Quantum electrodynamics and photon-assisted tunneling in long Josephson junctions

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We describe the interaction between an electromagnetic field and a long Josephson junction driven by a dc current. We calculate the amplitudes of emission and absorption of light via the creation and annihilation of quantized Josephson plasma waves (JPWs). Both the energies of JPW quanta and the amplitudes of light absorption and emission strongly depend on the junction's length and can be tuned by an applied dc current. Moreover, photon-assisted macroscopic quantum tunneling in long Josephson junctions shows resonances when the frequency of the outside radiation coincides with the current-driven eigenfrequencies of the quantized JPWs.

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I. INTRODUCTION

The miniaturization of electronic devices allows the observation of quantum effects which were impossible to measure in the past. Systems of Josephson junctions (JJ), characterized by high frequency (up to several terahertz), exhibit a crossover to the quantum regime at relatively high temperatures. Indeed, quantum oscillations and macroscopic quantum tunneling[1] (MQT) have been observed in charge, flux, and phase qubits.[2] Renewed interest in MQT occurred after the recent discovery of MQT in high-temperature layered superconductors.[3–6] The observed enhancement of MQT was attributed to the spatial structure of the tunneling fluxon.[7,8] It is important to develop a theory of quantum electrodynamics in long (about 1 μm in stacks of JJs, and about tens of microns in low-T_c junctions) JJs where the spatial distribution of the gauge-invariant phase difference is crucial. In this problem, the standard quantum-mechanical approach (where the phase difference is associated with the coordinate of a quantum particle tunneling through an effective potential barrier) becomes invalid, and a more advanced field-theoretical approach is needed.[9,10]

Here we consider a Josephson junction driven by a dc current near its critical value and exposed to terahertz electromagnetic (EM) waves. In this configuration, as known for pointlike contacts, the probability of MQT depends on the intensity and frequency ω of the incident EM waves. In contrast to the short-junction case, we predict several resonant enhancements of the MQT escape rate, when the frequency ω matches the eigenfrequencies of the JPWs. We also propose a full quantum electrodynamical description of long JJs, to calculate the probabilities of absorption and emission of light by JPW quanta.

In Sec. II we derive the model and quantize the field of the gauge-invariant phase difference φ. In Sec. III we consider the interaction of the quantized φ with photons and calculate the transition rates of absorption and emission of light by JPW quanta. This allows us to find the mean values of occupation numbers of JPW quanta and the mean energy of the system, which is pumped by external terahertz radiation. In Sec. IV we calculate the probability of photon-assisted macroscopic quantum tunneling.

II. SECOND QUANTIZATION OF THE PHASE DIFFERENCE FIELD

A. Lagrangian formulation

The geometry of the Josephson junction under study is shown in Fig. 1. Two superconducting bars overlap a length D in the x direction. An insulating layer of thickness s, about several nanometers, is placed between these two bars. A supercurrent with density i flows through the junction in the z direction. The width L of the JJ in the y direction is of the order of, or less than, the Josephson penetration depth λ_J, that is $l=L/\lambda_J \lesssim 1$. The dynamics of the gauge-invariant phase difference $\varphi(t,x,y)$ of such a junction is described by the action

$$\mathcal{S}[\varphi]=\frac{1}{\omega_p}\int dt(\mathcal{L}[\varphi]+\mathcal{L}_\Sigma[\varphi]),$$

$$\mathcal{L}[\varphi]=\frac{\lambda_J E_J}{L}\int dxdy\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2-\frac{1}{2}(\nabla\varphi)^2+\cos\varphi\right],$$

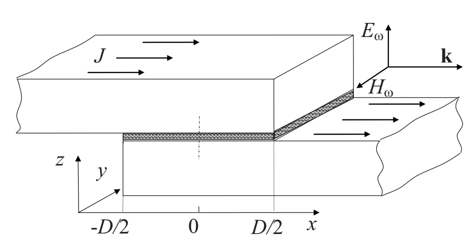

FIG. 1. Schematic of the Josephson junction. The wave vector of the externally applied polarized terahertz electromagnetic wave is directed along the x axis, while its electric (magnetic) field is directed along the z (y) axis.

$$\mathcal{L}_\Sigma[\varphi] = \frac{cE_J}{4\pi i_c L}\oint_\Sigma d\zeta\varphi[\mathbf{H}\times\mathbf{e}_z]_n. \tag{1}$$

In these equations, the x and y coordinates are normalized by λ_J, the time t is normalized by $1/\omega_p$, where ω_p is the Josephson plasma frequency, i_c is the critical current density, and

$$E_J = \hbar\omega_p\Lambda, \quad \Lambda = \frac{i_c\lambda_J L}{2e\omega_p},$$

where Λ is considered to be much larger than unity, $\Lambda \gg 1$. The integration in $\mathcal{L}_\Sigma[\varphi]$ is performed over a contour Σ around the junction's area, and the subscript n refers to the component normal to the contour Σ in the XY plane of the vector product of the magnetic field $\mathbf{H}$ and unit vector $\mathbf{e}_z$.

The classical equation of motion for φ with action (1) is the two-dimensional sine-Gordon equation

$$\frac{\partial^2\varphi}{\partial t^2} - \triangle\varphi + \sin\varphi = 0. \tag{2}$$

The surface term in action (1) depicts the boundary conditions to this equation,

$$\left.\frac{\partial\varphi}{\partial n}\right|_{\mathbf{r}\in\Sigma} = [\mathbf{H}\times\mathbf{e}_z]_n. \tag{3}$$

Representing the magnetic field in the form $\mathbf{H}=\mathbf{H}_J+\mathbf{H}^e$, where $\mathbf{H}^e$ is the external ac magnetic field and $\mathbf{H}_J$ is the field generated by the flowing current, we obtain

$$\left.\frac{\partial\varphi}{\partial x}\right|_{x=\pm d/2} = \pm\frac{I}{2} + \frac{cH_y^e}{4\pi i_c\lambda_J},$$

$$\left.\frac{\partial\varphi}{\partial y}\right|_{y=\pm l/2} = -\frac{cH_x^e}{4\pi i_c\lambda_J}, \tag{4}$$

where

$$I = \frac{iD}{i_c\lambda_J}, \quad d = \frac{D}{\lambda_J}, \quad l = \frac{L}{\lambda_J}.$$

