Generation of tunable terahertz out-of-plane radiation using Josephson vortices in modulated layered superconductors

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We show that a moving Josephson vortex in spatially modulated layered superconductors generates out-of-plane THz radiation. Remarkably, the magnetic and in-plane electric fields radiated are of the same order, which is very unusual for any good-conducting medium. Therefore, the out-of-plane radiation can be emitted to the vacuum without the standard impedance mismatch problem. Thus, the proposed tunable THz emitter for out-of-plane radiation can be more efficient than the standard one which radiates only along the *ab*-plane.

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I. INTRODUCTION

The recent growing interest in terahertz (THz) science and technology is due to its many important applications in physics, astronomy, chemistry, biology, and medicine, including THz imaging, spectroscopy, tomography, medical diagnosis, health monitoring, environmental control, as well as chemical and biological identification.[1] This range of the electromagnetic spectrum sits between 0.3 and 30 THz, which corresponds to 10–1000 μm (wavelength), 1.25–125 meV (energy) or 14–1400 K (temperature). The THz gap, which is still hardly reachable for both electronic and optical devices, covers temperatures of biological processes and a substantial fraction of the luminosity remanent from the Big Bang.[1]

High-temperature $Bi_2Sr_2CaCu_2O_{8+\delta}$ superconductors have a layered structure that allows the propagation of electromagnetic waves (called Josephson plasma oscillations[2–6]) with Josephson plasma frequency ω_J. This is drastically different from the strong damping of electromagnetic waves in low-temperature superconductors. The Josephson plasma frequency lies in the THz range (see, e.g., Refs. 7–9). Indeed, tunable filters of THz radiation have been proposed using the Josephson vortex lattice as a tunable photonic crystal.[10] Moreover, detectors of THz radiation have been very recently proposed using surface Josephson plasma waves.[11] A possible way to generate THz radiation in $Bi_2Sr_2CaCu_2O_{8+\delta}$ and related compounds is to apply an in-plane magnetic field H_{ab} and an external current $J_{\parallel c}$ perpendicular to the superconducting layers (i.e., along the c axis). Josephson vortices (JVs) induced by H_{ab} and driven fast by the c axis current emit THz radiation (e.g., Refs. 7 and 9). However, it was shown[12–14] that the radiation propagates *only* along the plane of motion of the JVs and decays in the c direction. This THz radiation is characterized by a huge impedance mismatch resulting in a very small fraction of THz wave intensity emitted from the sample.[9] This impedance mismatch is a very important problem restricting possible applications.[15]

To avoid this problem, we propose a new class of THz emitters based on JVs moving through in-plane modulated layered superconductors, including both the strongly anisotropic high-T_c $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals and artificial stacks of Josephson junctions (SJJ), e.g., Nb-Al-AlO_x-Nb. In-plane spatial variations of the Josephson maximum c-axis current J_c can be obtained by using either irradiation of a standard $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample covered by a modulated mask (see, e.g., Ref. 16) or pancake vortices controlled by an out-of-plane magnetic field.[17]

In order to pass through the superconductor-vacuum interface without a significant decrease of the amplitude, the electric and magnetic components of the propagating wave have to be of the same order of magnitude. This feature is inherent for the out-of-plane Josephson plasma waves (JPW), propagating both along and perpendicular to the layers with short wavelength along the c axis. For such waves, the transmission coefficient is about unity.[18] The out-of-plane JPW can be emitted, for instance, by a fast-moving Josephson vortex if its velocity V exceeds a certain threshold value $V_{\min}$. However, this out-of-plane Cherenkov-type radiation always completely reflects from the sample boundary and thus cannot be emitted into the vacuum. Indeed, the longitudinal wave vector q for the Cherenkov radiation is related to the wave frequency ω by $q=\omega/V$ and is much larger than the maximum possible value ω/c for waves in vacuum. This problem can be solved if the out-of-plane Cherenkov radiation propagates through a modulated layered superconductor. The out-of-plane Cherenkov wave interacting with periodic inhomogeneities generates new modes with wave vectors $q_m=q-2\pi m/a$, where a is the spatial period of the modulations and m is an integer. Thus, the wave vector $q_1=q-2\pi/a$ can meet the condition $q_1<\omega/c$ for vacuum waves and this mode is emitted from a sample without an impedance mismatch. It is important to stress that the Cherenkov radiation generated by any relativistic particle in any medium undergoes a complete internal reflection (since $q>\omega/c$) and, thus, we propose this general way of emitting any Cherenkov-type radiation to vacuum. Here, we predict this out-of-plane Cherenkov radiation, and derive the modes propagating in a modulated superconductor and emitted into the vacuum.

