Quantum noise in photothermal cooling

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We study the problem of cooling a mechanical oscillator using the photothermal (bolometric) force. Contrary to previous attempts to model this system, we take into account the noise effects due to the granular nature of photon absorption. We achieve this by developing a Langevin formalism for the motion of the cantilever, valid in the bad-cavity limit, which includes both photon absorption shot noise and the noise due to radiation pressure. This allows us to tackle the cooling problem down to the noise-dominated regime and to find reasonable estimates for the lowest achievable phonon occupation in the cantilever.

I. INTRODUCTION

The optomechanics of deformable cavities was born in the 1970s from the seminal works of Braginsky [1]. In recent years the first successful observation of self-cooling due to photothermal [2] and radiation pressure [3–5] forces started a race to reach the quantum regime of mechanical motion [6–17].

The self-cooling process usually involves an optical cavity whose mirrors feel an optically induced force proportional to the cavity population. This force depends on the mirror position because the cavity population depends on the cavity length. If this dependence is delayed in time, e.g., because the force depends on the position of the mirror at an earlier time, then self-cooling of the mirror motion can be achieved (given the right parameter regimes [18]).

These optomechanical self-cooling schemes can be classified into different categories according to the nature of the optically induced force that dominates the cooling process. The most commonly used self-cooling method is the radiation pressure force that exploits the pressure exerted by photons bouncing off the mirror. In this case, the delay in the force is given by the cavity storage time; that is, the time taken by the photon population inside the cavity to adjust when the cavity length is modified. The second kind of optically induced force exploits the thermal deformation of the mirror due to the absorption of photons [19]. In this case, the delay is given by the heat diffusion time through the mirror. This force, dubbed the photothermal (or bolometric) force, has been observed in multiple experiments and can be much stronger than the radiation-pressure force in appositely designed systems [2,20,21].

When either cooling mechanism brings the system toward the quantum regime, it also becomes necessary to consider (together with the dynamically induced cooling rate) the spontaneous heating rate given by the intrinsic quantum noise. This is because photons are reflected, or absorbed, one by one in both cooling schemes. In the case of radiation-pressure cooling, a quantum theory exists [22–26] showing that, with the ideal parameters, cooling to the ground state of the mirror motion is still possible. However, unlike the radiation-pressure force, until now the photothermal cooling process has not been investigated in the quantum limit. This is due to the intrinsic difficulty in building a fully quantum theory of a process that is

fundamentally dissipative and involves a macroscopic number of degrees of freedom (e.g., photon absorption, heat diffusion, and thermal deformation).

Our goal in this paper is to construct a theory (albeit a phenomenological one) rigorous enough to be able to firmly answer questions of practical importance for experiments that employ photothermal cooling. Mainly we want to know whether it is possible to exploit photothermal cooling to reach the quantum regime, and to identify the bounds on parameters necessary to reach this regime.

The theory we construct uses the model of a cantilever-mounted mirror (see Fig. 1) in order to be able to compare with specific experiments. However, in principle one can also apply our arguments to other systems, such as microdisk optomechanical systems [40,41].

This paper is structured as follows: in Sec. II we lay down our theory and analyze the results. After having described the issue of residual heating due to photon absorption in Sec. III, we will test our theory with numerical data taken from a well-documented experiment in Sec. IV.

II. THEORY OF PHOTOTHERMAL COOLING

A. Basic approach

Our aim is to describe the process of self-cooling of an oscillation mode of a cantilever (see Fig. 1). In the following, unless explicitly stated, we will always be considering this specific mode. It is usually the fundamental (lowest frequency) mode of the cantilever, but this depends on the geometry of the optical cavity and the cantilever [21].

In the specific model we consider here, the cantilever acts as one of the mirrors of a pumped photonic cavity. The cantilever mode couples to the photonic field of the cavity because the frequency of the cavity, and thus its equilibrium population, depend on the cantilever position. Assuming the pumping laser to be red detuned, and referring to the coordinate axes as shown in Fig. 1, we find that a shift of the cantilever toward positive x will cause a lowering of the cavity frequency and thus an increase in the photonic population (and vice versa).