When $\mathbf{H}^e=0$, the stationary solution to Eq. (2), corresponding to the lowest-energy minimum, does not depend on the y coordinate. Below we assume that the ac magnetic field of the incident radiation is directed along the y axis (see Fig. 1). In this case, only the plasma waves in the x direction are excited, and $\varphi=\varphi(t,x)$. When $D \ll \lambda_J$, the field $\varphi(t,x)$ only slightly depends on the x coordinate, and action (1) describes the dynamics of the particle in the washboard potential $V(\varphi)=-\cos\varphi - j\varphi$, where $j=i/i_c$. When $j<1$, this potential has an infinite number of minima, each one separated by a potential barrier of the order of $\sqrt{1-j^2}$. The probability of quantum tunneling from one minimum to the nearest minimum can be easily calculated in the semiclassical approximation.[11] When $D \gtrsim \lambda_J$, the spatial dependence of the field $\varphi(t,x)$ is essential, and the problem of quantum tunneling becomes more complicated. In the semiclassical approximation, the probability of tunneling can be written as[12]

$$\Gamma = \omega_0\sqrt{\frac{30B}{\pi}}\exp(-B),$$

where $B=2\mathcal{S}_E/\hbar$, and $\mathcal{S}_E$ is the action, defined in Eq. (1), in imaginary time $t=i\tau$ calculated along classical trajectories, and ω_0 is the oscillation frequency of the field $\varphi(t,x)$ near one of the energy minima $\varphi_0(x)$. In one of our previous papers[9] we proposed an approach for calculating the tunneling exponent B for a current $I=jd$ close to the critical value $I_c(d)$ (which now nonlinearly depends on d). Here we consider *the effect of external electromagnetic radiation* on the probability of tunneling.

B. Quantum regime

We consider the interaction of φ with electromagnetic waves as perturbations. First, we quantize the field $\varphi(t,x)$ near the energy minimum $\varphi_0(x)$ at $\mathbf{H}^e=0$, find the energy spectrum, and then calculate the transition rates of the field φ, from the ground state to its excited states and vice versa, due to the interaction with the electromagnetic field. The knowledge of the transition rates gives us the mean energy $\bar{E}(\omega,P)$ of the field φ in the presence of an external radiation as a function of its power P and frequency ω. Since the effective potential barrier decreases with the growth of $\bar{E}$, the external radiation enhances the tunneling. It is clear that a strong enhancement of the escape rate Γ should be at frequencies ω close to the eigenfrequencies ω_n of the φ field. The tunneling exponent B, as function of $\bar{E}$, is found here using the approach described in Ref. 9.

The static solution corresponding to an energy minimum satisfies the static sine-Gordon equation

$$\frac{d^2\varphi_0}{dx^2} = \sin\varphi_0 \tag{5}$$

with the boundary conditions

$$\left.\frac{d\varphi_0}{dx}\right|_{x=\pm d/2} = \pm\frac{I}{2}. \tag{6}$$

The solution to this equation exists for currents I less than the critical value $I_c(d)$. If $d \lesssim 4$, the current density in the JJ is approximately constant and the function $I_c(d)$ increases linearly with d; if $d \gg 1$, the current flows near the junction edges and $I_c(d)$ reaches the saturation value $I_c^{\max}=4$. In order to quantize φ we represent it in the form

$$\hat{\varphi}(t,x) = \varphi_0(x) + \hat{\psi}(t,x),$$

where the operator $\hat{\psi}$ satisfies the boundary conditions

$$\left.\frac{d\hat{\psi}}{dx}\right|_{x=\pm d/2} = 0, \tag{7}$$

and expand the Lagrangian $\mathcal{L}$ in powers of $\hat{\psi}$. We introduce the momentum (t in units of $1/\omega_p$)

$$\hat{\pi}(t,x) = \frac{\delta \mathcal{L}}{\delta\left(\omega_p \frac{\partial \hat{\psi}(t,x)}{\partial t}\right)} = \hbar\Lambda \frac{\partial \hat{\psi}(t,x)}{\partial t}, \tag{8}$$

and require the standard simultaneous commutation relation

$$[\hat{\psi}(t,x), \hat{\pi}(t,x')]_- = i\hbar\,\delta(x-x'). \tag{9}$$

The Hamiltonian of the system, $\hat{\mathcal{H}}$, has a form

$$\hat{\mathcal{H}} = \hat{\pi}\frac{\partial \hat{\psi}}{\partial t} - \hat{\mathcal{L}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}', \tag{10}$$

where

$$\hat{\mathcal{H}}_0 = E_J{:}\int_{-d/2}^{d/2} dx\left[\frac{\omega_p^2}{2E_J^2}\hat{\pi}^2 + \frac{1}{2}\hat{\psi}\hat{\mathcal{D}}\hat{\psi}\right]{:},$$

$$\hat{\mathcal{H}}' = E_J{:}\int_{-d/2}^{d/2} dx\left[-\frac{1}{6}\sin\varphi_0\hat{\psi}^3 - \frac{1}{24}\cos\varphi_0\hat{\psi}^4 + \dots\right]{:}. \tag{11}$$

In these equations, the colons ":" mean "normal ordering" and $\hat{\mathcal{D}}$ is a differential operator of the form

$$\hat{\mathcal{D}} = -\frac{\partial^2}{\partial x^2} + \cos[\varphi_0(x)]. \tag{12}$$

In the interaction representation, the operators $\hat{\psi}$ and $\hat{\pi}$ can be written as

$$\hat{\psi} = \sqrt{\frac{1}{2\Lambda}}\sum_n \frac{\psi_n(x)}{\mu_n^{1/4}}(e^{i\sqrt{\mu_n}t}\hat{b}_n^\dagger + e^{-i\sqrt{\mu_n}t}\hat{b}_n),$$

$$\hat{\pi} = i\hbar\sqrt{\frac{\Lambda}{2}}\sum_n \mu_n^{1/4}\psi_n(x)(e^{i\sqrt{\mu_n}t}\hat{b}_n^\dagger - e^{-i\sqrt{\mu_n}t}\hat{b}_n), \tag{13}$$

where μ_n, ψ_n are, respectively, the eigenvalues and orthogonal eigenfunctions of the operator $\hat{\mathcal{D}}$, that is,

$$\hat{\mathcal{D}}\psi_n = \mu_n\psi_n, \quad \int_{-d/2}^{d/2} dx\,\psi_n(x)\psi_m(x) = \delta_{nm}. \tag{14}$$

In Eq. (13), $\hat{b}_n^\dagger$ and $\hat{b}_n$ are the creation and annihilation operators of JPW quanta in the state n. Note that all μ_n's are positive when $I<I_c(d)$ because the $\varphi_0(x)$ corresponds to an energy minimum. In terms of the operators $\hat{b}_n^\dagger$ and $\hat{b}_n$, the Hamiltonian $\hat{\mathcal{H}}_0$ takes the form

$$\hat{\mathcal{H}}_0 = \hbar\omega_p\sum_n \sqrt{\mu_n}\hat{b}_n^\dagger\hat{b}_n. \tag{15}$$

The Hamiltonian $\hat{\mathcal{H}}'$ describes the self-interaction of the field φ. Since $\Lambda \gg 1$, $\hat{\mathcal{H}}'$ can be considered as a perturbation if the energy of the system [counting from the "vacuum" state corresponding to $\varphi_0(x)$] satisfies the condition $\bar{E} \ll \hbar\omega_p\Lambda$. In zeroth order, this energy is determined by the occupation numbers N_n, and reads

$$\bar{E} = \hbar\omega_p\sum_n \sqrt{\mu_n}N_n.$$

The correction to this result due to self-interactions can be found via perturbation theory.

