Testing quantum contextuality with macroscopic superconducting circuits

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We propose an experimental procedure to *macroscopically* test the Kochen-Specker theorem with superconducting qubits. This theorem, which has been experimentally tested with single photons and neutrons, elucidates the conflict between quantum mechanics and noncontextual hidden variable theories. Two Josephson charge qubits can be controllably coupled by using a two-level data bus built with a phase qubit. Then, by performing joint nondestructive quantum measurements of two distinct qubits, we show that the proposed circuits could demonstrate quantum contextuality at a macroscopic level.

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I. INTRODUCTION

There has been a long history of research on the statistical nature of quantum mechanics (QM). Much effort has been made to prove (or disprove) the difference between quantum statistics and classical statistics of hidden variables. Bell's theorem shows that predictions by quantum theory could contradict those by local hidden variable theories[1,2] if we look at correlations between spatially separated measurements.

Similarly, the Kochen-Specker (KS) theorem demonstrates the incompatibility between QM and *noncontextual* theories (NCTs).[1–5] Noncontextuality means that the measured value of an observable is independent of the choice of other comeasurable (commuting) observables that are measured previously or simultaneously. QM is contextual, i.e., outcomes depend on the context of measurements unlike our every day intuition (e.g., chilli peppers taste spicy regardless of how we eat it). It is an important complement to Bell's theorem: the test of the KS theorem can disprove noncontextual hidden variable theories without referring to locality. It would be very interesting if one could confirm such a counterintuitive phenomenon on a macroscopic scale, and indeed this is our primary motivation here, considering macroscopic quantum states[7] of superconductors.

Many experiments have been performed to show the nonlocal correlations that cannot be explained by any local hidden variable theories.[8] Yet, to our knowledge, only a few physical systems have been employed to carry out the test of the KS theorem, namely, photons,[9–11] neutrons,[12] and trapped ions.[13] A difficulty stems from the fact that the most feasible KS tests to date, which were proposed in,[14–16] require *joint* measurements (instead of the *independent* ones for Bell tests) on two or more quantum subsystems. As it is reviewed below, a quantum system consisting of two qubits facilitates the demonstration of the discrepancy between QM and NCT theories. The specific aim of this work is to propose a possible scheme to test the KS theorem at a macroscopic level, which has never been done before, with superconducting quantum circuits.[7] Thus, our proposal complements preceding studies on Bell's inequalities using superconducting qubits.[17–19]

The two quantum subsystems used in the previous KS tests[9–12] are two (quantum) degrees of freedom (i.e., the path and polarization) of *single* photons or neutrons. In this paper, we consider Josephson charge qubits (JCQs) that are realized with two *macroscopic* Cooper-pair boxes (CPBs) containing $\sim 10^9$ Cooper pairs.[7,20–23] Controllable interqubit couplings are necessary for state preparation and could be implemented by coupling the qubits to a common data bus, another macroscopic two-level quantum system: a Josephson phase qubit (JPQ).[20] The indirect coupling of JCQs has an advantage when performing independent measurements on the two qubits. In most of the schemes for indirect coupling of qubits, the interqubit interactions are usually mediated by *bosonic* modes, e.g., cavity modes for atomic qubits, the center-of-mass vibrational modes for trapped ions, or LC-oscillator modes for Josephson qubits.[21–23] Here we propose an alternative approach to indirectly couple JCQs by utilizing a different type of data bus, consisting of a two-level system such as a JPQ. Recently, the controllable coupling between a JCQ and a JPQ has been experimentally demonstrated.[24]

In our setting, the joint measurements will be achieved by combining two simultaneous measurements on two uncoupled CPBs. For example, an X measurement (σ_1^x) on the qubit 1 and a Z measurement (σ_2^z) on the qubit 2 could be combined as a joint measurement $J_1(=X_1Z_2)$ by using *only a single detector*.[25,26] By introducing a circuit with two dc superconducting quantum interference devices (dc-SQUIDs), joint measurements of two commuting observables (such as J_1 and $J_2=Z_1X_2$) could be simultaneously implemented. Such a circuit would enable a test of the KS theorem.