Even with a very reflective coating there will always be a residual photon absorption in the cantilever. When a photon is absorbed in the cantilever it will create an excess of heat
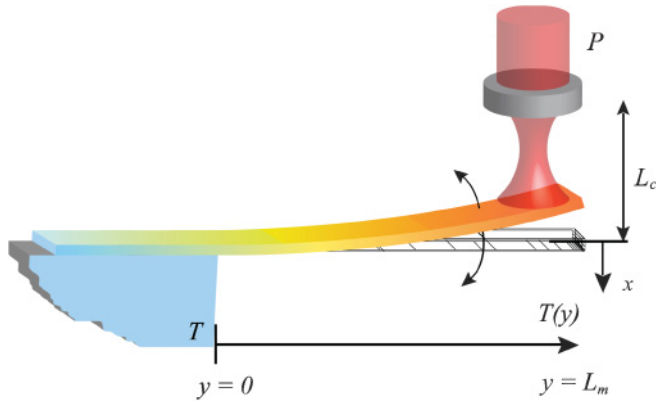
FIG. 1. (Color online) Schematic diagram of an optomechanical system. The mechanical system, or cantilever, is fixed on the (blue) left edge, and operates as a movable mirror (right side, in orange) forming one half of an optical cavity (shown by the red constriction). The gray disk is a nonmovable (i.e., fixed) half-mirror. The optical and mechanical modes are coupled via both radiation-pressure and photothermal forces. The optical cavity (bounded by the fixed gray disk and the (orange) right edge of the cantilever) has equilibrium frequency ω_c, quality factor Q_c, equilibrium length L_c, photon lifetime Γ_c, and is pumped by an external laser with power P and frequency ω_p. The cantilever has thermal conductivity κ, length L_m, and surface area s. The cantilever mode we consider for cooling has frequency ω_m, effective mass m, and quality factor Q_m. The right edge of the cantilever absorbs photons at the rate α, and has a thermal deformation coefficient χ.

(and thus an excess phonon population) in the region of the cantilever that serves as the optical cavity mirror. This heat will then diffuse through the cantilever following normal rules of heat transfer.

Anharmonicity in the interatomic potential in the cantilever gives rise to the phenomenon of thermal deformation [27]. As the average energies in the vibrational degrees of freedom of the atoms (and thus the phonon population) increase, the average interatomic distances change. If this deformation is not uniform through the whole cantilever, (e.g., if the cantilever is formed by layers of materials with different thermal expansion coefficients) then strains develop that exert an effective force on the entire cantilever.

The number of *absorbed* photons, and thus the corresponding excess heat, is proportional to the *total photon population* of the optical cavity. However, the deformation-generated force is exerted only after the heat has diffused through the cantilever. This implies that the cantilever is subject to a force dependent on its position at past times.

The cavity photons act on the cantilever also through the well-known radiation pressure effect, that is, they exert a force on the cantilever as they bounce on the movable mirror. In usual radiation-pressure cooling experiments the cavity response time is of the order of the cantilever mode frequency (the good cavity regime [23,24]), that is, the time the cavity population takes to adjust to the new cantilever position is of the same order of the cantilever period. This also results in a delayed force acting on the cantilever. However, in the systems optimized for photothermal cooling which we treat in this paper, the cavity response time is much shorter than the

cantilever oscillation period (the bad cavity limit) and we can thus safely neglect the retarded nature of such force.

As mentioned in the Introduction, to develop a microscopic quantum theory of the whole process is a formidable task, due to the intrinsic many-body and dissipative nature of the phenomena involved (photon absorption and heat transfer). However, given the time scales of the cantilever motion and of the thermal delay, we can assume that the quantized cantilever mode is evolving under the action of a semiclassical delayed feedback force. In other words, the whole system (with thermal delay) effectively measures the cantilever position (via the number of absorbed photons), stores it in a classical signal (the heat), and transmits it to a classical actuator (the cantilever deformation).