III. INTERACTION WITH AN ELECTROMAGNETIC FIELD: ABSORPTION AND EMISSION TRANSITION RATES

Now we consider the interaction of the field φ with electromagnetic waves, described by the vector potential $\mathbf{A}$ (we choose the gauge $A_0=0$, $\mathrm{div}\mathbf{A}=0$). Substituting $\varphi=\varphi_0+\psi$ into action (1) and expanding it in a power series of ψ, we derive the operator $\hat{V}$ describing the interaction of ψ with the electromagnetic field:[13]

$$\hat{V} = -\frac{cE_J}{4\pi i_c\lambda_J L}\oint_\Sigma d\zeta\hat{\psi}[\mathrm{rot}\,\mathbf{A}\times\mathbf{e}_z]_n. \tag{16}$$

Here we use the relation $\mathbf{H}^e=\mathrm{rot}\,\mathbf{A}/\lambda_J$ because we measure distances in units of λ_J. The vector potential $\mathbf{A}$ in Eq. (16) consists of two parts, describing both the incoming and outgoing radiation re-emitted by the JJ. We assume that the incident electromagnetic radiation is fully polarized and propagates along the x axis, as shown in Fig. 1. Below we measure the frequency ω in units of ω_p and the wavelength in units of λ_J. In this case, $\mathbf{A}$ can be written as

$$\hat{\mathbf{A}}(t,\mathbf{r}) = -i\mathbf{e}_z\frac{c}{\omega_p}\int\frac{d\omega}{2\pi}\frac{E_\omega}{\omega}e^{-i\omega(t-vx)} + \sqrt{\frac{4\pi c^2}{V\omega_p}}\sum_{\mathbf{k},\lambda}\left(\sqrt{\frac{\hbar}{2\omega_\mathbf{k}}}\mathbf{e}^\lambda(\mathbf{k})e^{-i\omega_\mathbf{k}t+i\mathbf{kr}}\hat{a}_{\mathbf{k}\lambda} + \sqrt{\frac{\hbar}{2\omega_\mathbf{k}}}\mathbf{e}^\lambda(\mathbf{k})e^{i\omega_\mathbf{k}t-i\mathbf{kr}}\hat{a}_{\mathbf{k}\lambda}^\dagger\right), \tag{17}$$

where $\hat{a}_{\mathbf{k}\lambda}^\dagger$ and $\hat{a}_{\mathbf{k}\lambda}$ are the creation and annihilation operators of a photon with wave vector $\mathbf{k}$ and polarization λ,

$$v = \frac{\omega_p\lambda_J}{c}$$

is the ratio of the Swihart velocity $\omega_p\lambda_J$ to the speed of light c, $\omega_\mathbf{k}=|\mathbf{k}|/v$, and V is the volume of space (dimensional) where the electromagnetic field exists. The first term in Eq. (17) corresponds to incoming radiation (which is here considered as classical), where E_ω is the electric field at frequency ω. The second term describes the photons appearing due to the interaction of the incoming electromagnetic waves with the JJ. In this term, $\mathbf{e}^\lambda$ is the vector of polarization, which satisfies the equality

$$\mathbf{k}\cdot\mathbf{e}^\lambda(\mathbf{k}) = 0. \tag{18}$$

Substituting Eq. (17) and the expansion (13) for $\hat{\psi}$ into Eq. (16), and performing the surface integration, we derive:

$$\hat{V} = \hat{V}_{\mathrm{ext}} + \hat{V}_{\mathrm{q}},$$

$$\hat{V}_{\text{ext}} = i\sqrt{\frac{\hbar c \lambda_L L v}{4\pi}} \sum_n \int \frac{d\omega}{2\pi} E_\omega \frac{\varkappa_n(\omega v)}{\sqrt{\omega_n}} e^{-i\omega t} \times (e^{i\omega_n t}\hat{b}_n^\dagger + e^{-i\omega_n t}\hat{b}_n) \quad (19)$$

$$\hat{V}_{\text{q}} = \hbar c \sqrt{\frac{\lambda_L L}{2V\lambda_J}} \sum_n \sum_{\mathbf{k}\lambda} \frac{S(k_y)}{\sqrt{\omega_{\mathbf{k}}\omega_n}} ([k_y h_x^\lambda(\mathbf{k})\chi_n(k_x) + h_y^\lambda(\mathbf{k})\varkappa_n(k_x)]e^{-i\omega_{\mathbf{k}} t}\hat{a}_{\mathbf{k}\lambda} + [k_y h_x^\lambda(\mathbf{k})\chi_n^*(k_x) + h_y^\lambda(\mathbf{k})\varkappa_n^*(k_x)]e^{i\omega_{\mathbf{k}} t}\hat{a}_{\mathbf{k}\lambda}^\dagger)(e^{i\omega_n t}\hat{b}_n^\dagger + e^{-i\omega_n t}\hat{b}_n), \quad (20)$$

where λ_L is the London penetration depth, which is related to i_c and λ_J by the relation (it is supposed here that $s \ll \lambda_L$)

$$\lambda_J^2 = \frac{\hbar c^2}{8\pi e(2\lambda_L + s)i_c} \cong \frac{\hbar c^2}{16\pi e \lambda_L i_c}.$$

In formulas (19) and (20)

$$\omega_n = \sqrt{\mu_n}, \quad (21)$$

$$\mathbf{h}^\lambda(\mathbf{k}) = \mathbf{k} \times \mathbf{e}^\lambda(\mathbf{k}), \quad (22)$$

and functions $\varkappa_n(k)$, $\chi_n(k)$, and $S(k)$ are the following:

$$\varkappa_n(k) = -i[\psi_n(d/2)e^{ikd/2} - \psi_n(-d/2)e^{-ikd/2}], \quad (23)$$

$$\chi_n(k) = \int_{-d/2}^{d/2} dx \psi_n(x) e^{ikx}, \quad (24)$$

$$S(k) = \frac{2\sin(kl/2)}{kl}. \quad (25)$$

A. Spontaneous photon emission

In first order of perturbation theory, there are only three possible processes: (i) spontaneous emission of a photon by the field φ, (ii) induced photon absorption, and (iii) induced emission.[14] Let us first consider the *spontaneous* emission, which is described by the operator $\hat{V}_q$. In the initial state, we have the set of occupation numbers of JPW quanta, $\{N_m\}$, and zero photons, and, in the final state, one of these numbers, say N_n, decreases by 1 and one photon appears in the system. We neglect the effect of thermal radiation, proceeding to the limit $T \to 0$. The probability per unit time of such a process, $w^{(-)}$, is proportional to N_n.