II. CONTROLLABLE COUPLING BETWEEN JOSEPHSON CHARGE QUBITS

We consider the quantum circuit shown in Fig. 1, wherein two SQUID- based CPBs are connected to a current-biased Josephson junction (CBJJ). The kth ($k=1,2$) CPB is biased by an external flux Φ_k and a gate voltage V_k and the CBJJ is

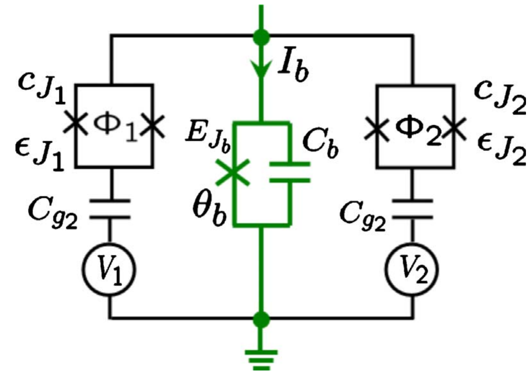

FIG. 1. (Color online) Two Josephson charge qubits are controllably coupled to a common CBJJ (denoted by the dark green part), which acts as a coupler.

biased by a dc current I_b. We assume that the two CPBs have equal junction capacitances (i.e., $c_{J1}=c_{J2}$), gate capacitances ($C_{g1}=C_{g2}$) and also are biased by the same external voltages: $V_1=V_2$. These two CPBs are coupled *indirectly* via the CBJJ. The coupling between the kth CPB and the CBJJ results from the voltage relation: $V_k=V_{Jk}+V_b+V_{gk}$, with V_{Jk}, V_{gk}, and V_b being the voltages across the junctions, the gate capacitance of the kth CPB, and the CBJJ, respectively. This circuit can be easily generalized to include more qubits, all coupled by a common CBJJ. The Hamiltonian of this circuit is[21]

$$\hat{H}=\sum_{k=1}^{2}\hat{H}_k+\hat{H}_b+\hat{H}_{1b}+\hat{H}_{2b}, \tag{1}$$

where

$$\hat{H}_k=\frac{2e^2(\hat{n}_k-n_{gk})^2}{C_k}-E_{Jk}(\Phi_k)\cos\,\hat{\theta}_k \quad \text{and}$$

$$\hat{H}_b=\frac{\hat{p}_b^2}{2\tilde{C}_b}\left(\frac{2\pi}{\Phi_0}\right)^2-E_{Jb}\left(\cos\,\hat{\theta}_b-\frac{I_b\hat{\theta}_k}{I_0}\right),$$

are the effective Hamiltonians describing the kth CPB and the CBJJ. Also,

$$\hat{H}_{kb}=\frac{2\pi C_{gk}E_{Jk}(\Phi_k)}{C_k\Phi_0}\hat{\theta}_b\,\sin\,\hat{\theta}_k, \tag{2}$$

describes the coupling between them. $E_{Jk}(\Phi_k)=2\epsilon_{Jk}\cos(2\pi\Phi_k/\Phi_0)$ and $C_k=2c_{Jk}+C_{gk}$ are the effective Josephson energy and capacitance of the kth CPB. The Josephson energy and effective capacitance of the CBJJ are denoted as E_{Jb} and $\tilde{C}_b=C_{Jb}+\Sigma_{k=1}^{2}(C_{Jk}^{-1}+C_{gk}^{-1})^{-1}$.

Suppose that the CPBs are biased such that $n_{gk}=C_{gk}V_k/(2e)\sim 1/2$ and thus they behave as effective two-level systems (with the basis $\{|0_k\rangle,|1_k\rangle\},k=1,2$) generating JCQs. By introducing Pauli operators with respect to this basis, the Hamiltonian for the kth JCQ becomes

$$\hat{H}_k=\frac{2e^2(n_{gk}-1/2)}{C_k}\hat{\sigma}_k^z-\frac{E_{Jk}(\Phi_k)}{2}\hat{\sigma}_k^x. \tag{3}$$

Although a CBJJ can be approximated as a harmonic oscillator,[21] when $I_b\ll I_0=2\pi E_{Jb}/\Phi_0$, we consider a different case, where $I_b\lesssim I_0$ so that the CBJJ has only a few bound states. The two lowest-energy states, $|0_b\rangle$ and $|1_b\rangle$, define a JPQ acting as a data bus. Under such a condition, the Hamiltonian of the CBJJ reduces to

$$\hat{H}_b=\hbar\omega_b\hat{S}_b^z, \tag{4}$$

with $\hat{S}_b^z=|0_b\rangle\langle 0_b|-|1_b\rangle\langle 1_b|$ being the standard Pauli operator and ω_b the eigenfrequency.