The cooling effect thus arises through a semiclassical feedback or backaction effect, not different in principle from electronically controlled active classical feedback mechanisms used in quantum optics [19,28–30]. In order to better understand this parallelism it is worthwhile to look at the orders of magnitude involved in usual photothermal experiments. For the cantilever in Ref. [20] the cantilever is 220 μm long and its thermal diffusion time is of the order of 0.5 ms. Comparing this with the lifetime of phonons in silicon, which are in the picosecond scale [31], it is easy to understand that no quantum coherence survives through the effective feedback loop.

Here, we thus construct a Langevin description of the cantilever under the action of the radiation-pressure force and of the time-delayed (effective feedback) photothermal force with an added noise source in the effective feedback loop due to the photon absorption shot noise.

B. Treatment of the photothermal force

It is well known that the photothermal force, in the classical regime, can be modeled using a time-delayed force, depending on the position of the cantilever at past times. This approach has been proven to describe well experiments not only in the cooling regime [2,20] but also in the regime where self-sustained oscillations develop [34]. The force can thus be written as a time integral over a memory kernel of the instantaneous force due to the thermal deformation

$$F_{\text{ph}}[x](t) = \int_{\mathbb{R}} d\tau \, h(t - \tau) \frac{dF_{\text{def}}(\tau, x(\tau))}{d\tau}, \qquad (1)$$

where the memory kernel associated to the thermal diffusion delay τ is taken to be

$$h(t) = [1 - \exp(-t/\tau)]\Theta(t), \qquad (2)$$

with $\Theta(t)$ the Heaviside function.

From Eqs. (1) and (2) we see that the instantaneous force F_{def} coincides with the actual photothermal force F_{ph} we would have for a vanishing delay time τ,

$$F_{\text{def}}(t, x) = \lim_{\tau \to 0} F_{\text{ph}}[x](t). \qquad (3)$$

As shown in Refs. [32] and [33], in a coherently pumped optical system with a closed feedback loop, it is sufficient to use a semiclassical theory in order to describe the photon absorption shot noise. We limit ourselves to a semiclassical description of the electromagnetic field in a coherent state. The

easiest way to do so is to start from a quantum Hamiltonian for the pumped photonic cavity [24]

$$\hat{H}_c = \hbar\omega_c\left(1 - \frac{x}{L_c}\right)\hat{a}^\dagger\hat{a} + Ee^{i\omega_p t}\hat{a} + Ee^{-i\omega_p t}\hat{a}^\dagger, \quad (4)$$

where $\hat{a}$ is the annihilation operator for a cavity photon, ω_c is the cavity frequency, L_c its equilibrium length, ω_p is the pump frequency, E is a pump term, and we assume the loss rate of cavity photons to be given by Γ_c. Substituting c numbers for the photonic operators ($\hat{a} \rightarrow a$), as usual for coherent states, allows us to calculate the population of the cavity. Given the large discrepancy between the frequency of the cantilever ω_m (usually in the kHz to MHz range) and of the photonic cavity response time Γ_c (usually in the GHz to THz range) we are not going to consider explicitly the dynamics of the photonic mode, but we will assume that it responds instantaneously to any change in the position of the cantilever.

We thus obtain the instantaneous photon population of the cavity as a function of the cantilever position x,

$$n_c(x) = \frac{E^2}{\left[\omega_c\left(1 - \frac{x}{L_c}\right) - \omega_p\right]^2 + \Gamma_c^2/4}. \quad (5)$$

Assuming that we have a well-defined cavity frequency ω_c, we can find the relation between the pump term E and the input power P by equating the number of incoming photons, $P/\hbar\omega_p$, with the number of photons lost, $n_c\Gamma_c$. We thus obtain

$$E = \sqrt{\frac{\Gamma_c P}{4\hbar\omega_p}}, \quad (6)$$

where Γ_c is the cavity photon lifetime.

In order to proceed in the derivation of a quantum Langevin equation for the cantilever, which takes into consideration photon shot noise, we need to fix the dependence of the photothermal force upon the intensity of the cavity field $n_c(x)$.

