due to the gap structure (see, e.g. Ref. [3]) of the Josephson plasma excitation spectra which has been observed via Josephson plasma resonance (e.g. [4]). The nonlinearity of Josephson plasma waves (JPWs) with frequency ω close to the Josephson plasma frequency ω_J becomes important even at small field amplitudes $\propto (1-\omega^2/\omega_J^2)^{3/2}, \omega < \omega_J$. In close analogy to nonlinear optics, nonlinear JPWs exhibit numerous remarkable features [1], including the slowing down of light, self-focusing effects, and the pumping of weaker waves by stronger ones. However, the nonlinearity of EMWs in layered superconductors is quite different from optical nonlinearities. This leads one to expect very different properties from known nonlinear EMWs.

Here we propose self-sustained JPWs propagating along a thin slab ($-d/2 < z < d/2$) of a layered superconductor symmetric with respect to the middle, $z = 0$, of the sample. The geometry of the problem considered here is shown in the inset of Fig. 1. For *thin* slabs, novel, essentially nonlinear, Josephson plasma waves (NJPWs) are predicted here. Surprisingly, even though the magnetic field H for NJPWs

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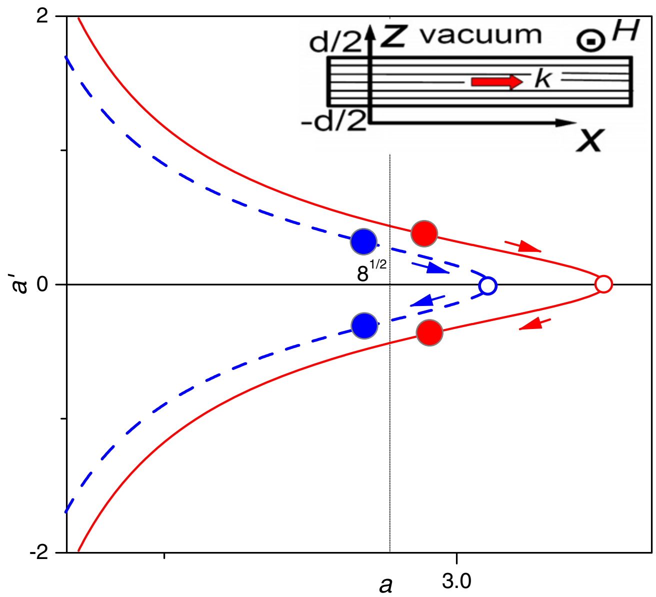

Fig. 1. Phase diagram of a' versus a. The blue dashed and red solid trajectories $a'(a)$ describe the symmetric strongly NJPWs in a superconducting slab surrounded by the vacuum (blue dashed) and metals (red solid), respectively. Moving along the trajectories between the solid circles corresponds to the change of coordinate z inside the sample ($-d/2 < z < d/2$). The inset shows the geometry of the problem. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

can be very small, the electric field E remains strong. In addition, the magnetic field of the NJPW at the sample surface can be much weaker than the one in the middle of the slab. For this case, the wave amplitude significantly affects the dispersion properties of the NJPWs. The non-monotonic dispersion relation, $\omega(k)$, results in the stop-light phenomenon, $\partial\omega(k,H)/\partial k = 0$, controlled by the magnetic field amplitude H. Note that NJPWs can exist in a superconducting slab either embedded inside a dielectric or surrounded by a vacuum. In the case of high enough wave amplitudes, the nonlinear waves can also propagate in a superconductor surrounded by metal plates. This case is considered here as well.

The NJPWs predicted here could be experimentally observed via resonance anomalies in the amplitude dependence of the reflectivity and transmissivity of the incident THz waves.

1. Model and phase trajectories of NJPWs

The Maxwell equations for EMWs in vacuum ($z > d/2$) determine the distributions of the magnetic and electric fields, $H(x, z > d/2, t)$, and $E_z(x, z > d/2, t) \propto \exp\{-q_v(z - d/2)\}\cos(kx - \omega t)$; also, $E_x(x, z > d/2, t) \propto \exp\{-q_v(z - d/2)\}\sin(kx - \omega t)$, with the spatial decrement $q_v = (k^2 - \omega^2/c^2)^{1/2}$. The impedance ratio,

$$Z_{\mathrm{vac}} = \frac{E_x(z = d/2)}{H(z = d/2)} = -\left(\frac{c^2k^2}{\omega^2} - 1\right)^{1/2} \tag{1}$$

should match the one obtained for the superconducting slab at the interface $z = d/2$. As for the electromagnetic field at $z < 0$, we only consider symmetric and antisymmetric solutions with respect to the middle of the sample $z = 0$.