The controllability of the present quantum circuit is due to the fact that the external flux and voltage biases for the JCQs are manipulable. For example, the charging energy $E_k^C(n_{gk})=4e^2(n_{gk}-1/2)/C_k$ of the kth JCQ can be switched off by setting the gate voltage V_k such that $n_{gk}=1/2$. Also, by adjusting the external flux Φ_k one can control the effective Josephson energy of the kth qubit and consequently its coupling to the JPQ. By setting $n_{g1}=n_{g2}=1/2$ and $E_{J1}(\Phi_1)\omega_b,E_{J1}(\Phi_1)\omega_b>0$, the Hamiltonian Eq. (1) reduces to (under the usual rotating wave approximation in the interaction picture)

$$\hat{H}'(t)=\sum_{k=1}^{2}\lambda_k(\Phi_k)\tilde{\sigma}_k^+\hat{S}_b^-e^{-i\Delta_k(\Phi_k)t}+H.c., \tag{5}$$

where

$$\lambda_k(\Phi_k)=2\pi i\frac{C_{gk}}{C_k\Phi_0}\theta_b^{01}E_{Jk}(\Phi_k),$$

$$\Delta_k(\Phi_k)=\omega_b-\frac{E_{Jk}(\Phi_k)}{\hbar}, \quad \text{and}$$

$$\theta_b^{kj}=\langle k_b|\hat{\theta}_b|j_b\rangle, \tag{6}$$

are the coupling strength, the detuning between the kth JCQ and the JPQ, and the "electric-dipole" matrix elements for the data bus ($k,j=0,1$), respectively. The ladder operators are defined as $\tilde{\sigma}_k^+=|+_k\rangle\langle -_k|$, $|\pm_k\rangle=(|0_k\rangle\pm|1_k\rangle)/\sqrt{2}$ and $\hat{S}_b^+=|1_b\rangle\langle 0_b|$. The JPQ can serve as a data bus to transfer information between the two JCQs. By switching on the Josephson energy of one of the JCQs, the JCQ can be tunably coupled to the data bus with *fixed* parameters.

The indirect coupling between the JCQs could also be designed to produce a direct *dynamical* interaction between them by adiabatically eliminating the state occupation of the data bus. This has been widely considered with bosonic data buses (e.g., Refs. 27 and 28), but never with the two-level data bus used here. By controlling the Josephson energies of the qubits [such that $E_{J1}(\Phi_1)\omega_b>0$, with $E_{J2}(\Phi_1)\omega_b<0$], the interaction Hamiltonian Eq. (5) becomes

$$\hat{H}''(t) = \lambda_1(\Phi_1)\tilde{\sigma}_1^{+}\hat{S}_b^{-}e^{-i\Delta_1(\Phi_1)t} + \lambda_2(\Phi_2)\tilde{\sigma}_2^{+}\hat{S}_b^{-}e^{-i\Delta_2(\Phi_2)t} + h.c. \quad (7)$$

We further assume that the external fluxes are properly set as $|E_{J1}(\Phi_1)|=|E_{J2}(\Phi_2)|=E_J$, yielding $|\lambda_1(\Phi_1)|=|\lambda_2(\Phi_2)|=\lambda, |\Delta_1(\Phi_1)|=|\Delta_2(\Phi_2)|=\Delta$. Here, we consider the large-detuning regime, $\lambda/\Delta \ll 1$, which can be easily satisfied with the typical experimental parameters (e.g.,[20] λ is usually less than a few hundred MHz, while Δ could be adjusted to a few GHZ). Thus, the Hamiltonian (7) can be approximated as

$$\tilde{H}'' \simeq \frac{\lambda^2}{\Delta}\hat{S}_b^{z}(\tilde{\sigma}_1^{+}\tilde{\sigma}_2^{+} + h.c.).$$