While the microscopic derivation of such a force would be extremely complex and sample dependent, we expect it to be a function of the heat absorbed by the cantilever. In other words, the instantaneous photon energy $\omega_c(1 - \frac{x}{L})$ times the current of absorbed photons $I_c(x,t)$. Here we will limit ourselves to the simple case of a linear dependence that, as we will see, gives good results when compared with experiments. We will thus write the instantaneous deformation force as

$$F_{\mathrm{def}}(t,x) = \chi\hbar\omega_c\left(1 - \frac{x}{L_c}\right)I_c(x,t), \quad (7)$$

where χ is a phenomenological deformation coefficient of the cantilever with a dimension of the inverse of a velocity.

We note that the current I_c of absorbed photons can be written as the sum of two terms: an average component and a fluctuating component due to shot noise, the average being proportional to the number of photons present in the cavity (itself a function of x), and the noise to a coefficient times a stochastic noise term $\eta(t)$ [32,33],

$$\begin{aligned} I_c(x,t) &= \alpha n_c(x) + \delta I_c(t,x) \\ &= \alpha n_c(x) + N(x)\eta(t), \end{aligned} \quad (8)$$

where α is the absorption rate of the cantilever.

In order to have a cooling effect, the noise term in Eq. (8) needs to be small compared with the average value. Assuming the cantilever oscillations to be much smaller than the equilibrium cavity length L_c, we can thus insert Eq. (8) into Eq. (7) and make a first-order expansion both x and η. We thus obtain

$$F_{\mathrm{def}}(t,x) \simeq F_0 + (\nabla F)x + N\eta(t). \quad (9)$$

As the different absorption events are completely uncorrelated we can, in a time interval short enough for neglecting the variation of x, consider the number of absorptions as a Poisson process. We thus define

$$n_p(t) = \int_0^t dt' I_c(x,t') \quad (10)$$

as the number of photons absorbed up to time t. Its momenta are given by

$$\langle n_p(t)\rangle = t\alpha n_c(x), \quad (11)$$

$$\langle n_p(t)^2\rangle = t^2\alpha^2 n_c(x)^2 + N^2\int_0^t dt'\int_0^t dt''\langle\eta(t')\eta(t'')\rangle.$$

Imposing that n_p has Poissonian, time-independent statistics over the time interval we consider here, we have

$$\langle n_p(t)\rangle = \langle n_p(t)^2\rangle - \langle n_p(t)\rangle^2. \quad (12)$$

We thus arrive at the following condition for the correlator of current fluctuations in the linear approximation:

$$N^2\langle\eta(t')\eta(t'')\rangle = \alpha n_c(0)\delta(t' - t''). \quad (13)$$

The current I_c is thus characterized by a white-noise spectrum (as in the semiclassical treatment of optical feedback [32,33]) and can be written as

$$I_c(x,t) = \alpha n_c(x) + \sqrt{\alpha n_c(0)}\eta(t), \quad (14)$$

where the white-noise term $\eta(t)$ has the correlator

$$\langle\eta(t')\eta(t'')\rangle = \delta(t' - t''). \quad (15)$$

Putting together Eqs. (14), (5), and (7) and applying basic analysis we find the following values for the coefficients ∇F and N:

$$\begin{aligned} \nabla F &= \frac{n_c(0)\alpha\chi\hbar\omega_c}{L_c}\frac{2\omega_c\Delta - \Delta^2 - \frac{\Gamma_c^2}{4}}{\Delta^2 + \frac{\Gamma_c^2}{4}}, \\ N &= \chi\hbar\omega_c\sqrt{\alpha n_c(0)}, \end{aligned} \quad (16)$$

where $\Delta = \omega_c - \omega_p$ is the cavity detuning. In Sec. II D we use these terms directly in the equation of motion for the cantilever.

C. Treatment of the radiation pressure force

The radiation pressure force lends itself to a microscopic quantum treatment and it has been studied in various publications [23,24]. These works show that the effect of radiation-pressure cooling can be easily modeled by an effective damping rate Γ_{rp} and an associated noise term F_{rp}.

The amplitude of Γ_{rp}, which is the cooling capacity of the radiation-pressure force, depends on the ratio between the cavity damping and the cantilever frequency and thus becomes negligible in the bad-cavity regime we consider here. The same

is not true for the noise term, which instead only depends on the intrinsic magnitude of the force and thus leads to a non-negligible equilibrium noise population.