Following the standard approach,[15] we derive for the probability of emission of a photon having wave vector $\mathbf{k}$ and polarization λ:

$$dw_{n\lambda}^{(-)}(\mathbf{k}) = N_n \frac{\pi c \lambda_L L}{v\lambda_J^3} \frac{|F_n^\lambda(\mathbf{k})|^2}{\omega_{\mathbf{k}}\omega_n} \delta(\omega_{\mathbf{k}} - \omega_n) \frac{d^3\mathbf{k}}{(2\pi)^3}, \quad (26)$$

where

$$F_n^\lambda(\mathbf{k}) = S(k_y)[k_y h_x^\lambda(\mathbf{k})\chi_n(k_x) + h_y^\lambda(\mathbf{k})\varkappa_n(k_x)]. \quad (27)$$

We introduce spherical coordinates in momentum space. Performing the integration over $\mathbf{k}$ and the summation over λ taking into account relation (18), finally, we derive

$$w_n^{(-)} = N_n \omega_p \gamma_n, \quad \gamma_n = \frac{\lambda_L L v}{2\pi\lambda_J^2}\nu_n, \quad (28)$$

$$\nu_n = \frac{1}{4\pi}\sum_\lambda \int d\mathbf{m} |F_n^\lambda(\omega_n v \mathbf{m})|^2, \quad (29)$$

where $\mathbf{m}$ is a unit vector in momentum space.

For relatively short junctions, it is possible to obtain an analytical expression for ν_n. The wavelength λ of the electromagnetic radiation under consideration is about

$$\lambda \sim c/\omega_p = \lambda_J/v \gg \lambda_J,$$

since the typical value of $v \sim 3\times 10^{-2} \ll 1$. Therefore, for $D \ll \lambda$, one can expand $F_n^\lambda(\omega_n v \mathbf{m})$ in Eq. (29) in powers of v. Doing so, we derive in the lowest order

$$\nu_{2m+1} \cong \frac{8}{3}\omega_{2m+1}^2 v^2 \psi_{2m+1}^2\left(\frac{d}{2}\right),$$

$$\nu_{2m} \cong \frac{4d^2}{15}\omega_{2m}^4 v^4 \left[\psi_{2m}^2\left(\frac{d}{2}\right) - \frac{1}{2}\bar{\psi}_{2m}\psi_{2m}\left(\frac{d}{2}\right) + \bar{\psi}_{2m}^2\right], \quad (30)$$

where

$$\bar{\psi}_n = \frac{1}{d}\int_{-d/2}^{d/2} dx \psi_n(x) \quad (31)$$

and $\nu_{2m} \ll \nu_{2m+1}$. The difference between ν_n with odd and even n comes from the symmetry properties of the JPW wave functions: $\psi_n(-x) = (-1)^n \psi_n(x)$.

The value of γ_n gives us the radiation width of the nth level in units of ω_p. When $d \sim \lambda_J$, we have from Eq. (30): $\nu_{2m+1} \sim v^2$ and $\nu_{2m} \sim v^4$. Considering L, $\lambda_J \sim 10^{-3}$ cm, $\lambda_L \sim 10^{-5}$ cm, and $v \sim 3\times 10^{-2}$ we obtain $\gamma_{2m+1} \sim 10^{-8}-10^{-7}$ and $\gamma_{2m} \sim 10^{-12}-10^{-10}$, that is $\gamma_{2m} \ll \gamma_{2m+1}$. Note that we do not consider here another possible mechanisms of dissipation, which can substantially increase the width of the JPW quanta energy levels.

B. Induced photon absorption and emission

Let us now consider processes of *induced* photon absorption and emission. These two processes are determined by the operator $\hat{V}_{\text{ext}}$. We denote by ${}^{\text{in}}w_n^{(+)}$ (${}^{\text{in}}w_n^{(-)}$) the probability per unit time of creation (annihilation) of a quantum of the φ field in the nth state due to induced photon absorption (emission). These two probabilities satisfy the following equality

$$\frac{{}^{\text{in}}w_n^{(+)}}{{}^{\text{in}}w_n^{(-)}} = \frac{N_n + 1}{N_n}.$$

Thus, to first order in perturbation theory, the probability per unit time of induced photon absorption and also accounting for induced emission, $w_n^{(+)} = {}^{\text{in}}w_n^{(+)} - {}^{\text{in}}w_n^{(-)}$, does not depend on N_n, and is only determined by the power and frequency of the external radiation. Making a similar calculation as for $w_n^{(-)}$, we derive

$$w_n^{(+)} = \frac{c\lambda_L L v}{2\hbar\omega_p}\int d\omega \frac{|E_\omega|^2}{2\pi}\frac{|\varkappa_n(\omega v)|^2}{\omega_n}\delta(\omega-\omega_n). \tag{32}$$

We assume that the incident radiation has a Gaussian distribution with central frequency $\bar{\omega}$ and width $\bar{\gamma}$, that is

$$\frac{|E_\omega|^2}{2\pi} = \frac{4\pi P}{c}\Delta(\omega-\bar{\omega}), \tag{33}$$

where P is the radiation power per unit area and

$$\Delta(\omega) = \frac{1}{\bar{\gamma}\sqrt{\pi}}\exp\{-\omega^2/\bar{\gamma}^2\}. \tag{34}$$

The probability $w_n^{(+)}$ then becomes

$$w_n^{(+)} = \frac{2\pi P\lambda_L L v}{\hbar\omega_p}\frac{f_n(\bar{\omega})}{\omega_n}, \tag{35}$$

where

$$f_n(\bar{\omega}) = \int d\omega\Delta(\omega-\bar{\omega})|\varkappa_n(\omega v)|^2\delta(\omega-\omega_n). \tag{36}$$

In equilibrium, the probabilities $w_n^{(+)}$ and $w_n^{(-)}$ coincide. This gives rise to a relation for the mean values of the occupation numbers $\bar{N}_n$:

$$\bar{N}_n = \frac{4\pi^2 P\lambda_J^2}{\hbar\omega_p^2}\frac{f_n(\bar{\omega})}{\omega_n\nu_n}. \tag{37}$$