This implies that the state occupation in the coupler (i.e., the data bus) could be adiabatically eliminated, since its excitation is virtual. Then, the above three-body Hamiltonian $\tilde{H}''$ can be effectively expressed as

$$\hat{H}_{\text{dyn}} = \frac{\lambda^2}{\Delta}(\tilde{\sigma}_1^{+}\tilde{\sigma}_2^{+} + h.c.), \quad (8)$$

which describes a dynamically induced direct interaction between the two JCQs.

In this circuit, single-qubit operations are relatively simple. For example, a σ_k^x-rotation $\tilde{R}_k^x(\beta)=\exp(i\beta\tilde{\sigma}_k^x)$, where $\beta=2e^2(n_{gk}-1/2)t/(\hbar C_k)$, can be implemented by decoupling the qubit from the data bus and varying the gate voltage V_k slightly from its degeneracy point ($V_k=e/C_{gk}$).

Before proceeding to the next section, let us remark on what we mean by the "macroscopic scale" in superconducting qubits. For a JCQ, the number of Cooper pairs in the CPB (the superconducting island) is a macroscopic variable. In the circuit we describe here, two Josephson junctions connect a CPB and a bulk superconductor. The phases of the CBP and the bulk superconductor are macroscopic variables. In the term H_k of Eq. (1), the charge variable n_k is the number of the so-called extra Cooper pairs, i.e., the number difference of the Cooper pairs in the box, while the conjugate variable θ_k is the average phase difference across the two Josephson junctions. Both the charge and phase variables, n_k and θ_k, can be macroscopic variables if the charging energy of the box is small. However, for the JCQ, this charging energy is designed to be large, so the charge variable n_k is well defined, while the phase variable θ_k fluctuates strongly. That is, the JCQ works in the charging regime (For more detailed discussions, see Ref. 6.) In this sense, the phase variable is a macroscopic variable, whereas the charge variable is not. Because the size of the superconducting circuit for the JCQ is larger than a micrometer, which is a macroscopic scale in the language of condensed matter physics, the circuit of JCQ can still be regarded macroscopic. For this reason, we say that the JCQ is an object at macroscopic scale, which behaves quantum mechanically.

III. JOINT MEASUREMENTS FOR TESTING THE KOCHEN-SPECKER THEOREM

Following the logic proposed in Refs. 14 and 15, for the KS test we need a composite quantum system (consisting of subsystems 1 and 2) or a single system with two degrees of freedom for which (*i*) one always finds the same outcomes for two sets of comeasurable dichotomic (e.g., ± 1) observables $\{Z_1,Z_2\}$ [i.e., $v(Z_1)=v(Z_2)$] and $\{X_1,X_2\}$ [i.e., $v(X_1)=v(X_2)$], and (*ii*) one can perform joint measurements

$$J_1 = Z_1X_2 \quad \text{and} \quad J_2 = X_1Z_2,$$

that are comeasurable as well.

In NCTs, each observable has a predetermined value. Therefore, the value v_1 (or v_2) of the measurement J_1 (or J_2) is given as the product of the values of each observable constituent, namely,

$$v_1 = v(Z_1)v(X_2) \quad [\text{or} \quad v_2 = v(Z_2)v(X_1)].$$

Also, the value of the measurement should be independent of the experimental context, i.e.,

$$\text{NCT:} \quad v_1v_2 = v(Z_1)v(Z_2)v(X_1)v(X_2) = 1. \quad (9)$$

However, in QM there exists a state $|\psi_{12}\rangle$ that gives the same outcomes for the observables $\{Z_1,Z_2\}$ and also for $\{X_1,X_2\}$, and is also an eigenstate of J_1J_2 with eigenvalue -1, i.e., $J_1J_2|\psi_{12}\rangle=-|\psi_{12}\rangle$. Thus, the measured value v_1 of the observable J_1 with this state will always have an opposite sign to that v_2 of J_2, i.e.,

$$\text{QM:} \quad v_1v_2 = -1. \quad (10)$$

Therefore, the noncontextuality in NCTs is incompatible with the contextuality in the standard QM.