Inside a layered superconductor, the magnetic and electric fields are determined by the gauge-invariant interlayer phase difference φ. We consider nonlinear JPWs with $|\varphi| \ll 1$, when the Josephson current $J_c \sin\varphi$ can be approximated by $J_c(\varphi - \varphi^3/6)$. In other words, we consider waves, excluding soliton or vortex-like solutions. In the continuum limit, the coupled sine-Gordon equations [5] for the gauge-invariant phase difference φ reduce to

$$\left(1 - \lambda_{ab}^2 \frac{\partial^2}{\partial z^2}\right)\left[\frac{1}{\omega_J^2}\frac{\partial^2 \varphi}{\partial t^2} + \varphi - \frac{\varphi^3}{6}\right] - \lambda_c^2 \frac{\partial^2 \varphi}{\partial x^2} = 0. \tag{2}$$

Here we omit the relaxation terms related to the quasiparticle currents. The spatial scales, λ_{ab} and λ_c, are the in-plane and c-axis London penetration depths. As was shown in Ref. [1], the nonlinearity plays a crucial role in wave propagation when $(1 - \Omega^2) \ll 1$, $\Omega = \omega/\omega_J$, since φ^3 is of the same order as $\omega_J^{-2}\partial^2\varphi/\partial t^2 + \varphi$. Below we focus on this frequency range.

We seek a solution of Eq. (2) in the form: $\varphi(x,z,t) \approx A(z)\sin(kx - \omega t)$. Substituting the last equation into Eq. (2) and introducing the dimensionless variables, $a = A/(1 - \Omega^2)^{1/2}$, $\kappa = \lambda_c k/(1 - \Omega^2)^{1/2}$, $\xi = \kappa z/\lambda_{ab}$, we derive the equation for the amplitude $a(\xi)$

$$\left[1 - \kappa^2 \frac{d^2}{d\xi^2}\right]\left(a - \frac{a^3}{8}\right) + \kappa^2 a = 0. \tag{3}$$

Here, the nonlinear term φ^3 was replaced by $\varphi^3 = (3/4)A^3\sin(kx - \omega t)$. Higher harmonics $(2n+1)\omega$ determine an additional damping mechanism related to the radiation friction of the NJPWs, which we do not study here. The amplitudes of these harmonics generated by φ^3 (as well the higher-order terms in the expansion of $\sin\varphi$ in Eq. (2)) decay as $\propto (1 - \Omega^2)^{n+1/2}$ and can be omitted in the main approximation, when expanding on $(1 - \Omega^2) \ll 1$ [1,6].

For symmetric solutions we use the boundary condition $a'(0) = 0$ in the middle of the sample. The electromagnetic fields are related to $a(\xi)$ as follows:

$$H(z) = -H_0(1 - \Omega^2)h(\xi)/\kappa, \quad h(\xi) = a(\xi) - a^3(\xi)/8, \tag{4}$$

$$E_x(z) = H_0 \frac{\lambda_{ab}\Omega}{\sqrt{\epsilon}\lambda_c}(1 - \Omega^2)h'(\xi),$$

$$E_z(z) = H_0 \frac{\Omega}{\sqrt{\epsilon}}(1 - \Omega^2)^{1/2}a(\xi). \tag{5}$$

Here, prime denotes $d/d\xi$, the scale of the magnetic field is $H_0 = \Phi_0/2\pi s\lambda_c$, Φ_0 is the flux quantum, s is the interlayer spacing, and ϵ is the interlayer dielectric constant. The matching of the impedance (continuity of $E_x(z)$ and $H(z)$) at the sample surface $z = d/2$ results in the dispersion relation of the NJPWs:

$$Z_{\rm vac} = \frac{\lambda_{ab}}{\sqrt{\epsilon}\lambda_c}\Omega\kappa f_{\rm s}(\Omega, \kappa, H/H_0) \quad (6)$$

with $Z_{\rm vac}$ given by Eq. (1). The factor $f_{\rm s} = h'/h(\xi = \kappa d/2\lambda_{ab})$ provides the amplitude dependence to the spectrum of the self-sustained waves. This factor has to be obtained by solving Eq. (3).

The decay of the wave in vacuum ($kc > \omega$), which is the necessary condition for waveguide modes (or NJPWs), implies the inequality $\kappa \gg 1$, at small $1 - \Omega^2$. In this limit, Eq. (3) yields $(a')^2 = -4/3 + G(8 - 3a^2)^{-2}$. The curves $a'(a)$ corresponding to waveguide modes is shown in Fig. 1. Solid circles mark the sample boundaries, while open circles indicate the middle of the slab, arrows show the direction of motion along the trajectories when ξ increases. In order to match the vacuum-superconductor boundary conditions, the starting and ending points of the trajectories should be within the interval from $-2\sqrt{2}$ to $2\sqrt{2}$.