Our proposal mainly concerns bulk layered superconductors, where it is important to have radiation propagating not only along the *ab* planes, but also along the *c* axis. For the case of thin films having a thickness much smaller than the in-plane London penetration depth (about 200 nm), the radiation damped along the *c* axis can also give some contribution to the waves emitted from the wide sample side, which is parallel to the *ab* plane. This can even increase the fraction of the out-of-plane radiation discussed in this paper. In other words, we show that there are two contributions to the out-of-plane radiation along the *c* axis: damped waves and propagating waves. The latter one is the real out-of-plane radiation. The former one (damped waves) can contribute to the out-of-plane radiation only if the sample is sufficiently thin. However, we will not consider this case in detail because the power of the emitted radiation decreases for decreasing thickness.

Brief summary of the results

For electromagnetic waves in any conducting media, the electric field E is very weak with respect to the magnetic field H: $E \ll H$. Also, for in-plane radiation: $E \ll H$. Thus, only a small fraction ($\sim E/H$) of the radiation can leave the sample. This is the so-called "impedance mismatch" problem that has severely limited progress in this field for years. Now, we are also considering out-of-plane radiation. This radiation has a strong enough in-plane electric field $E_{\parallel}$ to overcome the superconducting-vacuum interface. Indeed, $E_{\parallel}$ and the magnetic field both are of the same order of magnitude, similar to the one for waves propagating in the vacuum. This solves the impedance mismatch problem. Thus, we propose to use out-of-plane radiation, propagating in a *periodically modulated* $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample, to *overcome* the severe *impedance mismatch problem* which limits the application of layered superconductors for THz emitters.

The spatial modulations of the maximum Josephson current allows the emitted waves to shift their wavenumbers (within a wide range ω_J/V) towards the narrow spectral window $\sim\omega_J/c$ for waves propagating in the vacuum. Thus, the emitted waves can pass the superconductor-vacuum interface within a narrow frequency window $\sim\omega_J V/c$. This offers the possibility to select a narrow frequency window from the initially broad THz radiation produced by the JVs. Also, this should allow us to achieve superradiance via the stabilization of the square Josephson lattice moving in a periodically modulated $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample.[19] Other recent ways to control vortex motion are also attracting considerable attention.[20,21]

II. MODEL

We consider an infinite layered superconductor as in Fig. 1(b). Following Ref. 22, we assume that the superconducting layers are extremely thin, so that the spatial variations of the phase of the superconducting order parameter and the electromagnetic field inside the layers in the direction perpendicular to the layers can be neglected. We choose the *xy* plane to be parallel to the crystallographic *ab* plane and the *c* axis along the *z* axis. Superconducting layers are numbered by the subscript *l*. The electric $\vec{E}$ and magnetic $\vec{H}$ fields have components, $\vec{E}=\{E_x,0,E_z\}$, $\vec{H}=\{0,H,0\}$.