From Ref. [24] we find the following formulas for the optical damping term and for the equilibrium noise population n_m^{rp}:

$$\Gamma_{\text{rp}} = \frac{4 n_c(0) \hbar \Gamma_c \omega_c^2}{m L_c^2} \frac{\Delta}{\left(\Delta^2 + \frac{\Gamma_c^2}{4}\right)^2},$$

$$n_m^{\text{rp}} = \frac{\Delta^2 + \frac{\Gamma_c^2}{4}}{4 \omega_m \Delta}. \tag{17}$$

Having the damping rate Γ_{rp} and the noise population n_m^{rp} in this form, and because we have assumed the bad-cavity limit, means that we do not have to explicitly consider the dynamics of the optical cavity.

It is worthwhile to note that Eq. (17) has been derived neglecting absorption and it could thus become inaccurate if absorption becomes important (i.e., if a non-negligible fraction of the incident photons is absorbed by the cantilever). However, we neglect this possibility because, as we will see in Sec. III, this would lead to an extreme heating of the cantilever, making any cooling extremely problematic (and dependent on the microscopic details of the cantilever).

D. System dynamics: Feedback in a quantum Langevin equation

Using the results from the previous sections, we can describe the dynamics of the system with a set of coupled Langevin equations describing the cantilever mode coupled to a thermal bath and evolving under the conjunct effect of the photothermal and the radiation pressure force,

$$\dot{x} = \frac{p}{m},$$

$$\dot{p} = -m \omega_m^2 x - (\Gamma_m + \Gamma_{\text{rp}}) p + F_{\text{th}} + F_{\text{ph}}[x] + F_{\text{rp}}, \tag{18}$$

where Γ_m and F_{th} are the dissipation and the fluctuation terms given by the coupling with the bath [35–37].

Using the expression for the linearized time-delayed force from Eq. (9) in Eq. (18), we obtain the following second-order dynamical equation:

$$\ddot{x}(t) + \omega_m^2 x(t) + (\Gamma_m + \Gamma_{\text{rp}}) \dot{x}(t) = \frac{F_{\text{th}}(t)}{m} + \frac{F_{\text{rp}}(t)}{m}$$

$$+ \int_{\mathbb{R}} dt' h(t - t') \left[\frac{\nabla F}{m} \frac{d}{dt'} x(t') + \frac{N}{m} \frac{d}{dt'} \eta(t') \right]. \tag{19}$$

This is an integrodifferential equation, and the integral has a convolution. It thus becomes an algebraic equation in Fourier space

$$\left[\omega_m^2 - \omega^2 + i \omega (\Gamma_m + \Gamma_{\text{rp}}) - \frac{\nabla F}{m(1 + i \omega \tau)} \right] x(\omega)$$

$$= \frac{F_{\text{th}}(\omega)}{m} + \frac{F_{\text{rp}}(\omega)}{m} + \frac{N \eta(\omega)}{m(1 + i \omega \tau)}. \tag{20}$$

As we can see from Eq. (20), the photothermal force results in a frequency-dependent damping and a renormalization of the cantilever frequency (see also the discussion in Ref. [20]). In the usual (rigid cantilever) regime in which the frequency

of the cantilever, renormalized by the constant ($\omega = 0$) part of the photothermal force,

$$\tilde{\omega}_m = \sqrt{\omega_m^2 - \frac{\nabla F}{m}}, \tag{21}$$

does not differ much from ω_m, the essential part of the dynamics takes place at $\omega = \omega_m$, and we can thus take the photothermal-induced damping to be

$$\Gamma_{\text{ph}} = \frac{\tau \nabla F}{m(1 + \tau^2 \omega_m^2)}. \tag{22}$$

