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Generation of tunable terahertz radiation using Josephson vortices: Transition and Cherenkov radiation

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Abstract

We describe proposal on how to control the THz radiation generated by fast moving Josephson vortices in spatially modulated (either along the *c*-axis or the *ab*-plane) samples of $Bi_2Sr_2CaCu_2O_{8+\delta}$ and related superconducting compounds. We show that the JVs moving in a subset of slightly weaker junctions can generate *out-of-ab-plane* and *outside-the-cone* Cherenkov radiation. The *ab*-plane modulation of superconducting properties (achieved, for instance, by ion irradiation lithography) can result in transition radiation within certain frequency windows, i.e., allowing the design of tunable THz emitters and THz photonic crystals.

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Keywords: THz emitters; Layered superconductors; THz radiation

1. Introduction

The recent growing interest in terahertz (THz) science and technology is due to its many important applications in physics, astronomy, chemistry, biology, and medicine [1].

High-temperature $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212) and $Tl_2Ba_2CaCu_2O_{8+\delta}$ (Tl2212) superconductors have a layered structure that allows the propagation of electromagnetic waves with frequencies above the Josephson plasma frequency ω_J. These are called Josephson plasma oscillations [2–6]. This is opposite to the strong damping of electromagnetic waves in low temperature superconductors. It has been recognized (see, e.g., [7–9]) that the Josephson plasma frequency lies in THz range. A possible way to generate THz radiation in Bi2212, Tl2212, and related compounds is to apply an in-plane magnetic field H_{ab} and an external current $J_{\parallel c}$ perpendicular to the superconducting layers (i.e., along the *c*-axis). Josephson vortices (JVs) induced by H_{ab} and driven fast by the *c*-axis current emit THz radiation (e.g., [7,9]). However, it was shown [10–12] that the radiation propagates *only* along the plane of motion of the JVs and decays in the *c*-direction. This strong confinement of THz radiation, its rather restricted controllability, and the large required *c*-axis current, all limit potential applications.

Recent developments in sample fabrication can produce superconductors with alternating layers [13], as well as spatially modulated both pinning (e.g., [14]) and superconducting coupling along layers. We propose employing these capabilities to create modulated structures for a new generation of THz emitters [15] based on moving JVs.

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2. Radiation from a weak junction

A subset of weaker intrinsic Josephson junctions in $Bi_2Sr_2CaCu_2O_{8+\delta}$-based samples can be made using either

- the controllable intercalation technique [13],
- chemical vapor deposition (CVD) (see, e.g., [16]), or
- via the admixture of $Bi_2Sr_2Cu_2O_{6+\delta}$ and $Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$ [17].

The motivation for making such artificial samples (Fig. 1b) is that the radiation produced by the usual $Bi_2Sr_2CaCu_2O_{8+\delta}$ materials is confined to the *ab*-plane. In other words, if all planes (i.e., all junctions) are identical, the Cherenkov radiation generated by fast moving JVs [7,10–12] propagates only along the direction of the JV motion. Electromagnetic (EM) waves along the *c*-direction cannot be generated in identical-planes samples because the maximum velocity of the JVs is smaller than, or of the order of, the smallest velocity of the *c*-axis EM waves. Thus, previous proposals for pseudo-Cherenkov radiation [7,10–12] in standard samples do not refer to the usual Cherenkov radiation of a fast relativistic particle because the radiation generated by those JVs is narrowly-confined, i.e., it does not propagate within a Cherenkov cone. We propose using an artificial stack of thin $Bi_2Sr_2CaCu_2O_{8+\delta}$