2. Symmetric strong-amplitude NJPWs

The function $f_{\rm s}$ in Eq. (6) describing the deviation of the spectrum of the NJPWs from the "vacuum light line", has a very complicated structure with asymptotics: $f_{\rm s} \propto \kappa d/h_s\lambda_{ab}$ for $\kappa^2 d^2/\lambda_{ab}^2 \ll 1 - h_{\rm s}$ and $f_{\rm s} \propto \kappa^2 d^2/h_{\rm s}\lambda_{ab}$ for $\kappa d/\lambda_{ab} \gg 1$. This allows to construct a simple interpolation of the dispersion relation

$$1 - \Omega^2 = \frac{d^2}{3\epsilon\lambda_{ab}^2}\left[\left(\frac{H}{H_{\rm t}}\right)^2\left(1 - \frac{\omega_{\rm J}^2}{c^2k^2}\right) - \frac{c^2k^2}{4\omega_{\rm J}^2}\right] \quad (7)$$

where the threshold amplitude $H_{\rm t} \approx 0.8H_0 d^2/(\epsilon^{3/2}\lambda_{ab}^2\lambda_c^2)$ defines the lowest value of the magnetic field amplitude at the sample surface: at lower fields the predicted NJPWs do not exist. The interpolation formulae Eq. (7) is in perfect agreement with numerical results (see Fig. 2) obtained by the integration of Eq. (3).

The spectrum of the strong-amplitude NJPWs is non-monotonic (Fig. 2) and $\Omega(k)$ reaches the minimal value at $k = (\omega_{\rm J}/c)(2H/H_{\rm t})^{1/2}$. Thus, the stop-light phenomenon, $\partial\omega(k, H)/\partial k = 0$, occurs in the THz superconducting waveguide. This stop-light effect can be easily controlled by the magnetic field amplitude.

Note that the impedance ratio $E_x(z = d/2)/H(z = d/2)$ at the boundary of the superconductor changes its sign at $|a| = 2\sqrt{2}$. This means that the NJPWs with high enough amplitudes cannot propagate in the superconducting slab placed in the vacuum or a dielectric. However, nonlinear waves with $|a(z = d/2)| > 2\sqrt{2}$ can exist if the superconducting slab is surrounded by metals. Indeed, the impedance ratio $Z_{\rm met} = E_x(z = d/2)/H(z = d/2) \approx \omega/\omega_{\rm p}$ for metals, contrary to the vacuum and dielectrics, is positive. Here ω_p is the plasma frequency of a metal occupying the region $|z| > d/2$. The dispersion curve for such waves is described by the same formula, Eq. (6), as for the superconductor surrounded by the vacuum, where $Z_{\rm vac}$ should be changed by $Z_{\rm met}$. Fig. 2 shows this spectrum for a superconducting slab surrounded by semimetals. Note that the spectrum is now monotonic prohibiting the stop-light effect.

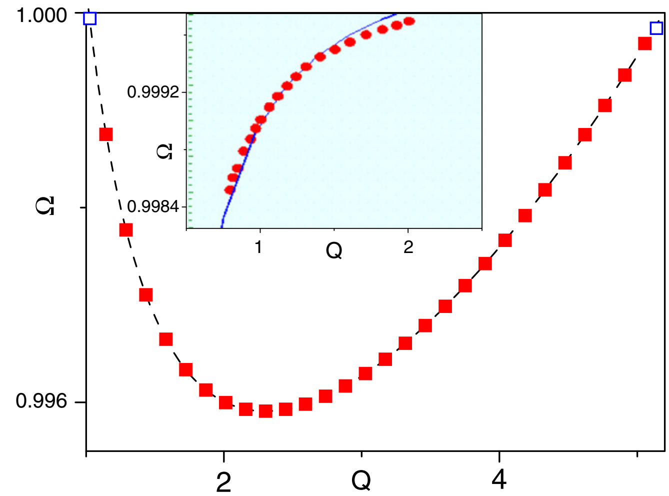

Fig. 2. Dispersion relation, $\Omega(Q = ck/\omega_J)$, for the strongly nonlinear waveguide mode: the solid red squares present the result of the numerical integration of Eq. (3); the dashed black line is obtained by the interpolation formula (7). The simulations and interpolation perfectly coincide. Here we use the following set of parameters: $d/\lambda_{ab} = 0.3$, $\lambda_c/\lambda_{ab} = 200$, $\varepsilon = 16$, $H/H_0 = 1.5 \cdot 10^{-5}$. Inset: dispersion curve for the NJPWs in a superconducting slab surrounded by semimetals with $\omega_p = 6 \times 10^{13}$ s^{-1}; other parameters are $d/\lambda_{ab} = 0.3$, $\varepsilon = 15$, and $\omega_J = 2 \times 10^{12}$ s^{-1} at $H(d/2)/H_0 = 2 \times 10^{-6}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3. Conclusions

We predict the existence of strongly nonlinear symmetric waveguide modes, which propagate in a thin slab of a layered superconductor, and decay fast enough away from it, into either the vacuum or a metal. The spectrum of strongly nonlinear symmetric waveguide modes is non-monotonic, resulting in a "stop-light" effect controlled by the magnetic field intensity. These nonlinear self-sustained waveguide modes could be observed via amplitude and angular anomalies in the reflectivity, transmissivity, and absorptivity of incident THz electromagnetic waves. The predicted THz modes could be potentially useful for the design of THz waveguides, detectors, filters, and other THz devices [7].

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