Notice that, as explained in Ref. [20], the value at which Eq. (21) vanishes corresponds to the onset of mirror instability and sets an upper limit to the strength of the photothermal force

$$\frac{\nabla F}{m \omega_m^2} < 1. \tag{23}$$

Neglecting quantum (zero-point) fluctuations in the photothermal force we can thus calculate the phonon equilibrium population due to the photothermal force alone [i.e., neglecting both the coupling to the thermal bath and the radiation pressure force in Eq. (20)] as

$$n_m^{\text{ph}} = \frac{N^2}{2 \hbar \omega_m \tau \nabla F}. \tag{24}$$

In principle we can also derive the total equilibrium phononic population due to the joint effect of the thermal bath, the radiation pressure, and the photothermal effect by integrating Eq. (20) (under the rigid-cantilever approximation). However, it is easier to calculate this total population via a simple rate equation, which gives

$$n_m^{\text{tot}} = \frac{\Gamma_m n_m^{\text{th}} + \Gamma_{\text{rp}} n_m^{\text{rp}} + \Gamma_{\text{ph}} n_m^{\text{ph}}}{\Gamma_m + \Gamma_{\text{rp}} + \Gamma_{\text{ph}}}, \tag{25}$$

where n_m^{th} is the equilibrium population at temperature T that, given the low frequencies of the cantilever modes, can be approximated as $kT / \hbar \tilde{\omega}_m$.

Inserting Eqs. (16), (17), and (22) into Eq. (25) and remembering that we are working in a regime where $\Gamma_{\text{rp}} \ll \Gamma_m \ll \Gamma_{\text{ph}}$, we can rewrite Eq. (25) as

$$n_m^{\text{tot}} = n_m^C + n_m^N, \tag{26}$$

where

$$n_m^C = \frac{kT m \omega_m^2}{\hbar \tilde{\omega}_m Q_m \nabla F} \frac{1 + \omega_m^2 \tau^2}{\omega_m \tau} \tag{27}$$

is the *classical* population due to photothermal cooling and

$$n_m^N = \left[\frac{\Gamma_c}{\alpha} \frac{\omega_c}{\chi L_c} \frac{1 + \omega_m^2 \tau^2}{\omega_m \tau} + \chi \omega_c L_c \frac{\Delta^2 + \frac{\Gamma_c^2}{4}}{2 \omega_m \tau} \right]$$

$$\times \left[2 \omega_c \Delta - \Delta^2 - \frac{\Gamma_c^2}{4} \right]^{-1} \tag{28}$$

is the population due to noise effects. The first term in Eq. (28) stems from the radiation-pressure noise contribution, and the second from the photothermal noise contribution.

E. Analysis of the results: The quantum regime

From Eq. (27) we can, using Eq. (23) and optimizing over ∇F, find (consistent with previously known results [2,20]) that the efficiency of classical photothermal cooling is maximal for $\omega_m \tau = 1$ and its ultimate value depends on the quality factor of the cantilever

$$Q_m = \frac{\omega_m}{\Gamma_m}. \tag{29}$$

The optimization of Eq. (27) gives us a *lower bound* on the phonon population

$$n_m^{C,\min} \geqslant \frac{3\sqrt{3}kT}{\hbar\omega_m Q_m}. \tag{30}$$

Thus our estimate for the minimal classical population for the cantilever is proportional to its thermal, equilibrium occupation $kT/\hbar\omega_m$ divided by the quality factor of the mechanical oscillator Q_m. Given the rather low frequencies of the mechanical modes involved, the condition of having an average occupation number lower than one puts a rather harsh requirement on the mechanical quality factor.

In Table I we compare some data taken from experimentally realized cantilevers. We see that for the stressed silicon cantilever studied in Ref. [38] the theory predicts, assuming optimal parameters and liquid-nitrogen temperatures, a thermal population almost in the quantum regime. Note that only the cantilever in Ref. [20] has actually been used for photothermal cooling, and thus it is the only cantilever for which we can estimate the deformation coefficient χ and the noise contribution to the final population (see Sec. IV).