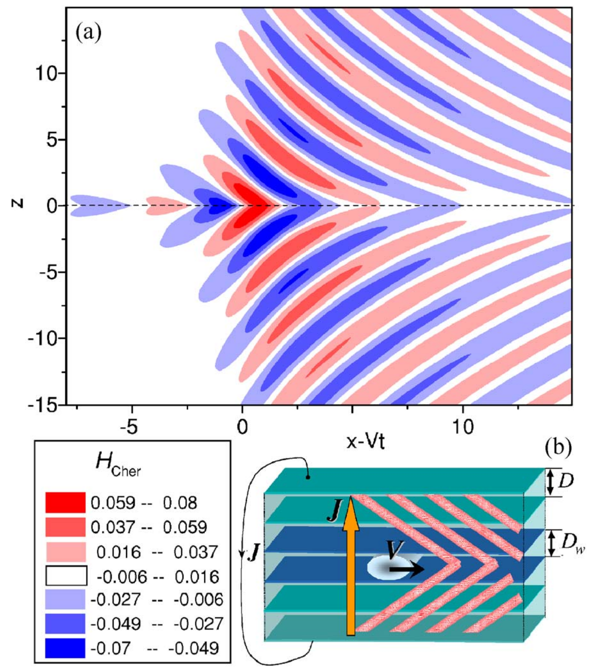

FIG. 1. (Color online) Cherenkov radiation generated by a fast Josephson vortex (located at $x=Vt$) moving in a weaker junction with the critical current $J_c^w < J_c$ and the junction thickness D_w. (a) Magnetic field distribution $H(x-Vt,z)$ in units of $\Phi_0/2\pi\lambda_c\lambda_{ab}$ ($\sim H_{c1}^J$=the first critical magnetic field for the Josephson vortices) for $J_c^w/J_c=0.2$, $D_w/D=1.2$, $V/V_{\max}=0.9$ is calculated using Eq. (17). The "running" coordinate, $x-Vt$, is measured in units of $\gamma D/(\pi\sqrt{v^2\eta^2-1})$, while the out-of-plane coordinate y is normalized by $D/(\pi\sqrt{v^2\eta^2-1})$, where $\eta=J_cD_w/J_c^wD$, and $v=V/V_{\min}$. The moving vortex emits radiation propagating forward. This radiation forms a cone determined by the vortex velocity V. (b) Geometry of the problem: in a weaker junction [located between the two blue superconducting planes in (b)], a *c* axis current $J_{\parallel c}$ drives a Josephson vortex with velocity V, which is higher than the minimum velocity $V_{\min}$ of the propagating electromagnetic waves. Red strips in (b) schematically show outside-the-cone Cherenkov radiation.

The gauge-invariant phase difference φ_l between $(l+1)$th and lth superconducting layers is described by coupled sine-Gordon equations,[18,22,23]

$$\left(1-\frac{\lambda_{ab}^2}{D^2}\Delta_l\right)\left(\frac{\partial^2\varphi_l}{\partial t^2}+\omega_J^2[1+\mu(x)]\sin(\varphi_l)\right) - \frac{c^2}{\varepsilon}\frac{\partial^2\varphi_l}{\partial x^2}=0, \quad \mu \ll 1. \qquad (1)$$

Here D is the spatial period of the layered structure, λ_{ab} is the London penetration depth along the *c* axis, the operator Δ_l is defined as

$$\Delta_l f_l = f_{l+1} + f_{l-1} - 2f_l,$$

and

$$\omega_J = \sqrt{\frac{8\pi e D J_c}{\hbar \varepsilon}} \qquad (2)$$

is the Josephson plasma frequency, ε is the interlayer dielectric constant, and the modulation factor $\mu(x)=\mu(x+a)$ is a periodic function with spatial period a. For simplicity, we neglect the relaxation term related to the quasiparticle current, which is very small at low temperatures.

The coupled sine-Gordon equations (1) describe both the Josephson vortices in layered superconductors and the emitted Josephson plasma waves. In the latter case, Eq. (1) should be linearized, i.e., $\sin(\varphi_l)$ replaced by φ_l.

III. NONLOCAL SINE-GORDON EQUATION

In zero approximation with respect to the modulation μ, the traveling wave solution of Eq. (1), $\varphi_l=\phi_l(\zeta=x-Vt)$, represents the JV moving with a constant velocity. The main phase difference $\phi=\phi_0(\zeta)$ occurs at the central junction, where nonlinearity plays a crucial role, while equations for junctions with $l\neq 0$ can be linearized. However, the magnetic flux of a JV spreads over a large number of these junctions, $l\sim \lambda_{ab}/D \gg 1$. Thus, the equation for the central junction cannot be decoupled from others. This determines a complicated nonlocal structure of the JV in layered superconductors.

Following the approach by Gurevich,[24] the closed-form nonlocal sine-Gordon equation for $\phi_0(\zeta)$ can be derived (see the appendix),

$$\frac{V^2}{\omega_J^2}\frac{\partial^2 \phi}{\partial \zeta^2} + \sin\phi = \frac{\lambda_J^2}{\pi \lambda_c}\int_{-\infty}^{\infty} d\zeta' \, K_0\left(\frac{|\zeta'-\zeta|}{\lambda_c}\right)\frac{\partial^2 \phi}{\partial \zeta'^2}, \qquad (3)$$