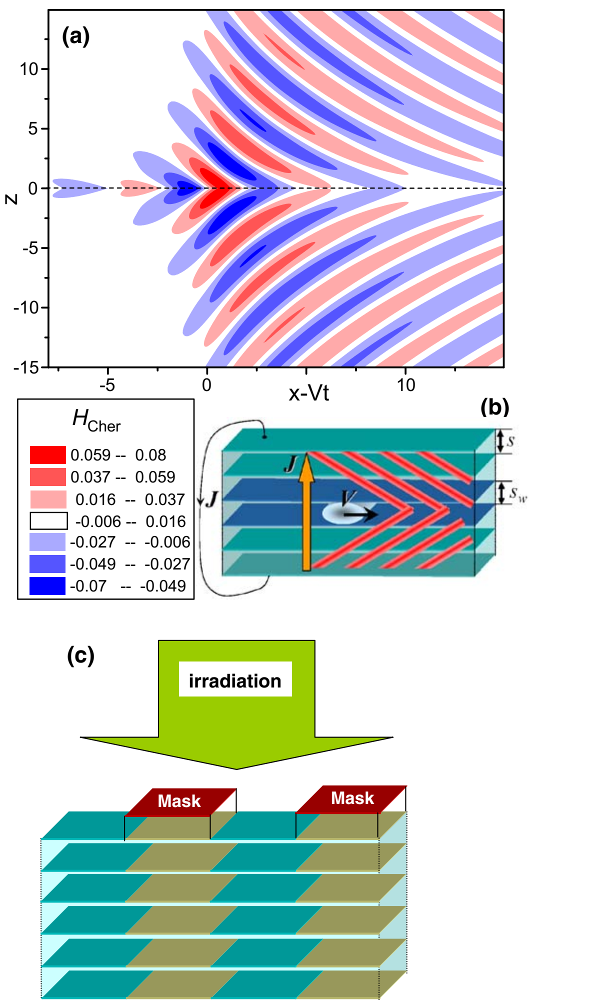

Fig. 1. Cherenkov radiation generated by a fast Josephson vortex (located at $x = Vt$) moving in a slightly weaker junction. (a) Magnetic field distribution $H(x - Vt, y)$ in units of $\Phi_0/2\pi\lambda_c\lambda_{ab}$ for $J_c^w/J_c = 0.2$, $s_w\varepsilon/(s\varepsilon_w) = 1.2$, $V/V_{max}^w = 0.9$. The "running" coordinate, $x - Vt$, is measured in units of $\gamma s \Big/ \left(\pi\sqrt{v^2\beta^2 - 1}\right)$, while the out-of-plane coordinate y is normalized by $s \Big/ \left(\pi\sqrt{v^2\beta^2 - 1}\right)$, where $\beta = \pi J_c \varepsilon s_w / 2J_c^w \varepsilon_w s$. The moving vortex emits radiation propagating forward. This radiation forms a cone determined by the vortex velocity V. (b) A suggested experimental set up: in a weaker junction an out-of-plane current $J_{\parallel c}$ drives a Josephson vortex with velocity V, which is higher than the minimum velocity c_{min} of the propagating electromagnetic waves. (c) Schematic diagram of a sample preparation technique producing a spatially modulated J_c.

