Preparation of macroscopic quantum superposition states of a cavity field via coupling to a superconducting charge qubit

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(Received 15 June 2004; published 28 June 2005)

We propose how to generate macroscopic quantum superposition states using a microwave cavity containing a superconducting charge qubit. Based on the measurement of charge states, we show that the superpositions of two macroscopically distinguishable coherent states of a single-mode cavity field can be generated by a controllable interaction between a cavity field and a charge qubit. After such superpositions of the cavity field are created, the interaction can be switched off by the classical magnetic field, and there is no information transfer between the cavity field and the charge qubit. We also discuss the generation of the superpositions of two squeezed coherent states.

DOI: 10.1103/PhysRevA.71.063820 PACS number(s): 42.50.Dv, 42.50.Ct, 74.50.+r

I. INTRODUCTION

The principle of linear superposition is central to quantum mechanics. However, it is difficult to create and observe superposed states because the fragile coherence of these states can be easily spoiled by the environment. Typical examples are the macroscopic quantum superposition states [Schrödinger cat states (SCS's)] [1]. Many theoretical schemes [2] have been proposed to generate SCS's and superpositions of macroscopic states (SMS) in optical systems. Also, much experimental progress has been made to demonstrate SMS and SCS's: in superconducting systems (e.g., Ref. [3]), laser-trapped ions [4], optical systems constructed by Rydberg atoms, and superconducting cavities in the microwave regime [1,5]. The SMS, which are formed by two optical coherent states—e.g., in Ref. [6]—have been investigated for applications in quantum information processing [6–8]. These states can be used as a robust qubit encoding for a single bosonic mode subject to amplitude damping. They can also be used to study both the measurement process and decoherence by coupling the system to the external environment [6–8]. Thus, generating and measuring SMS and SCS's are not only important to understand fundamental physics, but also to explore potential applications.

Superconducting quantum devices [9–13] allow one to perform quantum-state engineering including the demonstration of SCS's and SMS. Theoretical schemes to generate superpositions that are different from the above experiments [3] have also been proposed [14–16] in superconducting quantum devices. For example, the scheme in Ref. [14] generates superpositions of Bloch states for the current of a Josephson junction. Marquardt and Bruder [15] proposed ways to create SMS for a harmonic oscillator approximated by a large superconducting island capacitively coupled to a smaller Cooper-pair box. Armour *et al.* [16] proposed a similar scheme as in Ref. [15] but using a micromechanical resonator as the harmonic oscillator. A review paper on micromechanical resonators [17] can be found in Ref. [18]. In Ref. [19]; a scheme was proposed to generate SMS and squeezed states for a superconducting quantum interference device (SQUID) ring modeled as an oscillator. Since then, several proposals have been made which focus on superconducting qubits interacting with the nonclassical electromagnetic field [20–26].

Optical states allow a fast and convenient optical transmission of the quantum information [30] which is stored in charge qubits. Compared with the harmonic system [15,16] formed by the large superconducting junction and the micromechanical resonator, optical qubits can easily fly relatively long distances between superconducting charge qubits. Moreover, the qubit formed by SMS enables a more efficient error correction than that formed by the single-photon and vacuum states, and the generation and detection of coherent light are easy to be implemented.

In contrast to [15,16], here we aim at generating SCS's in the interaction system between a single-mode microwave cavity field and a superconducting charge qubit, and then creating SMS by virtue of the measurements of the charge states. The generation of such states has been studied theoretically [27] and demonstrated in optical cavity QED experiments [5]. However, in these cases (i) several operations are needed because atoms must pass through three cavities and (ii) the interaction times are tuned by the controlling velocity of the atoms flying through the cavity. In our proposal, we need only one cavity, and interaction times are controlled by changing the external magnetic field.

Although our scheme is similar to that proposed in Ref. [16], the interaction between the box and resonator in Ref. [16] is not switchable. Due to the fixed coupling in Ref. [16], the transfer of information between the micromechanical resonator and the box still exists even after the SCS's or SMS are produced. In our proposal, the interaction between the cavity field and the qubit can be switched off by a classical magnetic field after the SCS's or SMS are generated.