Our method for the KS test with the macroscopic circuit in Fig. 2 consists of the following three steps:

(1) Prepare a state of a composite system for which the measured results of Z_1 and Z_2 are always found to be equal to each other, and the same for X_1 and X_2.

The effective Hamiltonian in Eq. (8) can directly deliver such a quantum state and the dichotomic observables can be defined as: $X_k=\tilde{\sigma}_k^x=\hat{\sigma}_k^z, Z_k=\tilde{\sigma}_k^z=\hat{\sigma}_k^x (k=1,2)$. The time evolution operator generated by $\hat{H}_{\text{dyn}}$ in Eq. (8) can then be expressed as

$$\tilde{U}_{\text{dyn}}(\alpha) = \cos\alpha(|--\rangle\langle--| + |++\rangle\langle++|) + i\sin\alpha(|--\rangle\langle++| - |++\rangle\langle--|),$$

with $\alpha=\lambda^2 t/\hbar\Delta$. Thus, starting with the initial state $|\psi(0)\rangle=|--\rangle$, the application of $U_{\text{dyn}}(\alpha)$ with $\alpha=3\pi/4$, followed by a $\tilde{\sigma}_k^z$ rotation, can generate the desired entangled state,

$$|\psi_{12}\rangle = \tilde{R}_1^z\left(\frac{\pi}{4}\right)\tilde{U}_{\text{dyn}}\left(\frac{3\pi}{4}\right)|--\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle). \quad (11)$$

(2) Perform the measurement of Z_1 and Z_2, and also of X_1 and X_2, to confirm the above requirement (*i*), i.e.,

$$v(Z_1) = v(Z_2) \quad \text{and} \quad v(X_1) = v(X_2), \quad (12)$$

from which the NCTs deduce

$$v(Z_1X_2) = v(X_1Z_2), \quad (13)$$

because Eq. (12) leads to $v(Z_1)v(X_2)=v(X_1)v(Z_2)$.

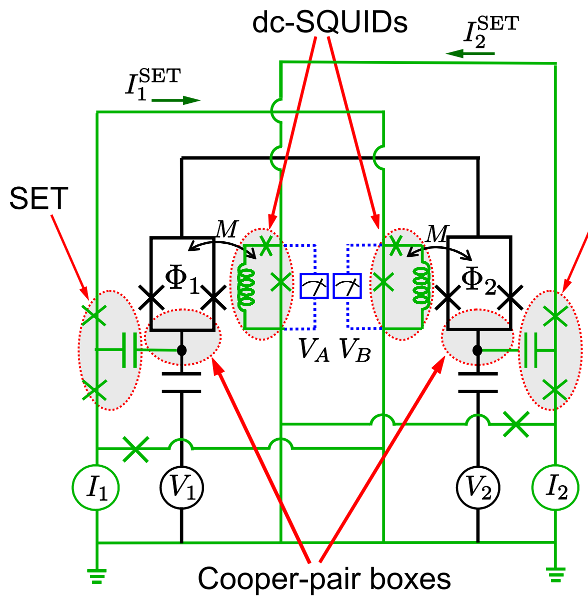

FIG. 2. (Color online) An example of a circuit for the joint measurements on two JCQs. The green-colored parts, which consist of SETs and dc-SQUIDs, indicate the proposed detectors, while the black parts are the JCQs. Two rf-SETs, coupled capacitively to the CPBs, probe the charge states of the qubits. The information about the observable X_j is then transferred to the current I_j^{SET} that biases the dc-SQUID, which probes the circulating current in the $k(\neq j)$th JCQ (i.e., the Z_k measurements) through the inductive coupling M. Voltmeters, V_A and V_B, detect if the neighboring Josephson junction collapses to its normal state. The green parts have a short-cut path to limit the amount of current going through the SETs.