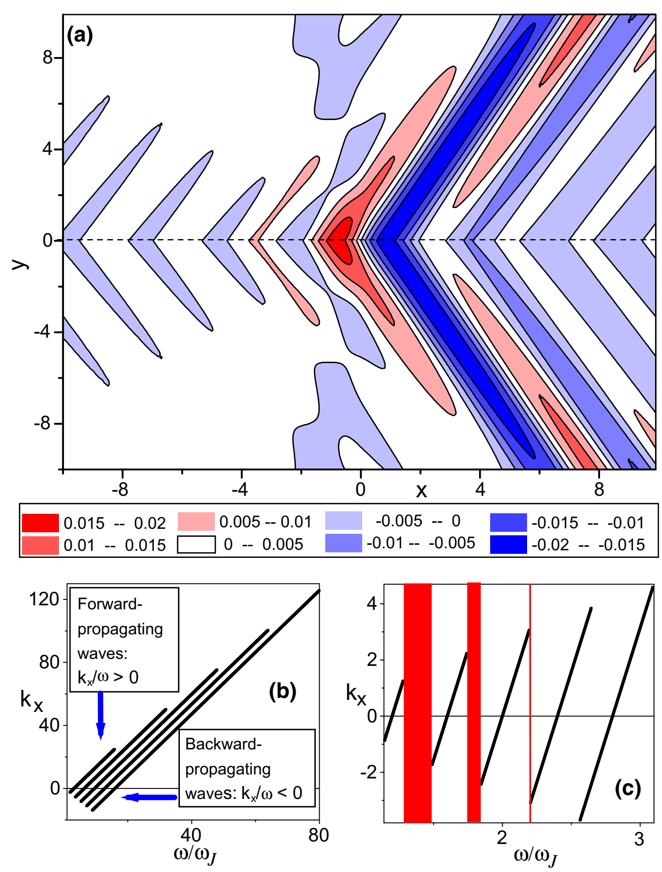

Fig. 2. Transition radiation emitted by a Josephson vortex (located at $x = 0$) moving through a spatially modulated (along the *ab*-plane) layered superconducting sample, as shown in Fig. 1c. (a) Magnetic field distribution $H(x, y)$ (at a certain time moment, say $t = 0$) in units of $\mu_1\Phi_0 s/2\pi\lambda_{ab}^2 a_{var}$ for $V/c_{min} = 0.8$, $a_{var}/l(V = 0) = 1$. The in-plane and out-of-plane coordinates x and y are normalized by the core size l of a static Josephson vortex and $2s$, respectively. (b, c) The x-component k_x of the wave-vector of the radiation versus frequency. In contrast to the Cherenkov radiation (Fig. 1), the phase velocity k_x/ω of the transition radiation could be positive or negative (b), resulting in waves propagating both forward and backward with respect to the Josephson vortex motion. The radiation frequency has forbidden zones, shown by red strips in (c), when the vortex moves relatively slow. This suggests the remarkable possibility of *THz photonic crystals* in modulated layered superconductors, which could be potentially important for applications. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)

sheets having several Josephson junctions which are slightly weaker than the other intrinsic ones. The JVs in the slightly weaker junctions can move much faster than the Josephson plasma waves in $Bi_2Sr_2CaCu_2O_{8+\delta}$ materials, producing electromagnetic waves propagating both parallel and perpendicular to the CuO_2 layers (Fig. 1a and b). Tuning the c-axis current changes both the JV speed and the radiation power.

3. Transition radiation

A completely different design involves samples with periodically modulated [18] (in the ab-plane) superconducting properties which are now uniform along the c-axis. For example, spatial (in-plane) variations of the Josephson maximum c-axis current J_c can be obtained by using irradiation of a standard $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample covered by a modulated mask (see, e.g., [14] and also Fig. 1c). For such a sample, the Josephson vortices can produce *transition* terahertz radiation (Fig. 2a), even if the vortex velocity is slower than the Josephson plasma waves. The terminology "transition radiation" [19] refers to the radiation produced when the vortex alternatively transits from one "layered medium" to another. The spatial periodicity in the ab-plane controls the radiation frequency.

4. Josephson vortex as a fast relativistic particle

Besides potential applications to THz technology, the motion of a JV in layered superconducting materials has significant scientific interest because it can mimic some properties of fast relativistic particles. Indeed, it is well-known that the sine-Gordon equation, describing the motion of a JV in a conventional Josephson junction, is invariant under a Lorentz transform, where the speed of light is replaced by the Swihart velocity [20]. For a standard Josephson junction, the Swihart velocity restricts both the maximum speed of small magnetic field perturbations and the maximum vortex velocity. Following approach [21], the equation describing the Josephson vortex in layered structures and its magnetic field $H(x,y,t)$ becomes nonlocal [15]

$$H-\lambda_{ab}^2\frac{\partial^2 H}{\partial y^2}-\lambda_c^2\frac{\partial^2 H}{\partial x^2}+\omega_J^{-2}\frac{\partial^2}{\partial t^2}\left(1-\lambda_{ab}^2\frac{\partial^2}{\partial y^2}\right)H=0, \quad (1)$$