Furthermore, three operations, with different approximations made in every operation, are required in Ref. [16]. In addition, in order to minimize the environmental effect on the prepared state, the number of operations and instruments should be as small as possible; *one* operation is enough to generate SCS's or SMS in our proposal. Thus our proposed scheme offers significant advantages over the pioneering proposals in Refs. [15,16].

II. MODEL

We consider a SQUID-type qubit superconducting box with n excess Cooper-pair charges connected to a superconducting loop via two identical Josephson junctions with capacitors C_{J} and coupling energies E_{J}. A controllable gate voltage V_{g} is coupled to the box via the gate capacitor C_{g} with dimensionless gate charge $n_{\mathrm{g}}=C_{\mathrm{g}}V_{\mathrm{g}}/2e$. The qubit is assumed to work in the charge regime with $k_{\mathrm{B}}T \ll E_{\mathrm{J}} \ll E_{\mathrm{C}} \ll \Delta$, where k_{B}, T, E_{C}, and Δ are the Boltzmann constant, temperature, charge, and superconducting gap energies, respectively. For known charge qubit experiments—e.g., in Ref. [10]—$T\sim 30$ mK which means $k_{\mathrm{B}}T\sim 3\ \mu$eV, $E_{\mathrm{J}} \sim 51.8\ \mu$eV, $E_{\mathrm{C}}\sim 117\ \mu$eV, and $\Delta \sim 230\ \mu$eV, so the above inequalities are experimentally achievable. We consider a gate voltage range near a degeneracy point $n_{\mathrm{g}}=1/2$, where only two charge states, called $n=0$ and $n=1$, play a leading role. The other charge states with a much higher energy can be neglected, which implies that the superconducting box can be reduced to a two-level system or qubit [28]. This superconducting two-level system can be represented by a spin-$\frac{1}{2}$ notation such that the charge states $n=0$ and $n=1$ correspond to eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of the spin operator σ_z, respectively. If such a qubit is placed into a single-mode superconducting cavity, the Hamiltonian can be written as [22,29]

$$H = \hbar\omega a^{\dagger}a + E_z\sigma_z - E_{\mathrm{J}}\sigma_x \cos\frac{\pi}{\Phi_0}(\Phi_c I + \eta a + \eta^* a^{\dagger}), \quad (1)$$

where the first term represents the free Hamiltonian of the single-mode cavity field with frequency ω and $E_z=-2E_{\mathrm{ch}}(1-2n_{\mathrm{g}})$ with the single-electron charging energy $E_{ch}=e^2/(C_{\mathrm{g}}+2C_{\mathrm{J}})$. Here Φ_0 is the flux quantum, Φ_c is the flux generated by the classical magnetic field through the SQUID, and I is the identity operator. The last term in Eq. (1) is the nonlinear photon-qubit interaction. The parameter η has units of magnetic flux and its absolute value represents the strength of the quantum flux inside the cavity. We will later on assume this "quantum magnetic flux" η to be small, becoming our perturbation parameter. The parameter η can be written as

$$\eta = \int_S \mathbf{u}(\mathbf{r}) \cdot d\mathbf{s}, \quad (2)$$

where $\mathbf{u}(\mathbf{r})$ is the mode function of the single-mode cavity field, with annihilation (creation) operators $a(a^{\dagger})$, and S is the surface defined by the contour of the SQUID. For convenience, hereafter, we denote $|\downarrow\rangle$ and $|\uparrow\rangle$ by $|e\rangle$ and $|g\rangle$, respectively. The cosine in Eq. (1) can be decomposed into classical and quantized parts; Eq. (1) can then be expressed as

$$H = \hbar\omega a^{\dagger}a - E_{\mathrm{J}}\sigma_x \cos\left(\frac{\pi\Phi_c}{\Phi_0}\right)\cos\frac{\pi}{\Phi_0}(\eta a + \eta^* a^{\dagger}) + E_z\sigma_z + E_{\mathrm{J}}\sigma_x \sin\left(\frac{\pi\Phi_c}{\Phi_0}\right)\sin\frac{\pi}{\Phi_0}(\eta a + \eta^* a^{\dagger}). \quad (3)$$