With the quantum circuit proposed above, the Z_k- and X_k-measurements can be performed by individually detecting the circulating current I_k^{SQUID} (i.e., $\hat{I}_k^{SQUID} \lesssim I_c \hat{\sigma}_k^x = I_c \tilde{\sigma}_k^z, I_c = 2\pi\varepsilon_J/\Phi_0$) along the kth SQUID loop and the excess charge n_k (i.e., $\tilde{\sigma}_k^x = \hat{\sigma}_k^z = |0_k\rangle\langle 0_k| - |1_k\rangle\langle 1_k|$) on the kth CPB, respectively. Although the present qubits work in the charge regime, the above critical current I_c could still reach a measurable value, e.g., ~ 8 nA for a typical Josephson junction with $\epsilon_J \sim 25$ μeV.

(3) Perform the two joint measurements, J_1 and J_2, in a row (in any order). The first measurement needs to be done in a nondestructive manner, but the second one does not. Thus the second one does not even have to be a joint measurement; multiplying the outcomes of two separate single-qubit measurements suffices to obtain the value of, say, $v(X_1Z_2)$.

The prepared state $|\psi_{12}\rangle$ can be rewritten as

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}(|\chi_{1,-1}\rangle + |\chi_{-1,1}\rangle), \tag{14}$$

where $|\chi_{1,-1}\rangle$ and $|\chi_{-1,1}\rangle$ are the two eigenstates of the commuting operators J_1 and J_2. The first (second) index of χ indicates the eigenvalue with respect to J_1 (J_2). More explicitly,

$$|\chi_{1,-1}\rangle = \frac{1}{\sqrt{2}}(|0+\rangle - |1-\rangle) = \frac{1}{\sqrt{2}}(|-0\rangle + |+1\rangle), \tag{15}$$

and $|\chi_{-1,1}\rangle$ can be expressed similarly. Thus, when we perform one of the above joint measurements (J_1 or J_2), the state $|\psi_{12}\rangle$ will collapse to either $|\chi_{1,-1}\rangle$ or $|\chi_{1,-1}\rangle$, depending on the outcome.

It can be clearly seen from the superposition in Eq. (14) that the J_1 and J_2 measurements always give outcomes of opposite signs. This implies $v(J_1) = -v(J_2)$, namely,

$$v(Z_1X_2) = -v(X_1Z_2), \tag{16}$$

which contradicts with the noncontextual theories' prediction Eq. (13).

An example of a circuit for the KS test by performing two joint measurements Z_1X_2 and X_1Z_2 on two JCQs is shown in Fig. 2.

The joint measurements on two qubits can be implemented by connecting two individual measurements on single qubits. The trick is in connecting the two measurements, for example, so that the signal for the outcome "0" (or "+") from one qubit will cancel out that for "1" (or "−") from the other, and vice versa. Then, we cannot distinguish the two qubit states, $|01\rangle$ and $|10\rangle$, and the net projection is onto the subspace spanned by these two vectors. Therefore, if the state before the measurement was a superposition in this subspace, such as $\alpha|01\rangle + \beta|10\rangle$, the joint measurement projects it onto itself in a nondestructive manner. In the case of the initial state of Eq. (14), we design the joint measurement Z_1X_2 so that it projects the state onto the subspace spanned by $|0+\rangle$ and $|1-\rangle$, and similarly for X_1Z_2. We cannot project onto the other eigensubspace of Z_1X_2, i.e., the one spanned by $|0-\rangle$ and $|1+\rangle$, however, this projection is unnecessary to rule out the possibility of NCTs.

Let us now describe the measurements in detail. The X_k measurement is achieved, e.g., by a rf-SET (radio-frequency single-electron transistor)[29–31] coupled capacitively to the kth CPB. Suppose that the applied rf-SET is sufficiently sensitive to nondestructively distinguish two charge states $|0\rangle$ and $|1\rangle$ of a CPB. Here, the term "nondestructive" means that the charge in the CPB does not leak/tunnel out in the measurement process. The measured result is then transferred to the current I_k^{SET} (or $-I_k^{SET}$) if the measured state is $|0\rangle$ (or $|1\rangle$).[39] Next, the induced current I_k^{SET} biases the $j(\neq k)$th dc-SQUID (at the center of Fig. 2), which is coupled inductively to the jth qubit. Detecting the circulating currents in the SQUID achieves the Z_j measurement:[32] the clockwise and anticlockwise currents I_j^{SQUID} and $-I_j^{SQUID}$ in the jth SQUID-loop correspond to the states $|+\rangle$ and $|-\rangle$, respectively.