Assuming that we have a cantilever quality factor Q_m large enough to bring n_m^C into the quantum regime, we are thus confronted with the noise contributions in Eq. (28). The noise in Eq. (28) depends on various parameters, and the two noise contributions (the radiation pressure in the first line and the photothermal in the second) often have inverse dependencies upon the parameters. Thus naively minimizing one noise term can enlarge the other. We are thus obliged to carefully optimize the parameters in order to calculate the minimal noise population. We start by rewriting Eq. (28) as

$$n_m^N = \left[\frac{\Gamma_c}{\alpha} \frac{1+\omega_m^2\tau^2}{\omega_m^2\tau^2} Q_c^2 A + \frac{1}{2A}\left(\tilde{\Delta}^2 + \frac{1}{4}\right)\right]$$

$$\times \left[2Q_c\tilde{\Delta} - \tilde{\Delta}^2 - \frac{1}{4}\right]^{-1}, \tag{31}$$

where we have defined

$$A = \frac{\omega_m \tau}{\chi \omega_c L_c}, \tag{32}$$

and introduced the renormalized detuning

$$\tilde{\Delta} = \Delta / \Gamma_c, \tag{33}$$

and the cavity quality factor

$$Q_c = \omega_c / \Gamma_c. \tag{34}$$

The first term in the numerator of Eq. (31) is from the radiation pressure noise, and the second is from the photothermal noise. Thus, the new parameter A encapsulates the way in which the two noise terms are inversely proportional to one another. Optimizing Eq. (31) over A we obtain

$$n_m^N = \sqrt{2\frac{\Gamma_c}{\alpha}\frac{1+\omega_m^2\tau^2}{\omega_m^2\tau^2}} \frac{Q_c\sqrt{\tilde{\Delta}^2 + \frac{1}{4}}}{2Q_c\tilde{\Delta} - \tilde{\Delta}^2 - \frac{1}{4}}, \tag{35}$$

where the optimal value of A is given by

$$A_{\mathrm{opt}} = \sqrt{\left(\tilde{\Delta}^2 + \frac{1}{4}\right)\frac{\alpha}{2\Gamma_c Q_c^2}\frac{\omega_m^2\tau^2}{1+\omega_m^2\tau^2}}. \tag{36}$$

This optimal value of A makes the two noise terms equal and minimum. The second factor in Eq. (35) can be further optimized over Q_c, yielding its minimum for large Q_c,

$$n_m^N = \sqrt{\frac{\Gamma_c}{\alpha}\frac{1+\omega_m^2\tau^2}{2\omega_m^2\tau^2}}\sqrt{1+\frac{1}{4\tilde{\Delta}}}. \tag{37}$$

Choosing a large enough detuning $\tilde{\Delta}$, we can thus ignore the second factor in Eq. (37) and we obtain our final estimate for the minimum noise population

$$n_m^{N,\min} \geqslant \sqrt{\frac{\Gamma_c}{\alpha}\frac{1+\omega_m^2\tau^2}{2\omega_m^2\tau^2}}. \tag{38}$$

We see that, given a large enough cavity quality factor Q_c, and the optimal value for χ from Eq. (36), two factors determine the minimum noise population: the delay parameter $\omega_m\tau$ and Γ_c/α. The second one, Γ_c/α, is the ratio between the total photon loss rate (Γ_c, the sum of the mirror absorbtion α and other radiative losses), and the loss part due to absorption in the moving mirror alone (thus this ratio is always larger than one).

Equation (38) shows us that, given appropriate parameters, it is indeed possible to reach the "quantum regime" for the cantilever mode, characterized by a population lower than one. In order to reach such quantum regime it is necessary not only to limit all the losses other than the cantilever

TABLE I. Length L_m, frequency ω_m, quality factor Q_m, thermal equilibrium population n_m^{th} at 77 K, and final population $n_m^{C,\min}$ from Eq. (30) for different cantilevers reported in the literature.

Reference	$L_m(\mu\mathrm{m})$	ω_m	Q_m	n_m^{th}	$n_m^{C,\min}$, Eq. (30)
[38]	275	6.5 MHz	1.5×10^6	1.2×10^6	5
[20]	220	46 KHz	2.2×10^3	1.7×10^8	6.4×10^5
[39]	3.9	3.4 MHz	2.9×10^3	2.2×10^6	7.8×10^3

absorption in order to lower Γ_c/α, but also to design samples with rather large values of $\omega_m\tau$. Furthermore, while large values of $\omega_m\tau$ help lower the noise population, Eq. (27) tells us that they correspondingly reduce the classical cooling efficiency, thus increasing the classical phonon population. To reiterate, to reach the quantum regime we require the following conditions:

(1) The feedback cooling term, Eq. (30), tells us that we need a large cantilever quality factor Q_m, large cantilever frequency ω_m, and low initial temperature T, so that $Q_m \gg kT/\hbar\omega_m$.