$$\omega_c^{-2}\frac{\partial^2\phi}{\partial t^2}+\sin\phi=\frac{l}{\pi}\int_{-\infty}^{\infty}\mathrm{d}\zeta K_0\left(\frac{|\zeta-x|}{\lambda_c}\right)\frac{\partial^2\phi}{\partial\zeta^2}, \quad (2)$$

$$\frac{\partial\phi}{\partial x}=\frac{2\pi\lambda_{ab}^2}{\Phi_0}\left\{\frac{\partial H(x,y=+0)}{\partial y}-\frac{\partial H(x,y=-0)}{\partial y}\right\}. \quad (3)$$

Here, the Josephson plasma frequency ω_J is $\sim$1 THz, depending on doping and temperature; the two length scales $\lambda_c=\gamma\lambda_{ab}$ and $\lambda_{ab}=2000\Big/\sqrt{1-T^2/T_c^2}$ Å, with $T_c=90$ K and $\gamma\sim 300$, determine the characteristic scales of magnetic field variations parallel (along the x-axis) and perpendicular (along the y-axis) to the superconducting layers. Eq. (2) describes the propagation of the gauge invariant phase $\phi=\chi_1-\chi_2+2\pi A_y s_w/\Phi_0$ along the junction where the Josephson vortex moves, where χ_1 and χ_2 are the phases of the wave functions of the superconducting condensate of the CuO_2 layers forming the junction, and A_y is the y-component of the vector potential. Here, s_w denotes the thickness of this junction and Φ_0 is the magnetic flux quantum. Eq. (2) has its own space and time scales: l and ω_c^{-1}. There K_0 denotes the modified Bessel function usually employed for the magnetic field distribution in superconductors.

When all Josephson junctions are the same (e.g., as in standard Bi2212 or Tl2212 samples), $\omega_c=\omega_J$, $s_w=s$, and the size l of the soliton at $V=0$ is defined as a product of the anisotropy parameter γ and the spacing s between CuO_2 planes ($l\approx\gamma s$). For a slightly weaker Josephson junction, the Josephson soliton is more elongated, $l=l_w=\gamma sJ_c/J_c^w$, and $\omega_c=\omega_w$ is determined by the parameters of the slightly weaker junction: $\omega_w=\omega_J\sqrt{s_wJ_c^w\varepsilon/sJ_c\varepsilon_w}$ with J_c, ε are the maximum superconducting current and dielectric constant of the intrinsic junctions; the index w refers to the weaker Josephson junction.

5. Cherenkov radiation

Eq. (1) for the magnetic field H allows us to obtain the spectrum of EM waves propagating in the layered structure

$$\omega^2=\omega_J^2+c_J^2(k_y)k_x^2, \quad c_J(k_y)=\frac{\omega_J\lambda_c}{\sqrt{1+\lambda_{ab}^2k_y^2}}. \quad (4)$$

The minimum vortex velocity c_{min} needed for Cherenkov radiation ($c_{min}=c_J(k_y=\pi/s)=\gamma s\omega_J/\pi$), and the characteristic angle

$$\theta=\arctan\left(\frac{\pi\sqrt{V^2-c_{min}^2}}{\omega_J s}\right) \quad (5)$$

of the propagating EM wave are determined by three conditions:

- Eq. (4),
- $\omega=k_xV$,
- and the minimum wavelength $k_y\lesssim\pi/s$.

However, the JVs cannot move above the maximum speed V_{max} determined by the nonlocal equation (2): $V_{max}\approx\omega_J\gamma s/2$ for identical-junctions sample and $V_{max}^w\approx\omega_w l_w$ for samples with some slightly weaker junctions. Therefore, the radiation propagating within the Cherenkov cone with $\theta\neq 0$ cannot be generated using standard $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples, while it can be achieved using the proposed artificial stack or c-axis-modulated $Bi_2Sr_2CaCu_2O_{8+\delta}$ samples.