In order to implement the joint measurement described above, we utilize *asymmetric* SQUIDs, in which two Josephson junctions have different areas (See Fig. 3). Let us label the smaller junction as 'A' and the larger "B," and let R_x and I_c^x denote the resistance and its critical current of junction $x = \{A,B\}$, respectively. Then, because of the difference in areas, $R_A > R_B$ and $I_c^A < I_c^B$.

Let us focus on the X_1Z_2 measurement for clarity, i.e., the measurement by SET for qubit 1 and SQUID for qubit 2. The current that biases the SQUID, I_1^{SET}, is split into two currents,

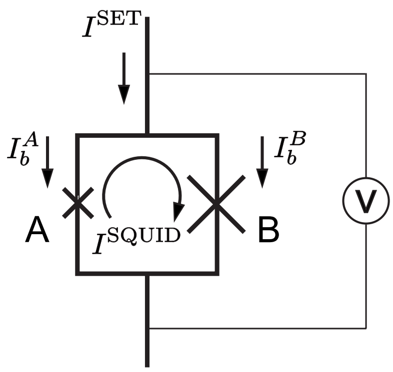

FIG. 3. Asymmetric SQUID as a part of the joint measurement circuit. Two junctions (A and B) have different sizes so that only the smaller one is sensitive to the combination of the directions of $I^{\rm SET}$ and $I^{\rm SQUID}$.

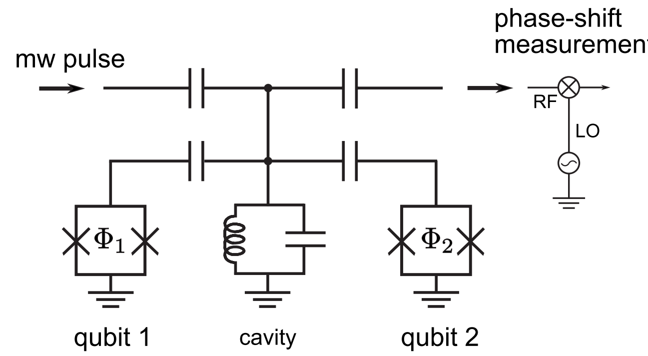

FIG. 4. Schematic circuit diagram for the joint measurement Z_1Z_2 on two superconducting qubits. The transmission line, or cavity, is represented as an *LC* resonator between the qubits.

I_b^A and I_b^B, which go through junctions A and B, respectively. Note that due to the difference in resistance, $I_b^A < I_b^B$. Now, suppose that $I_1^{\rm SET} > 0$ flows downward in Fig. 3 and let $I_2^{\rm SQUID}$ be the current that is inductively induced in the SQUID by the qubit 2 ($I_2^{\rm SQUID} > 0$ when flowing clockwise). Further, we design the SET and the SQUID so that $I_b^x < I_c^x (x = \{A, B\})$, $|I_b^A - I_2^{\rm SQUID}| < I_c^A$, $|I_b^A + I_2^{\rm SQUID}| > I_c^A$, and $|I_b^B \pm I_2^{\rm SQUID}| < I_c^B$. Also, the positive values of these currents I_b^A and $I_2^{\rm SQUID}$ correspond to qubit states $|0\rangle$ and $|+\rangle$, respectively, as we have described above.

Hence, the current going through the junction A exceeds its critical current depending on the combination of the states of qubits 1 and 2. Namely, the collapse of the junction A to the normal state is detected by the voltmeter when the directions of the two currents, I_b^A and $I_2^{\rm SQUID}$, coincide. If the two currents flow in the opposite directions, then the junction A stays superconducting as well as the junction B. Clearly, the combinations of the currents, either $I_b^A - I_2^{\rm SQUID}$ or $-I_b^A + I_2^{\rm SQUID}$, cannot be distinguished, so the desired projection onto the space spanned by $|0+\rangle$ and $|1-\rangle$ is attained without destructing the state of the qubits.