The mean value $\bar{E}$ of the system energy (to zeroth order in $\hat{\mathcal{H}}'$) then reads

$$\bar{E} = \frac{4\pi^2 P\lambda_J^2}{\omega_p}\sum_n \frac{f_n(\bar{\omega})}{\nu_n}. \tag{38}$$

If the frequency band of the radiation source is large enough, that is, $\bar{\gamma} \gg \gamma_n$, we can easily perform an integration in Eq. (36). As a result, the mean energy becomes

$$\bar{E} = \frac{4\pi^2 P\lambda_J^2}{\omega_p}\sum_n \frac{|\varkappa_n(\omega_n v)|^2}{\nu_n}\Delta(\bar{\omega}-\omega_n). \tag{39}$$

In the opposite case of near-monochromatic radiation, $\bar{\gamma} \ll \gamma_n$, we should take into account that the energy levels of JPW quanta have finite width γ_n (in units of ω_p). Replacing the delta function in Eq. (36) by

$$\delta(\omega-\omega_n) \to \frac{1}{\pi}\frac{\gamma_n}{(\omega-\omega_n)^2+\gamma_n^2}$$

and using $\Delta(\omega-\bar{\omega})=\delta(\omega-\bar{\omega})$, we obtain

$$\bar{E} = \frac{2P\lambda_L L v}{\omega_p}\sum_n \frac{|\varkappa_n(\omega_n v)|^2}{(\bar{\omega}-\omega_n)^2+\gamma_n^2}, \tag{40}$$

where we take into account that $\gamma_n \ll 1$.

Note that Eqs. (38)–(40) are valid only when the radiation power of the electromagnetic waves is not too high: $\bar{E} \ll \hbar\omega_p\Lambda$. Otherwise, we should take into account anharmonic terms in Hamiltonian (11). Note also that here we only consider single-photon processes. Multiphoton processes can be calculated in higher orders of perturbation theory with respect to $\hat{V}$. It can be shown that the amplitudes of these processes are negligible when the following condition is met

$$g_{\rm ext}^2(P) \equiv \frac{vP\lambda_L L}{\hbar\omega_p^2} \ll 1,$$

where $g_{\rm ext}(P)$ can be considered as an effective coupling constant of the junction interacting with an external electromagnetic field.

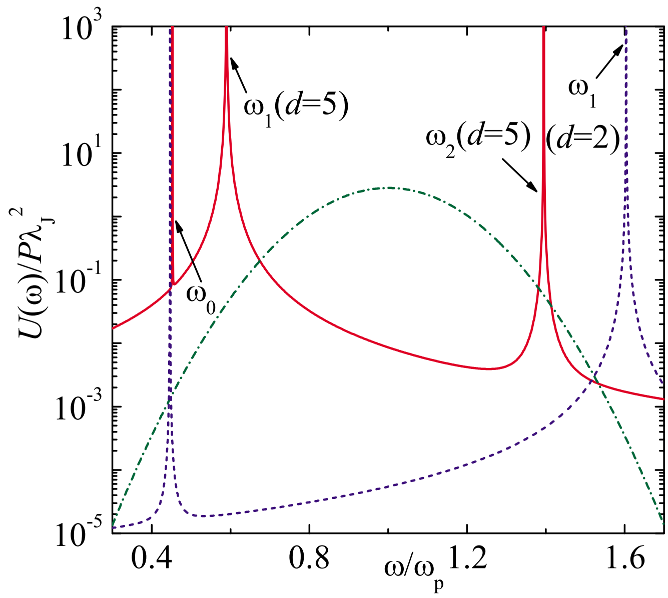

FIG. 2. (Color online) The frequency dependence of the intensity of re-emission, $U(\omega)$, calculated for $d=5$ (red solid curve) and $d=2$ (blue dashed curve). Other parameters are: $I/I_c(d)=0.98$, $\lambda_L/\lambda_J=4\times10^{-3}$, and $L/\lambda_J=2$. The green dot-dashed curve corresponds to Gaussian distribution of the intensity of incoming radiation with central frequency $\bar{\omega}=1$ and the width $\bar{\gamma}=0.2$ (in units of ω_p).

C. Response of the junction to a wave packet

Consider now the response of a JJ to a wideband terahertz wave packet. We now assume that the central frequency of the incoming radiation $\bar{\omega}$ is about ω_p and that the width $\bar{\gamma}$ of the wave packet is large enough. In this case, the first several energy levels of the system will be excited. The intensity $U(\omega)$ of light re-emission at frequency ω is given by the sum $\hbar\omega_p\Sigma_{n\lambda}\omega_n dw_{n\lambda}^{(-)}(\mathbf{k})$, with $dw_{n\lambda}^{(-)}(\mathbf{k})$ from Eq. (26), integrated over all directions of $\mathbf{k}$. Taking into account relation (37) for the mean values of the occupation numbers $\bar{N}_n$, and replacing again the delta function in Eq. (26) by a Lorentzian curve, we obtain

$$U(\omega) = 2P\lambda_L L\sum_n \frac{|\varkappa(\omega_n v)|^2\Delta(\bar{\omega}-\omega_n)}{(\omega-\omega_n)^2+\gamma_n^2}. \tag{41}$$

The function $U(\omega)$, for relatively short ($d=2$) and long JJs ($d=5$), is shown in Fig. 2. The wave packet of the incident radiation has a central frequency $\bar{\omega}=1$ and width $\bar{\gamma}=0.2$ (in units of ω_p). In this case, the first two ($d=2$) or three ($d=5$) energy levels are excited. For short junctions, $d \lesssim 1$, the eigenfrequencies are, approximately

$$\omega_0 \approx (1-j^2)^{1/4}, \quad \omega_n \approx \frac{\pi n}{d} \gg \omega_0, \quad n>0. \tag{42}$$

For increasing values of d, ω_1 tends to ω_0, and when $d \gtrsim 4$, we have

$$\omega_0 \approx \omega_1 \ll \omega_n, \quad n>1. \tag{43}$$

The relation $\omega_1 \approx \omega_0$ is essential for the properties of macroscopic quantum tunneling in JJs. Namely, in this case we have two channels of tunneling, corresponding to fluxons arising near the junction's edges. This situation is considered in Sec. IV.