where $K_0(x)$ is the modified Bessel function,

$$\lambda_J^2 = \frac{c\Phi_0}{16\pi^2 \lambda_{ab} J_c}$$

is the Josephson length, $\lambda_c = c/\omega_J\sqrt{\varepsilon}$ is the penetration depth of the magnetic field along layers, and Φ_0 is the flux quantum. Strictly speaking, Eq. (3) is derived for a JV moving in a weaker junction with a critical current $J_c^w < J_c$, but describes qualitatively even the stack of identical junctions (see the appendix). Equation (3) is reduced to the usual local sine-Gordon equation only for $\lambda_c \ll \lambda_J$, i.e., if the kernel K_0 in the integral in Eq. (3) is a sharper function of ζ' than $\partial^2\phi/\partial\zeta'^2$. For $Bi_2Sr_2CaCu_2O_{8+\delta}$ as well as for artificial layered superconductors, the opposite strongly nonlocal limit, $\lambda_c \gg \lambda_J$, is realized. In this case, a solitonlike solution,

$$\phi = \pi + 2\arctan\left(\frac{2x}{L}\right), \qquad (4)$$

of Eq. (3) was obtained in Ref. 24 for a fixed JV, $V=0$, with the soliton size

$$L = \frac{2\lambda_J^2}{\lambda_c} = \frac{\lambda_c D}{\lambda_{ab}} \ll \lambda_J. \qquad (5)$$

It is important to stress that the soliton size L (rather than λ_J) coincides with the well-known estimate of the JV core ($L=\gamma D$, $\gamma=\lambda_c/\lambda_{ab}$) in a layered superconductor. Analytical and numerical analysis proves that the size $L(V)$ of the moving JV remains of the same order at any allowed vortex velocities,

$$V < V_c \sim \omega_J L = \frac{cD}{\lambda_{ab}\sqrt{\varepsilon}}. \qquad (6)$$

Note that, due to nonlocality, the maximum vortex velocity V_c in a layered superconductor is much smaller than the maximum vortex velocity $c_{sw}=\lambda_J\omega_J$ (the so-called Swihart velocity) in a single junction.[25]

IV. JOSEPHSON PLASMA WAVES

In homogeneous ($\mu=0$) layered superconductors, the linearized coupled sine-Gordon equations admit wave solutions of the form,

$$\varphi_l = \varphi_0 \exp\{i[qx-\omega t + k(q,\omega) l D]\}, \qquad (7)$$

with the dispersion relation,

$$\sin^2\left(\frac{kD}{2}\right) = \frac{D^2}{4\lambda_{ab}^2}\left(\frac{c^2 q^2}{\varepsilon(\omega^2-\omega_J^2)} - 1\right), \qquad (8)$$

for the transverse wave vector $k(q,\omega)$. This relation coincides with the spectrum obtained in Ref. 18 in the particular limit where the breaking of the charge neutrality effect (that we can easily take into account) is neglected.

The electric and magnetic fields exhibit the same spatiotemporal dependence as in Eq. (7) for the phase difference φ_l, while their amplitudes, H_0, E_{x0}, and E_{z0}, are related to φ_0 via

$$\left[1 + \frac{4\lambda_{ab}^2}{D^2}\sin^2\left(\frac{kD}{2}\right)\right] H_0(q,\omega) = \frac{iq\Phi_0}{2\pi D}\varphi_0(q,\omega), \qquad (9)$$

$$E_{z0}(q,\omega) = -\frac{i\omega\Phi_0}{2\pi c D}\varphi_0(q,\omega), \qquad (10)$$

$$E_{x0}(q,\omega) = \frac{i\omega\lambda_{ab}^2}{cD}[1-\exp(-ikD)]H_0(q,\omega). \qquad (11)$$

According to Eq. (8), the JPW can propagate if $\omega > \omega_J$. For a homogeneous layered superconductor, JVs moving with constant velocity V can excite waves with $q=\omega/V$ (Cherenkov radiation[7,12–14]). Substituting $\omega=qV$ in Eq. (8) and noticing that the right-hand side of this equation is less than unity, we obtain the limiting vortex velocity

$$V_{\min} = \frac{cD}{2\lambda_{ab}\sqrt{\varepsilon}}. \qquad (12)$$

If the vortex velocity is larger than this threshold value, $V > V_{\min}$, then $\mathrm{Im}(k)=0$ and the *out-of-plane* waves can be excited. The characteristic angle θ of the propagating radiation (Cherenkov cone)

$$\tan\theta = \left(\frac{\pi\sqrt{V^2 - V_{\min}^2}}{D\omega_J}\right) \tag{13}$$

is determined by three standard conditions: (i) Eq. (8), (ii) $\omega = qV$, and (iii) the minimum wavelength $k^{-1} \lesssim D/\pi$. For $V = V_{\min}$ the Cherenkov cone is obviously closed, $\theta = 0$. At lower vortex velocities, $V < V_{\min}$, the Cherenkov radiation with $\mathrm{Im}(k) \neq 0$ (decaying from the central junction)[12] can propagate only along the *ab* plane[26] (see also Ref. 27 where the Cherenkov radiation in two coupled junctions is studied).