The factors $\sin[\pi(\eta a+\mathrm{H.c.})/\Phi_0]$ and $\cos[\pi(\eta a+\mathrm{H.c.})/\Phi_0]$ can be further expanded as a power series in $a(a^{\dagger})$. For the single-photon transition between the states $|e,n\rangle$ and $|g,n+1\rangle$, if the condition

$$\frac{\pi|\eta|}{\Phi_0}\sqrt{n+1} \ll 1 \quad (4)$$

is satisfied, all higher orders of $\pi|\eta|/\Phi_0$ can be neglected in the expansion of Eq. (3). To estimate the interaction coupling between the cavity field and the qubit, we assume that the single-mode cavity field is in a standing-wave form

$$B_x = -i\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}(a - a^{\dagger})\cos(kz),$$

where V, ε_0, c, and k are the volume of the cavity, permittivity of the vacuum, light speed, and wave vector of the cavity mode, respectively. Because the superconducting microwave cavity is assumed to only contain a single mode of the magnetic field, the wave vector $k=2\pi/\lambda$ is a constant for the given cavity. The magnetic field is assumed to propagate along the z direction and the polarization of the magnetic field is along the normal direction of the surface area of the SQUID. If the area of the SQUID is, e.g., of the order of 100 $(\mu\mathrm{m})^2$, then its linear dimension—e.g., approximately of the order of 10 μm—should be much less than the microwave wavelength of the cavity mode. Thus, the mode function $u(\mathbf{r})$ can be considered to be approximately independent of the integral area and the factor $\cos(kz)$ only depends on the position z_0 where the qubit is located. So the parameter η can be expressed as

$$|\eta| = S\sqrt{\frac{\hbar\omega}{\varepsilon_0 V c^2}}|\cos(kz_0)|,$$

which shows that the parameter $|\eta|$ depends on the area S and the position z_0 of the SQUID, the wavelength λ of cavity field, and the volume V of the cavity. It is obvious that a larger S for the SQUID corresponds to a larger $|\eta|$. If the SQUID is placed in the middle of a cavity with full wavelength, that is, $z_0=L/2=\lambda/2$. Then $kz_0=(2\pi/\lambda)(\lambda/2)=\pi$, the interaction between the cavity field and the qubit reaches its maximum, and

$$3.28\times 10^{-9} \leqslant \pi|\eta|/\Phi_0 \leqslant 7.38\times 10^{-5} \ll 1 \quad (5)$$

in the microwave region with 15 cm$\geqslant\lambda\geqslant$0.1 cm. For a half- or quarter-wavelength cavity, the condition $\pi|\eta|/\Phi_0 \ll 1$ can also be satisfied. Therefore, the approximation in Eq. (4) can be safely made in the microwave regime, and then Eq. (3) can be further simplified (up to first order in $\xi=\pi\eta/\Phi_0$) as

$$H_1 = \hbar\omega a^\dagger a + E_z\sigma_z - E_J\sigma_x \cos\left(\frac{\pi\Phi_c}{\Phi_0}\right) + E_J\sigma_x \sin\left(\frac{\pi\Phi_c}{\Phi_0}\right)(\xi a + \xi^* a^\dagger), \tag{6}$$

where ξ is a dimensionless complex number with its absolute value equal to the dimensionless quantum magnetic flux, and it is defined by

$$\xi = \frac{\pi}{\Phi_0}\int_S \mathbf{u}(\mathbf{r})\cdot d\mathbf{s} = \frac{\pi}{\Phi_0}\eta. \tag{7}$$