IV. PROBABILITY OF PHOTON-ASSISTED TUNNELING

A. Field-theoretic approach

Now we calculate the probability per unit time of quantum tunneling to another vacuum state, stimulated by external electromagnetic radiation, using the approach[9] proposed in one of our previous papers. In the semiclassical approximation, we can consider the quantum field $\hat{\psi}$ as a classical field $\psi(\tau,x)$ in imaginary time $t=i\tau$. The probability, Γ, then reads[12]

$$\Gamma(\bar{E}) = \omega_p \sqrt{\frac{30B(\bar{E})\mu_0}{\pi}} \exp[-B(\bar{E})], \quad B(\bar{E}) = \frac{2S_E}{\hbar}, \tag{44}$$

where $\mathcal{S}_E$ is the action (1) in imaginary time. Substituting

$$\varphi(\tau,x) = \varphi_0(x) + \psi(\tau,x)$$

into Eq. (1) and expanding the action in powers of ψ, we obtain

$$B(\bar{E}) = 2\Lambda \int_0^{\tau_0} d\tau \left\{ \int_{-d/2}^{d/2} dx \left[\frac{1}{2}\psi\left(\hat{\mathcal{D}} - \frac{\partial^2}{\partial\tau^2}\right)\psi - \frac{1}{6}\sin\varphi_0 \psi^3 - \frac{1}{24}\cos\varphi_0\psi^4 - \dots \right] - \frac{\bar{E}}{E_J} \right\}. \tag{45}$$

The last term in Eq. (45) originates from the matching condition for the wave function Φ of the quantum field $\hat{\psi}$ inside $[\Phi_{\text{in}} \propto \exp(-\mathcal{S}_E/\hbar)]$ and outside $[\Phi_{\text{out}} \propto \exp(-i\bar{E}t/\hbar)]$ the barrier. The field ψ in Eq. (45) satisfies the equation $\delta B(\bar{E})/\delta\psi=0$, that is

$$\frac{\partial^2\psi}{\partial\tau^2} - \hat{\mathcal{D}}\psi = -\frac{1}{2}\sin\varphi_0\psi^2 - \frac{1}{6}\cos\varphi_0\psi^3 - \dots, \tag{46}$$

with the following initial and boundary conditions

$$\left.\frac{\partial\psi}{\partial\tau}\right|_{\tau=0,\tau_0} = 0, \qquad \left.\frac{\partial\psi}{\partial x}\right|_{x=\pm d/2} = 0. \tag{47}$$

In Eqs. (45) and (47), τ_0 is the final imaginary time of the tunneling process. The value of τ_0 depends on the energy $\bar{E}$ of the system and can be found using the approach described below.

We seek a solution of the Eq. (46) in the form

$$\psi(\tau,x) = \sum_n c_n(\tau)\psi_n(x). \tag{48}$$

Multiplying Eq. (46) by ψ_n and performing space integration and using Eq. (14), we obtain the system of equations for $c_n(\tau)$

$$\ddot{c}_n - \mu_n c_n = -\frac{1}{2}\sum_{mk} U^{(3)}_{nmk}c_m c_k - \frac{1}{6}\sum_{mkl} U^{(4)}_{nmkl}c_m c_k c_l - \dots \tag{49}$$

with initial conditions

$$\dot{c}_n(0) = \dot{c}_n(\tau_0) = 0. \tag{50}$$

Here, the dot means "imaginary-time derivative," and

$$U^{(i)}_{n\ldots k} = -\int_{-d/2}^{d/2} dx \frac{\partial^i(\cos\varphi_0)}{\partial\varphi_0^i}\psi_n \ldots \psi_k. \tag{51}$$

The tunneling exponent $B(\bar{E})$, Eq. (45), can be expressed as

$$B(\bar{E}) = \Lambda \int_0^{\tau_0} d\tau \left[\frac{1}{6}\sum_{nmk} U^{(3)}_{nmk}c_n c_m c_k - \frac{2\bar{E}}{E_J} + \frac{1}{12}\sum_{nmkl} U^{(4)}_{nmkl}c_n c_m c_k c_l + \dots \right]. \tag{52}$$

When the current I is close to the critical value $I_c(d)$, we have $\mu_0 \ll 1$ and $c_n \ll 1$. So, we can neglect all terms in the right-hand side of Eq. (49), except the first one. Our analysis shows that when $d \gtrsim 4$, $\mu_1 \approx \mu_0$, and we have the following relation for the eigenvalues of the operator $\hat{\mathcal{D}}$

$$\mu_0 \approx \mu_1 \ll \mu_n, \quad n>1.$$

In this case, c_0, $c_1 \gg c_n$ ($n>1$), and we can consider only the first two equations of the system [Eq. (49)], taking $c_n=0$ for all $n>1$ (for details, see Ref. 9).

We now introduce new variables

$$\alpha_i(\eta) = \frac{\sqrt{u_0 u_i}c_i(\tau)}{3\mu_0}, \quad i=0,1, \tag{53}$$

where

$$\eta = \sqrt{\mu_0}\tau, \quad u_i = U^{(3)}_{0ii}. \tag{54}$$

The system of Eq. (49) takes the form

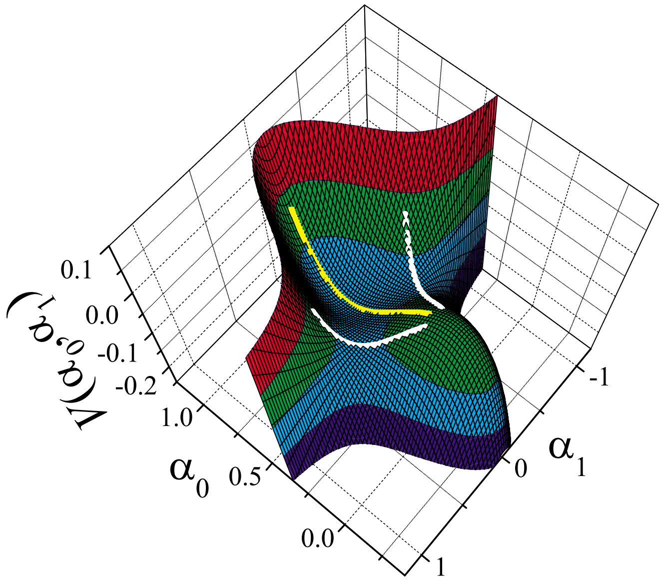

FIG. 3. (Color online) The potential $V(\alpha_0,\alpha_1)$ calculated when $d=6$, $I/I_c(d)=0.98$ ($\lambda/u_{01}=1.18$). A particle tunnels from its initial position near $\alpha_i=0$. The α_i are collective coordinates for the tunneling fluxon. The curves correspond to three possible imaginary-time trajectories of the particle: $\alpha_i^{(0)}(\eta)$ and $\alpha_i^{(\pm)}(\eta)$ (see text below). Note that the real-time potential equals to $-V(\alpha_0,\alpha_1)$.