As was shown above, the maximum vortex velocity $V_c \sim \omega_J L$, at which the moving soliton is stable, is of the same order as $V_{\min}$. Below, the out-of-plane Cherenkov radiation generated by a vortex moving in a junction weaker than others is derived (while the question if such radiation exists in a system of identical junctions remains open[7]). A subset of weaker intrinsic Josephson junctions in $Bi_2Sr_2CaCu_2O_{8+\delta}$-based samples can be made using either (i) the controllable intercalation technique,[28] (ii) chemical vapor deposition (CVD) (see, e.g., Ref. 29), or (iii) via the admixture of $Bi_2Sr_2Cu_2O_{6+\delta}$ and $Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$.[30] Also, such a system can be easily created using artificial stacks of layers of low-temperature superconductors.

V. OUT-OF-PLANE CHERENKOV RADIATION

In order to find the relation between the amplitudes of the emitted waves inside a superconductor and the phase difference, ϕ, in the central junction, we use the standard equation,[24,25]

$$\frac{d\phi}{d\zeta} = \frac{8\pi^2\lambda_{ab}^2}{c\Phi_0}\{J_x^+(\zeta) - J_x^-(\zeta)\}, \tag{14}$$

where $J_x^\pm$ are the currents at the top and bottom edges of the central contact. Equation (14) is valid if $D \ll 2\lambda_{ab}$, that is, if the magnetic flux through the central junction is small compared to Φ_0. In other words, the gradient of the gauge-invariant phase difference ϕ along the central junction occurs due to the gradient of the phase $\chi^\pm$ of the superconducting order parameter in the layers forming the central junction rather than the trapped magnetic flux. Using Eq. (14) and the Maxwell equation $\partial H/\partial z = -4\pi J_x/c$, we obtain

$$H_0(q,\omega = qV) = -\frac{i\Phi_0 D q}{4\pi\lambda_{ab}^2\{1 - \exp[-ik(q,qV)D]\}}\phi(q), \tag{15}$$

where $\phi(q)$ is the Fourier transform of $\phi(\zeta)$.

When $\phi(q)$ is known, Eq. (15) determines the magnetic field distribution of the emitted out-of-plane Cherenkov radiation. For simplicity, we use the Fourier transform $\phi(q) = \phi_0(q)$ of the solution for a fixed vortex,

$$\phi(q) = -2\pi\frac{i\exp(-|q|L)}{q}. \tag{16}$$

Using this last equation and integrating $H_0(q,qV)\exp[iqx - iqVt + ik(q,qV)z]$ over $q_{\min} < q$ for the traveling out-of-plane waves, we derive the expression for the magnetic field H_{Cher} of the radiation,

$$H_{\mathrm{Cher}}(x,z,t) = \frac{i\Phi_0 D}{2\pi\lambda_{ab}^2}\int_{q_{\min}}^{\infty}\frac{dq}{1 - \exp[-ik(q)D]} \times\exp(-qL)\sin[q(x - Vt) + k(q,qV)|z|], \tag{17}$$

where

$$q_{\min} = \frac{\omega_J}{\sqrt{V^2 - V_{\min}^2}}. \tag{18}$$

The magnetic field distribution (17) in the emitted Cherenkov waves is shown in Fig. 1(a).

Due to the rather unusual dispersion relation (8), i.e., the decrease of $k(q,\omega = qV)$ with increasing q, as well as due to the spatial extension of a vortex, the generated electromagnetic waves are located *outside* the Cherenkov cone [Fig. 1(a)], which is drastically different from the Cherenkov radiation of a fast (pointlike) relativistic particle. The type of radiation predicted here could be called *outside-the-cone* Cherenkov radiation.