III. GENERATION OF CAT STATES

We assume that the qubit [30] is initially in the ground state $|g\rangle=(|+\rangle+|-\rangle)/2$ where $|+\rangle(|-\rangle)$ is eigenstate of the Pauli operator σ_x with the eigenvalue $1(-1)$. The cavity field is assumed initially in the vacuum state $|0\rangle$. Now let us adjust the gate voltage V_g and classical magnetic field such that $n_g=1/2$ and $\Phi_c=\Phi_0/2$, and then let the whole system evolve a time interval τ. The state of the qubit-photon system evolves into

$$\begin{aligned}|\psi(\tau)\rangle &= \exp\{-i[\omega a^\dagger a + \sigma_x(\Omega^* a + \Omega a^\dagger)]\tau\}|0\rangle|g\rangle \\ &= \frac{1}{2}[A(\Omega)|0\rangle|+\rangle + A(-\Omega)|0\rangle|-\rangle] \\ &= \frac{1}{2}(|\alpha\rangle + |-\alpha\rangle)|g\rangle + \frac{1}{2}(|\alpha\rangle - |-\alpha\rangle)|e\rangle,\end{aligned} \tag{8}$$

where the complex Rabi frequency $\Omega=\xi^* E_J/\hbar$, $A(\pm\Omega)=\exp\{-i[\omega a^\dagger a \pm (\Omega^* a+\Omega a^\dagger)]\tau\}$, and a global phase factor $\exp[-i(\xi^* E_J/\hbar\omega)^2 \sin(\omega t)+i\xi^{*2}E_J^2 t/\hbar^2\omega]$ has been neglected. $|\pm\alpha\rangle$ denotes coherent state

$$|\pm\alpha\rangle \equiv e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{(\pm\alpha)^n}{\sqrt{n!}}|n\rangle, \tag{9}$$

with

$$\alpha = \frac{\xi^* E_J}{\hbar\omega}(e^{-i\omega\tau}-1).$$

In the derivation of Eq. (8), we use the formula $\exp[\theta(\beta_1 a+\beta_2 a^\dagger a+\beta_3 a^\dagger)]=\exp[f_1 a^\dagger]\exp[f_2 a^\dagger a]\exp[f_3 a]\exp[f_4]$ with the relations $f_1=\beta_3(e^{(\beta_2\theta)}-1)/\beta_2$, $f_2=\beta_2\theta$, $f_3=\beta_1(e^{(\beta_2\theta)}-1)/\beta_2$, and $f_4=\beta_1\beta_3(e^{(\beta_2\theta)}-\beta_2\theta-1)/\beta_2^2$. After the time interval τ, we impose $\Phi_c=0$ by adjusting the classical magnetic field; thus, the interaction between the charge qubit and the cavity field is *switched off* [e.g., the last term in Eq. (6) vanishes]. Equation (8) shows that entanglement of the qubit and the microwave cavity field can be prepared for an evolution time $\tau\neq 2m\pi$ with the integer number m; then, Schrödinger cat states can be created [1]. If the condition $e^{-i\omega\tau}\neq 1$ is satisfied in Eq. (8), the SMS [31] of the cavity field denoted by $|\text{sms}\rangle_\pm$,

$$|\text{sms}\rangle_\pm = \frac{1}{\sqrt{2\pm e^{-2|\alpha|^2}}}(|\alpha\rangle \pm |-\alpha\rangle), \tag{10}$$

can be obtained by measuring the charge state $|e\rangle$ or $|g\rangle$, by using, for example, a single-electron transistor (SET).