Using Eqs. (1) and (2) we calculate perturbatively the Cherenkov radiation (see Fig. 1a):

$$H_{\rm Cher}(x,y,t) = \frac{\Phi_0}{2\pi\lambda_{ab}^2}\int_{q_{\min}}^{\infty}\frac{\mathrm{d}k_x}{k_y(k_x)}\exp[-k_x l(V)] \times \sin[k_x(x-Vt)+k_y(k_x)|y|], \quad (6)$$

where the dynamical-soliton size $l(V)$ decays when the vortex velocity V [22] increases as

$$l(V) = l_0\left(1+\sqrt{1-V^2/(V_{\max}^{\rm w})^2}\right)\Big/2. \quad (7)$$

The y-component k_y of the wave-vector is determined by the dispersion relation (4), while

$$q_{\min} = \omega_{\rm J}\Big/\sqrt{V^2-c_{\min}^2}. \quad (8)$$

Due to the rather unusual dispersion relation (4), i.e., the decrease of $k_y(k_x,\ \omega = k_x V)$ with increasing k_x, the electromagnetic waves are located *outside* the Cherenkov cone (Fig. 1a), which is drastically different from the Cherenkov radiation of a fast relativistic particle. The new type of radiation predicted here could be called *outside-the-cone* Cherenkov radiation.

6. Zone structure of the transition radiation

Here we consider standard Bi2212 or Tl2212 materials with identical junctions. As mentioned above, previously proposed pseudo-Cherenkov-like radiation [7,10–12] is confined to the *ab*-plane, and requires a rather strong *c*-axis current. In contrast to this, we consider an alternative approach to generate THz radiation using modulated $Bi_2Sr_2CaCu_2O_{8+\delta}$ materials which allows to *control the spectrum* of the radiation. This proposal might be the easiest to experimentally implement, among the ones listed here, since it does *not* require samples with slightly weaker junctions.

The modulation of superconducting properties along the *ab*-plane could be done by using, e.g., either lithography (see Fig. 1c) or by controlling the JV dynamics via their interactions with other type of vortices (pancake vortices; with its density determined by the out-of-plane field H_c). Indeed, the *c*-axis field, H_c, creates another type of vortices, called pancake vortex stacks, which have slow dynamics and attract Josephson vortices [23,24]. The interaction of these two types of vortices changes the dynamics of the JVs [25,26] and affects their motion in a similar manner as spatially modulating the critical-current with a period which can be easily tuned by the out-of-plane field H_c.

To simplify the problem, we study the radiation of Josephson vortices with modulated $\omega_{\rm c}(x)$. Note that the modulation of any other parameters (e.g., $l(x)$ or both $l(x)$ and $\omega_{\rm c}(x)$) in Eq. (2) results in the same qualitative results. For periodically-varying properties (with spatial period $a_{\rm var}$) of intrinsic junctions, the frequency ω and the in-plane wave-vector k_x of the emitted EM waves are related by the condition

$$\omega = (k_x + 2\pi m/a_{\rm var})V, \quad (9)$$

with $m = 1,2,\ldots$ This is the momentum conservation law: the momentum of a moving JV can be transferred not only to the EM waves but also to the modulated medium, in full analogy with electron motion in a periodic potential. Following this analogy, we derived the "zone structure" of the radiated waves

$$H = \sum_m \int_{\omega_{\min}(m)}^{\omega_{\max}(m)} H_m(\omega)\sin(k_x x + k_y(\omega,k_x)|y| - \omega t)\mathrm{d}\omega, \quad (10)$$

where

$$H_m(\omega) = \frac{\mu_m\Phi_0}{2\pi\lambda_{ab}^2 a_{\rm var}}\frac{\omega k_x(m)}{k_y(\omega,k_x(m))}\frac{\exp(-|\omega|l(V)/V)}{|k_x(m)|V_{\max}\omega_{\rm J}-\omega^2}. \quad (11)$$