VI. IMPEDANCE MISMATCH

It is important to emphasize that a fast-moving vortex emits mostly JPWs with short out-of-plane wavelengths, i.e., with $k(q)$ about $k(q_{\min}) = \pi/D$ [see Fig. 1(a)]. According to Eq. (11), the ratio E_x/H is about

$$\beta = \frac{2\omega_J\lambda_{ab}^2}{cD} \sim 1.1 \tag{19}$$

for these short waves. For this estimate we use commonly accepted parameters for Bi2212 samples ($\omega_J/2\pi \sim 1$ THz, depending on doping and temperature, $\lambda_{ab} = 2000$ Å, and $D = 15$ Å). This result, unusual for conducting media, suggests that there is no impedance mismatch for the out-of-plane radiation.

However, as was mentioned above, the Cherenkov radiation can never pass through the sample boundary because it has a large longitudinal wave vector $q = \omega/V \gg \omega/c$. In order to decrease the longitudinal wave vector q, the spatial modulations of the critical current can be used. To analyze this analytically we use the perturbation expansion $H = H^{(0)} + \mu H^{(1)} + \cdots$, with $\mu(x) = \mu\cos(2\pi x/a)$, $\mu \ll 1$, in Eq. (1).

In zeroth-order approximation, at the sample boundary, the mode

$$H_0^+(x,z,t) = H_{0,0}^+\exp[iqx - iqVt + ik(q,qV)z], \tag{20}$$

generated by a vortex, completely reflects back to the superconductor sample as

$$H_0^-(x,z,t) = H_{0,0}^-\exp[iqx - iqVt - ik(q,qV)z], \tag{21}$$

generating the decaying wave in vacuum

$$H_{\mathrm{damp}}(x,z,t) = H_{\mathrm{vac}}^{\mathrm{damp}}\exp[iqx - iqVt - \kappa_v z] \tag{22}$$

with a damping coefficient

$$\kappa_v = q\sqrt{1-\frac{V^2}{c^2}}. \tag{23}$$

The amplitudes $H_{0,0}^{+}$, $H_{0,0}^{-}$, and $H_{\mathrm{vac}}^{\mathrm{damp}}$ of the modes are determined by the continuity of the electric and magnetic fields at the sample boundary.

To first approximation, using the linear relations between ϕ_l and H_l, as well as Eq. (1), we obtain the following equation:

$$\left(1-\frac{\lambda_{ab}^2}{D^2}\Delta_l\right)\left(\frac{\partial^2 H_l^{(1)}}{\partial t^2}+\omega_J^2 H_l^{(1)}\right)-\frac{c^2}{\varepsilon}\frac{\partial^2 H_l^{(1)}}{\partial x^2} = -\mu\cos\left(\frac{2\pi x}{a}\right)\left(1-\frac{\lambda_{ab}^2}{D^2}\Delta_l\right)\omega_J^2(H_0^{+}+H_0^{-}). \tag{24}$$

The solution of this equation consists of the sum of the following modes:

$$H_{n,m}^{\pm}(x,z,t) = H_{n,m}^{\pm}\exp\left[i\left(q-\frac{2\pi n}{a}\right)x - iqVt \pm ik\left(q-\frac{2\pi m}{a},qV\right)z\right], \tag{25}$$

where $H_{1,0}^{\pm}$ and $H_{-1,0}^{\pm}$ are forced waves and $H_{1,1}^{\pm}$ and $H_{-1,-1}^{\pm}$ are eigensolutions. The modes $H_{-1,0}^{\pm}$ are also reflected from the boundary, while the modes $H_{1,0}^{\pm}$ generate the wave

$$H_{\mathrm{vac}}^{\mathrm{prop}}\exp\left[i\left(q-\frac{2\pi}{a}\right)x - iqVt + ik_v z\right], \tag{26}$$

propagating in vacuum with wave vector

$$k_v = \sqrt{\frac{q^2V^2}{c^2}-\left(q-\frac{2\pi}{a}\right)^2}. \tag{27}$$

The continuity of E_x and H for the longitudinal wave vector $q-2\pi/a$ defines the amplitudes of the waves $H_{1,0}^{\pm}$, $H_{1,1}^{-}$, and $H_{\mathrm{vac}}^{\mathrm{prop}}$, as well as the transition coefficient T for the emitted waves from the sample,

$$T = \frac{E_{x\ \mathrm{vac}}^{\mathrm{prop}}}{H_{0,0}^{+}} = -\mu\beta\frac{v(v^2-1)p^2\sqrt{p^2-1}}{[1+v^2(p^2-1)]^2} \tag{28}$$

for $|p-2\pi/aq_{\min}|<V/c$ and $T=0$ otherwise. Here, $p=q/q_{\min}$ and $v=V/V_{\min}$ are the dimensionless longitudinal wave vector and vortex velocity.