(2) To simultaneously minimize the photothermal and radiation pressure noises, we need a high-quality optical cavity $Q_c \gg 1$, a large enough detuning Δ, and a deformation constant χ given by Eq. (36).

(3) These conditions give us Eq. (38), the final equilibrium phonon population due to noise, which is then minimized by choosing Γ_c/α small. Again, this can further be minimized by increasing $\omega_m\tau$, at the cost of reducing the feedback cooling efficiency, and thus requiring a higher cantilever quality factor Q_m.

III. RESIDUAL HEATING FROM PHOTON ABSORPTION

Since the photothermal force is due to photon absorption, the same process that gives the cooling is simultaneously heating up the cantilever as a whole. This is a rather intriguing aspect of photothermal cooling that we have, until now, neglected.

This heating mechanism does not modify the fundamental bounds we derived, accounting simply for an increased temperature in Eq. (27). However, the photothermal heating effect is important if we want to quantitatively fit our model with experimental data, as it can noticeably influence the observed temperature.

The exact description of such residual heating on the cantilever motion would depend on the microscopic structure of the cantilever and it is thus not within the scope of this paper. Here we will limit ourselves to a phenomenological model, able to mimic the interesting physics for realistic absorption and heating rates, and only depending on one adjustable parameter. The simplest way to take into account this phenomenon is to use, inside Eq. (27), not the environmental temperature T, but an average equilibrium temperature under constant illumination (average because the temperature profile will in general vary over the cantilever).

We now consider a rectangular cantilever, characterized by a constant thermal conductivity (constant in space and in temperature), ignoring radiative processes and assuming that the absorption processes happen homogeneously over one of the surfaces (see Fig. 1 for a schematic illustration). Calling s the cantilever surface, L_m its length, and κ its thermal conductivity, from our approximations and Fourier's law we obtain a linear temperature profile,

$$T(y) = T + \frac{\alpha n_c(0)\hbar\omega_c y}{s\kappa}. \tag{39}$$

The effective temperature to be used in Eq. (27) will thus be an average between $T(0)$ and $T(L_m)$, taking into account

that the mechanical mode profile is not distributed uniformly over y,

$$\bar{T} = T + \frac{\alpha n_c(0)\hbar\omega_c L_m}{\epsilon s\kappa}, \tag{40}$$

where the adjustable parameter ϵ is of the order of unity ($\epsilon = 2$ for an arithmetic mean). We include this effect in our estimates of χ in the next section.

IV. EXISTING EXPERIMENTS ON PHOTOTHERMAL COOLING: ESTIMATING χ

In order to test our model and to gain some insight in the value of the key parameter χ, we analyze the results from Ref. [20] that, to the best of our knowledge, is the only publication reporting the observation of photothermal cooling with enough experimental data to allow a comparison with our theory. In Ref. [20] the temperature of the cantilever mode is measured for different laser powers, and for temperatures between 300 and 32 K. All the material parameters, besides Γ_c/α, can be retrieved directly from the aforementioned reference.

Fitting the temperature for each different laser power allows us to fix an estimate for the value of χ around the value 2×10^{-5} s m^{-1}. With these parameters, we obtain a phonon noise population of 1.4×10^4, that is a phonon temperature of 5 mK.

V. CONCLUSIONS

In summary, we have extended the classical model of photothermal cooling by Metzger *et al.* [2] to take into account the effect of quantum noise on cooling efficiency. Rather than build up a quantum model from a microscopic Hamiltonian, we treated the complex many-body thermal cantilever deformation effect as an effective time-retarded force. As one cools the mechanical motion to the ground state, noise due to both photon absorbtion and radiation pressure begins to play a role, and we explicitly included these, and heating due to absorbtion, in our model.

In the future it will be interesting to test the predictive power of our model against a larger experimental dataset than the one currently available.

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