$$\begin{cases} \dfrac{d^2\alpha_0}{d\eta^2}-\alpha_0=-\dfrac{3}{2}(\alpha_0^2+\alpha_1^2), \\ \dfrac{d^2\alpha_1}{d\eta^2}-\lambda\alpha_1=-3u_{01}\alpha_0\alpha_1, \end{cases} \tag{55}$$

where

$$\lambda=\frac{\mu_1}{\mu_0}, \quad u_{01}=\frac{u_1}{u_0}. \tag{56}$$

The system [Eq. (55)] has the first integral

$$\frac{9\mu_0^3}{u_0^2}\left[\left(\frac{d\alpha_0}{d\eta}\right)^2+\frac{1}{u_{01}}\left(\frac{d\alpha_1}{d\eta}\right)^2+V(\alpha_0,\alpha_1)\right]=-\frac{\bar{E}}{E_J}, \tag{57}$$

where we introduce a potential

$$V(\alpha_0,\alpha_1)=\alpha_0^3+3\alpha_0\alpha_1^2-\alpha_0^2-\frac{\lambda}{u_{01}}\alpha_1^2. \tag{58}$$

Taking into account the initial conditions [Eq. (50)], we have, at the turning points:

$$V[\alpha_0(\sqrt{\mu_0}\tau), \quad \alpha_1(\sqrt{\mu_0}\tau)]|_{\tau=0,\tau_0}=-\bar{\varepsilon}, \tag{59}$$

where

$$\bar{\varepsilon}=\frac{u_0^2\bar{E}}{9\mu_0^3E_J}, \quad 0<\bar{\varepsilon}<\varepsilon_0=\frac{4}{27}. \tag{60}$$

Equation (59) defines the value of τ_0 as a function of system energy $\bar{E}$.

Thus, we reduce the problem of quantum tunneling of the field φ to the problem of tunneling a quantum particle in two dimensions, where the α_i's play the role of the particle generalized "coordinates." The potential $V(\alpha_0,\alpha_1)$ is shown in Fig. 3.

When $d>d_c(I,\bar{\varepsilon})\approx 4$, there are three solutions of the system of Eq. (55) with the conditions [Eq. (59)], $\alpha_i^{(0)}(\eta)$ and $\alpha_i^{(\pm)}(\eta)$, which are characterized by the following relations

$$\alpha_1^{(0)}(\eta)=0, \quad \alpha_1^{(-)}(\eta)=-\alpha_1^{(+)}(\eta).$$

The trajectories $\alpha_i^{(0)}(\eta)$ and $\alpha_i^{(\pm)}(\eta)$ are shown in Fig. 3. The solution $\alpha_i^{(+)}(\eta)$ [$\alpha_i^{(-)}(\eta)$] corresponds to the formation of vortex (antivortex) nucleus at left (right) junction's edge, while the solution $\alpha_i^{(0)}(\eta)$ describes the tunneling of φ as a whole.[9] The minimum of $B(\bar{E})$ corresponds to the solutions $\alpha_i^{(\pm)}(\eta)$. The tunneling exponent then reads

$$B(\bar{E})=\frac{24\Lambda\mu_0^{5/2}}{5u_0^2}b(\bar{\varepsilon}), \tag{61}$$

where

$$b(\bar{\varepsilon})=\frac{15}{16}\int_0^{\eta_0}d\eta[\alpha_0^3+3\alpha_0\alpha_1^2-2\bar{\varepsilon}], \quad \eta_0=\sqrt{\mu_0}\tau_0. \tag{62}$$

Note, that we should multiply the probability $\Gamma(\bar{E})$, Eq. (44), by a factor of 2, since we have two channels for tunneling.

When $d<d_c(I,\bar{\varepsilon})$, all three solutions coincide, $\alpha_1^{(0)}(\eta)=\alpha_1^{(-)}(\eta)=\alpha_1^{(+)}(\eta)=0$, and the second equation of the system [Eq. (55)] becomes trivial, while the first one can be easily integrated. As a result, we obtain

$$b(\bar{\varepsilon})=\frac{15}{16}\int_{\alpha_1(\bar{\varepsilon})}^{\alpha_2(\bar{\varepsilon})}d\alpha\frac{\alpha^3-2\bar{\varepsilon}}{\sqrt{\alpha^2(1-\alpha)-\bar{\varepsilon}}}, \quad b(0)=1, \tag{63}$$

where $\alpha_{1,2}(\bar{\varepsilon})$ are the smaller and larger positive roots of the cubic equation

$$\alpha^2(1-\alpha)-\bar{\varepsilon}=0.$$

B. Results and discussion

The analysis of the tunneling exponent B on the junction's width d and current I was carried out in one[9] of our previous papers. Now we are interested in the effect of electromagnetic radiation on $B(\bar{E})$. Using formulas (39), (40), (60), (62), and (63), we calculate the dependence of $\{b(0)/b[\bar{\varepsilon}(\bar{\omega},P)]-1\}$ as a function of the radiation's central frequency $\bar{\omega}$, for short $d<d_c$, and long $d>d_c$ junctions. The results of the calculations, both for wideband ($\bar{\gamma}\gg\gamma_n$) and monochromatic ($\bar{\gamma}\ll\gamma_n$) radiations, are shown in Fig. 4. It is clear that we have several resonances at frequencies $\bar{\omega}=\omega_n=\sqrt{\mu_n}$ (in units of ω_p).

For relatively short junctions, $d\lesssim 1$, resonance peaks are well separated from each other even for wideband terahertz radiation, as it can be seen from Fig. 4; while for $d\gtrsim d_c$, we have $\omega_0\approx\omega_1\ll\omega_n$, and the first two peaks can merge into a single peak. Note, that the inequality $\omega_0\approx\omega_1\ll\omega_n$ is valid for not too large junction's width $d\lesssim 20$. In the opposite case we should consider the large number of equations in the