Because of $\beta\sim 1$, the transmission coefficient $T\sim 1$ for out-of-plane radiation in a narrow frequency region

$$\Delta\omega \sim \frac{\omega_J V}{c} \ll \omega_J. \tag{29}$$

The quite narrow window of the transmitted waves occurs due to the broad spectrum,

$$\Delta\omega\sim\omega_J, \quad \Delta q_{\mathrm{Cherenkov}}\sim\Delta\omega/V, \tag{30}$$

of the Cherenkov radiation emitted by a Josephson vortex with respect to the spectrum of waves propagating in the vacuum,

$$\Delta q_{\mathrm{vac}} \sim \omega_J/c \ll \Delta q_{\mathrm{Cherenkov}}. \tag{31}$$

A periodic spatial modulation can shift all the wave vectors towards the spectral window of the vacuum waves, while it cannot affect the width of the spectrum. Thus, the surface cuts a narrow strip from the broad Cherenkov radiation, allowing these waves to pass the interface. Note that changing the period a (e.g., changing the distance between pancake vortices via the c-axis magnetic field) allows one to tune the frequency window for the emitted radiation. In order to make the frequency window $\Delta\omega$ wider, one can employ, e.g., appropriate aperiodic modulations $\mu(x)=\int_{\kappa_1}^{\kappa_2}\cos\kappa x\, d\kappa$ with κ_1 and κ_2 incommensurate. Also, $\mu(x)$ could be modulated as a one-dimensional (1D) quasicrystal.[31]

VII. CONCLUSIONS

We propose how to generate out-of-plane THz radiation in a controllable frequency range. We show that the standard severe mismatch problem can be overcome here for out-of-plane radiation using spatially modulated samples. Moreover, recent studies[32,33] of Josephson vortex arrangements in small samples (there, the interaction of vortices with sample boundaries acts similar to an additional potential) suggest a way to obtain a square vortex lattice, which is important for superradiance.[34] Thus, spatial modulations of J_c can result in a more ordered vortex flow or even a flowing square vortex lattice generating superradiance. Of course, this problem requires more detailed studies, which will be presented in the future.

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APPENDIX: DERIVATION OF THE NONLOCAL EQUATION FOR A JOSEPHSON VORTEX

We consider the distributions of the gauge-invariant phase differences and the magnetic field of a single vortex located in the central junction. We assume that the main phase difference is across this junction, whereas ϕ_l are small across other junctions. Such an approximation is, strictly speaking, correct for a weaker contact with $J_c^w<J_c$. This approach is also applicable for qualitative analysis for the set of identical junctions. In particular, it provides correct asymptotic behavior for ϕ_l when $l\gg 1$. Thus, we can use the linearized Eqs. (1) for all junctions except the junction where a JV is localized. It is important to stress that the magnetic field and the corresponding phase difference distributions generated by the vortex are extended over many junctions in the layered medium. Therefore, we can use Eqs. (1) in the continuum form, which reads

$$\left(1-\lambda_{ab}^2\frac{\partial^2}{\partial z^2}\right)\left(\omega_J^2+V^2\frac{\partial^2}{\partial\zeta^2}\right)\phi(\zeta,z)-\frac{c^2}{\varepsilon}\frac{\partial^2\phi}{\partial\zeta^2}=0. \tag{A1}$$

Using the Fourier transform

$$\phi(q,z) = \int_{-\infty}^{\infty} \mathrm{d}\zeta \exp(-iq\zeta)\phi(\zeta,z), \tag{A2}$$

we obtain the solution of Eq. (A1),

$$\phi(q,y) = \phi(q,0)\exp[\mathrm{i}\ \mathrm{sgn}(q)k(q)|z|], \tag{A3}$$

where $\mathrm{sgn}(q)=1$ if $q>0$ and -1 if $q<0$, the wave vector $k(q)$ is defined by Eq. (8) for $kD \ll 1$, $\omega = qV$, and $V \ll c/\varepsilon$

$$k(q) = \frac{1}{\lambda_{ab}}\left(\frac{\omega_J^2 + c^2q^2/\varepsilon}{q^2V^2 - \omega_J^2}\right)^{1/2}. \tag{A4}$$