IV. DISCUSSIONS

The major experimental challenges toward the experimental demonstration of our proposal would be the fabrication and manipulation of the JPQ as a switchable coupler and the simultaneous detection of the charge and current states of the JCQ. In particular, fast measurements of the qubits within their decoherence times are a common challenge for almost all coherent manipulations in the solid state systems.

Our measurements are based on the monitoring of the voltage change over the junction of each dc-SQUID, which is biased by the current induced by the rf-SET for measuring the charge states of the JCQ. Suppose that a rf-SET with a demonstrated sensitivity[31] of $\delta q = 3.2 \times 10^{-6} e/\sqrt{\rm Hz}$ is weakly coupled (the corresponding dimensionless coupling strength is $k \sim 0.01$) to the JCQ, the measurement time t_m^c needed to separate two charge states of the JCQ is $t_m^c \geq (\delta q / ke)^2 \sim 30$ ns.[31] The decoherence times in JCQs have been lengthened to the order of a few microseconds.[33,34] Therefore, the physical limit due to decoherence for testing quantum contextuality could be overcome with such JCQs so that our proposal presented here can be feasible.

For the joint measurements, we can employ an alternative method that has been proposed very recently.[35–38] Figure 4 depicts the setting for the joint measurement Z_1Z_2 on two superconducting qubits that are coupled to a common transmission line resonator. Note that the joint observables $J_1' = Z_1Z_2$ and $J_2' = X_1X_2$ can also be employed for the KS test, because the entire protocol can be made equivalent up to only a local transformation. Namely, with an initial state

$$|\psi_{12}'\rangle = \frac{1}{\sqrt{2}}(|0+\rangle - |1-\rangle), \tag{17}$$

in QM, the successive measurements Z_1Z_2 and X_1X_2 always give the same outcomes, i.e., $v(Z_1Z_2) = v(X_1X_2)$. On the other hand, NCTs predict that they are always of the opposite sign, $v(Z_1Z_2) = -v(X_1X_2)$.

The transmission line plays a role of the (leaky) cavity in the typical setting of cavity QED, and the qubits correspond to two-level atoms placed in the cavity. Microwave pulses are injected from one end of the transmission line and the phase of the pulses are measured at the other end with respect to some phase reference, which is denoted as LO (local oscillator) in Fig. 4.

If the qubits are largely detuned from the cavity, the photonic state of the transmission line acquires a phase shift depending on the state of the qubits.[28] The phase shift can take four distinct values, corresponding to four different combinations of qubit states in the $\{|0\rangle, |1\rangle\}$ basis, i.e., $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$. By tuning the system parameters, such as the energy gap in each qubit and the coupling strengths between the cavity and qubits, the phase shifts for two of these two-qubit states, $|01\rangle$ and $|10\rangle$, can be made equal, say $\Delta\varphi_0$, in principle. Thus, observing the phase shift $\Delta\varphi_0$ is equivalent to projecting the two qubits onto the subspace spanned by $|01\rangle$ and $|10\rangle$. This means that if we obtain the value $\Delta\varphi_0$ the joint measurement succeeds, otherwise it does not.

Even with this probabilistic, but conclusive, joint measurement, showing $v(Z_1Z_2) = v(X_1X_2)$ suffices to rule out the

NCTs because such an occurrence cannot be explained by NCTs.

V. CONCLUSIONS

We have presented a possible method to perform joint measurements on two distinct superconducting qubits, with which the KS theorem can be tested at a macroscopic level. Our method is within the reach of present technology. The contextuality is a distinctive feature of quantum mechanics that contrasts sharply with our standard recognition in the classical mechanical world. The Josephson junctions are essentially macroscopic objects, containing a huge number of atoms and electrons. If the contextuality could be observed in our setting, it will be a good example where the intuitive classical notion (NCTs) cannot explain the outcome at all, not even as a (classical) probabilistic rare occurrence. Our scheme would provide a unique test of quantumness in a solid-state-based system involving a large number of particles.

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