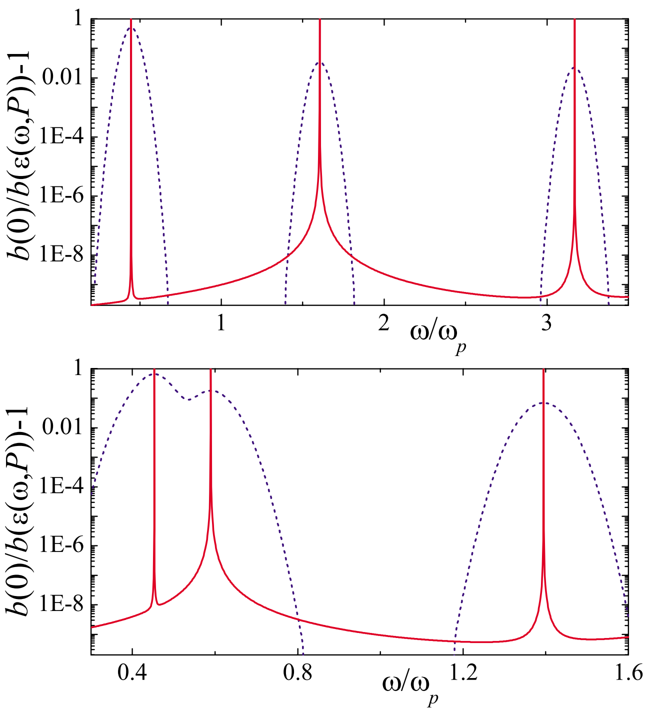

FIG. 4. (Color online) The frequency dependence of $b(0)/b[\bar{\varepsilon}(\omega,P)]-1$, calculated for $d=2$ (upper graph) and $d=5$ (lower graph). Other parameters are $I/I_c(d)=0.98$, $L=2\lambda_J$, and $v=1/30$ for both cases. Solid curves correspond to monochromatic incoming radiation, while the dashed curves describe the response on wideband terahertz radiation with $\bar{\gamma}=5\times10^{-2}$ (in units of ω_p). The radiation power P is the same for all curves and is chosen such that $\max[\bar{\varepsilon}(\omega,P)]=0.3\varepsilon_0=2/45$ for wideband radiation.

system [Eq. (49)] to calculate the tunneling exponent.

When $\bar{\gamma}=0$, the resonance peaks are very narrow, and one can switch the JJ to the resistive state ($B=0$) at small radiation power, if $\bar{\omega}=\omega_n$. Note, that the condition $\bar{\omega}=\omega_n$ can be achieved by changing the applied dc current I, since ω_n depend on I. In other words, if the frequency of the incoming radiation lies near one of the ω_n, one can observe a resonance behavior of the tunneling exponent B as a function of the dc current. Such a resonant behavior of the tunneling exponent $B(I)$ and photon-assisted MQT escape rate $\Gamma(I)$ on the applied dc current I was experimentally observed in low-T_c (Nb/AlO_x/Nb JJ, Refs. 16 and 17) and high-T_c (Bi2212 stacks of intrinsic JJs, Refs. 6) junctions. In particular, Ref. 17 reported several resonances in the tunneling probability when the dc current satisfies the equality

$$\bar{\omega}=\frac{\omega_p}{m}(1-I^2/I_c^2)^{1/4},$$

where m is an integer ($m=1,2,3,4,5$). The case $m=1$ corresponds to the resonance $\bar{\omega}=\omega_0(I)$ considered here. Since the junction studied has a width $D\ll\lambda_J$, its resonance frequencies satisfy the Eq. (42), and the levels with $n>0$ (considered here) cannot be excited. The authors of Ref. 17 attributed the resonances with $m>1$ to multiphoton absorption, corresponding to resonances at $m\bar{\omega}=\omega_0(I)$. Such processes can be considered in the framework of our theory, in higher orders of perturbation with respect to $\hat{V}$. Note that the *quasiclassical* theory of photon-assisted MQT in short ($D\ll\lambda_J$) junctions, based on multiphoton phenomena, was developed in Ref. 18. Here we provide a *quantum* theory, instead of a quasiclassical study. Also, here we study long junctions, while Ref. 18 focuses on short junctions.

The second peak, corresponding to the resonance at $\bar{\omega}=\omega_1$, is much wider than the first (ω_0) and the third (ω_2) peaks, in the case of monochromatic incoming radiation (shown by the red curves in Fig. 4). Here we consider the case when the width of the nth peak is defined by the radiation width of the nth energy level, γ_n. Due to the symmetry of the JPW wave functions $\psi_n(x)$, the following condition is met $\gamma_{2m}\ll\gamma_{2m+1}$ (see discussion at the end of Sec. III A), and, therefore, the second peak (the peak with $n=1$) turns out to be much wider than the first ($n=0$) and the third ($n=2$) peaks. However, the widths of the peaks can be wider by several orders of magnitude due to other possible mechanisms of dissipation (which we do not consider here). Thus, the height of the peaks, that is the maximum enhancement of the photon-assisted MQT, could be much smaller and determined by the phenomenological parameters γ_n measured by independent experiments. For example, from the experiment Ref. 17 on photon-assisted MQT, we obtain the following estimate for $\gamma_{\rm exp}\sim10^{-2}$.

V. CONCLUSION

In conclusion, we proposed a quantum field theory for Josephson plasma waves interacting with external electromagnetic radiation. We also calculated the macroscopic quantum tunneling of a fluxon, stimulated by terahertz light, in a long Josephson junction driven by a dc current. The probability of absorption and emission of terahertz light depends on the current and the length of the Josephson junction. The MQT escape rate shows several resonance maxima as a function of the frequency, corresponding to eigenfrequencies of Josephson plasma-wave quanta. This could be potentially useful for a variety of superconducting quantum terahertz devices. Classical terahertz devices are discussed in Ref. 19.

Note that the approach proposed here for calculating photon-assisted quantum tunneling is somewhat reminiscent of the quantum-mechanical approach considered in Ref. 20, where a system of master equations for the probabilities of a quantum particle to occupy discrete energy levels is used. However, the method developed in Ref. 20 describes the tunneling of a *single*-quantum particle from a potential well, and cannot be applied for *distributed* systems such as *continuum fields* and, thus, fails to describe MQT in the long Josephson junctions considered here. In our case, instead of a system of master equations, we have a set of detailed-balance equations for the transition rates, $w_n^{(+)}=w_n^{(-)}$, which provide a set of occupation numbers $\bar{N}_n$ describing the state $\Phi(\bar{E})$ with the energy $\bar{E}$ of the quantum field $\hat{\psi}$. The MQT escape rate $\Gamma(\bar{E})$ of the quantum field in this state is then calculated using the approach developed in our previous paper Ref. 9.

Our approach can be easily generalized to the case of a system of intrinsic JJs. It would allow to calculate the probability of photon-assisted tunneling in high-T_c superconductors, which can be considered as a stacks of coupled intrinsic Josephson junctions. Note that such calculations were carried out in Ref. 21, but only for very short stacks and only for capacitively-coupled junctions, which is not the case for experimentally realistic superconducting samples. The collective excitations in the system of JJs considered in Ref. 21 correspond to JPWs in our theory. However, the spatial distribution of phase differences inside junctions was not taken into account in Ref. 21. But the spatial distribution of a tunneling fluxon is essential[10] for samples of a micron size. Indeed, our theory successfully explains the large enhancement of the MQT escape rate observed in recent experiments[6] on MQT in $Bi_2Sr_2CaCu_2O_{8+\delta}$ stacks of intrinsic JJs (for details, see Ref. 10).

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