Using the Maxwell equation and considering the Josephson and displacement currents, we derive relation between H and ϕ,

$$-\frac{\partial H}{\partial \zeta} = \frac{4\pi}{c}\left(J_c\phi + \frac{\hbar\varepsilon V^2}{8\pi e s}\frac{\partial^2\phi}{\partial\zeta^2}\right). \tag{A5}$$

The Fourier transform of Eq. (A5) results in

$$H(q,z) = -\frac{8\pi i}{cq}\left(J_c - \frac{\hbar\varepsilon V^2 q^2}{8\pi e s}\right)\phi(q,z). \tag{A6}$$

Thus, the dependence of the magnetic field $H(q,z)$ on the transverse coordinate z obeys the same law as the phase difference $\phi(z)$,

$$H(q,z) = H(q,0)\exp[i\ \mathrm{sgn}(q)k(q)|z|]. \tag{A7}$$

Next, using Eq. (14) and the Maxwell equation $\partial H/\partial z = -4\pi/cJ_x$, we obtain the relation between Fourier components of the magnetic field and the phase difference ϕ,

$$H(q,0) = -\frac{\Phi_0}{4\pi\lambda_{ab}^2}\frac{q}{k(q)}\phi(q). \tag{A8}$$

In order to express the z component of the current in layered media in terms of the phase difference ϕ in the central junction, we use Eqs. (A4) and (A6)–(A8). Performing the reverse Fourier transformation, we obtain

$$J_z(\zeta,z) = \frac{c\Phi_0}{16\pi^2\lambda_{ab}}\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{2\pi}q^2\phi(q) \times \sqrt{\frac{\omega_J^2 - q^2V^2}{\omega_J^2 + q^2c^2/\varepsilon}}\exp(iq\zeta + i\ \mathrm{sgn}(q)k(q)|z|). \tag{A9}$$

Below we consider the case where the main contribution to the integral in Eq. (A9) comes from the region

$$q^2 \ll \frac{\omega_J^2}{V^2}, \tag{A10}$$

which is valid if the vortex velocity is smaller than $\lambda_J^2\omega_J/\lambda_c$.

Substituting instead of $\phi(q)$ its coordinate Fourier transform $\phi(\zeta')$, we find the expression for the current component J_z at the edge of the central junction, $z=0$. Performing the integration over q and taking into account the inequality Eq. (A10), one gets

$$J_z(\zeta,0) = \frac{c\Phi_0}{16\pi^3\lambda_{ab}\lambda_c}\int_{-\infty}^{\infty}\mathrm{d}\zeta' K_0\left(\frac{|\zeta'-\zeta|}{\lambda_c}\right)\frac{\partial^2\phi}{\partial\zeta'^2}, \tag{A11}$$

where $K_0(x)$ is the modified Bessel function of the zero order. Equating the current Eq. (A11) to the sum of Josephson and displacement currents in the central junction, we obtain the nonlocal sine-Gordon equation for the fluxon presented in the text. Note that the obtained equation for ϕ is similar to the nonlocal sine-Gordon equation obtained by Gurevich for the fluxon in a single Josephson junction between isotropic superconductors.[24]

[1] See, e.g., the special issue of Philos. Trans. R. Soc. London **362**, No. 1815 (2004); Proceedings of the first international conference on biomedical imaging and sensing applications of THz technology, edited by M. Chamberlain and M. Smith [Phys. Med. Biol. 47, No. 21 (2002)]; M. C. Beard, G. M. Turner, and C. A. Schmuttenmaer, J. Phys. Chem. B **106**, 7146 (2002); D. M. Mittleman, M. Gupta, R. Neelamani, R. G. Baraniuk, J. V. Rudd, and M. Koch, Appl. Phys. B **68**, 1085 (1999); R. Köhler, A. Tredicucci, F. Beltram, H. E. Beere, E. H. Linfield, A. G. Davies, D. A. Ritchie, R. C. Iotti, and F. Rossi, Nature **417**, 156 (2002); H. Murakami, Y. Tominari, M. Tonouchi, T. Uchiyama, I. Iguchi, and Z. Wang, Jpn. J. Appl. Phys., Part 1 **44**, 2998 (2005); Y. Tominari, T. Kiwa, H. Murakami, M. Tonouchi, H. Wald, and P. Seidel, Appl. Phys. Lett. **80**, 3147 (2002); H. Murakami, T. Kiwa, N. Kida, M. Tonouchi, T. Uchiyama, I. Iguchi, and Z. Wang, Europhys. Lett. **60**, 288 (2002).

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