Here $k_x(m)$ is related to the momentum $2\pi m/a_{\rm var}$ transferred to the modulated medium by

$$k_x = \omega/V - 2\pi m/a_{\rm var} \quad (12)$$

and μ_m is the Fourier component of $2[1-\omega_{\rm c}(x)/\omega_{\rm J}]$. For a chosen zone-number m, the radiation is emitted in a certain frequency window $\omega_{\min}(m) < \omega < \omega_{\max}(m)$, with

$$\omega_{\min,\max}(m) = \frac{2\pi m c_{\min}}{a_{\rm var}}\frac{V^2}{c_{\min}^2 - V^2} \times\left[\frac{c_{\min}}{V}\mp\sqrt{1-\omega_{\rm J}^2\left(\frac{a_{\rm var}}{2\pi m c_{\min}}\right)^2\frac{(c_{\min})^2-V^2}{V^2}}\right]. \quad (13)$$

Since the frequencies $\omega_{\min,\max}$ change with changing $a_{\rm var}$, the generation of THz radiation in a certain frequency range could be designed by adjusting the period of the spatial modulations of ion irradiation during the sample preparation (Fig. 1c). When the out-of-plane magnetic field H_c is applied, the transition THz radiation might be tuned by varying H_c.

In contrast to the outside-the-cone Cherenkov-like radiation (Fig. 1a), our proposed transition radiation (Fig. 2a) propagates both *forward and backward* in space. Indeed, the in-plane component k_x of the wave-vector and, thus, the corresponding phase velocity both change sign (Fig. 2b and c). Also, the EM waves running backward can be directly seen from the magnetic field distribution shown in Fig. 2a. For fast Josephson vortices moving with velocity close to $c_{\min}$, the frequency zones overlap for different zone number m (Fig. 2b). However, for slower speeds, we obtain the forbidden frequency ranges $\omega_{\max}(m) < \omega < \omega_{\min}(m+1)$ (see, Fig. 2c) of radiated electromagnetic waves. Thus, such materials might be used for making tunable THz photonic crystals controlled, e.g., by the changing out-of-plane magnetic field H_c.

For out-of-plane radiation, the electric field is of the same order of the magnetic field, which is very unusual for conducting media. This allows to overcome the important impedance mismatch problem [15].

7. THz detectors and tunable THz filters

To achieve a selective filtering of THz waves, we have proposed [27] tunable filters of THz radiation using the Josephson vortex lattice as a tunable photonic crystal. Indeed, the Josephson vortex (JV) lattice is a periodic array that scatters electromagnetic waves in the THz frequency range. We show that JV lattices can produce a controllable photonic band gap structure [27,28] (THz photonic crystal) with easily tunable forbidden zones controlled by the in-plane magnetic field. The scattering of electromagnetic waves by JVs results in a strong magnetic-field dependence of the reflection and transparency [27]. Fully transparent or fully reflected frequency windows can be conveniently tuned by the in-plane magnetic field.

Our suggested design for novel THz detectors [29] employs the predicted surface Josephson plasma wave, which can propagate along the superconductor–vacuum interface when the wave frequency is below ω_J. We derive that the incident THz wave can resonantly excite the surface wave at certain angles between the incident wave and the sample surface. This results in a strong increase of absorption of THz wave in the sample, a resonant peak of the sample resistance, and the strong suppression of the specular wave (so-called Wood's anomalies). The position of the peak allows to measure frequency and direction of the incident THz wave.

8. Conclusions

A grand challenge is to controllably generate electromagnetic waves in Bi2212 and Tl2212 and other layered superconducting compounds because of its terahertz frequency range [7–9]. Considering recent advances in sample fabrication [13,14], we propose [15,27,29,30] several experimentally realizable systems for generating, filtering, detecting THz radiation.

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