If we initially inject a coherent light $|\alpha'\rangle$, then by using the same method as in the derivation of Eq. (8), we can also obtain the entanglement of two different optical coherent states $|\alpha_\pm\rangle$ and qubit states with the evolution time τ_1:

$$|\varphi(\tau_1)\rangle = \frac{1}{2}[\exp(i\varphi)|\alpha_+\rangle + \exp(-i\varphi)|\alpha_-\rangle]|g\rangle + \frac{1}{2}[\exp(i\varphi)|\alpha_+\rangle - \exp(-i\varphi)|\alpha_-\rangle]|e\rangle, \tag{11}$$

where

$$\varphi = \text{Im}\left[\frac{\xi E_J}{\hbar\omega}\alpha'(1-e^{i\omega t})\right]$$

and

$$\alpha_\pm = \alpha' e^{(-i\omega\tau_1)} \pm \kappa[1-e^{(-i\omega\tau_1)}],$$

with

$$\kappa = \frac{\xi^* E_J}{\hbar\omega}.$$

After a time interval τ_1, we can switch off the interactions between the charge qubit and the cavity field by setting $\Phi_c=0$ and $n_g=1/2$. Measuring the charge states, we can obtain another SMS denoted by $|\text{SMS}\rangle$

$$|\text{SMS}\rangle = N_\pm^{-1}(e^{i\varphi}|\alpha_+\rangle \pm e^{-i\varphi}|\alpha_-\rangle),$$

with normalized constant

$$N_\pm = \sqrt{2\pm(e^{-i2\varphi}\langle\alpha_+|\alpha_-\rangle + e^{i2\varphi}\langle\alpha_-|\alpha_+\rangle)},$$

where $\langle\alpha_\mp|\alpha_\pm\rangle$ can be easily obtained [32] by the above expression of $\alpha_\pm$—for example,

$$\langle\alpha_+|\alpha_-\rangle = \exp\{-4\kappa^2[1-\cos(\omega\tau_1)] - i2\kappa\alpha'\sin(\omega\tau_1)\};$$

here, we assume that the injected coherent field has a real amplitude α'. In Eq. (11), we entangle two different superpositions of coherent states with the ground and excited states of the qubit. We can also entangle two different coherent states $|\alpha_\pm\rangle$ with the qubit states by applying a classical flux such that $\Phi_c=\Phi_0$. Then with the time evolution $t=\pi/4E_J$, we have

$$|\psi(\tau_1)\rangle = \frac{1}{2}(e^{-i\varphi}|\alpha_-\rangle|g\rangle + e^{i\varphi}|\alpha_+\rangle|e\rangle). \tag{12}$$

It should be noticed that a global phase factor $\exp[-i(\xi^* E_J/\hbar\omega)^2\sin(\omega t)+i\xi^{*2}E_J^2 t/\hbar^2\omega]$ has been neglected in Eqs. (11) and (12).

From a theoretical point of view, if we can keep the expansion terms in Eq. (3) up to second order in $\xi=\pi\eta/\Phi_0$, we can also *prepare a superposition of two squeezed coherent states, which could be used to encode an optical qubit* [31]. To obtain this superposition of two squeezed coherent states,

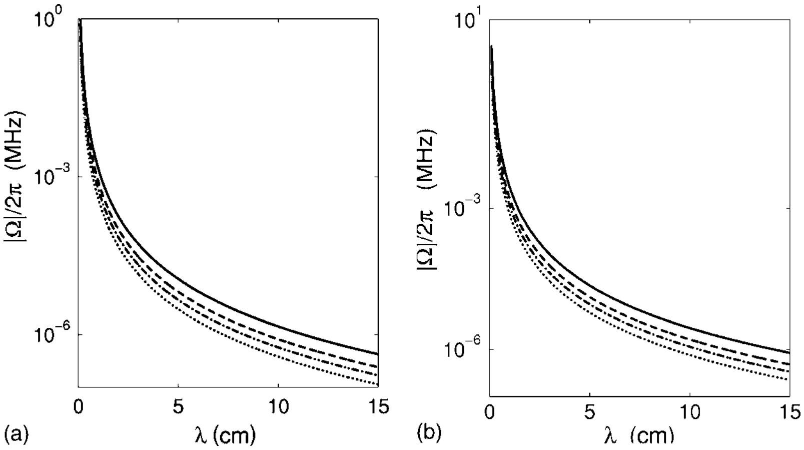

FIG. 1. Rabi frequency $|\Omega|$ versus the microwave wavelength λ for a full-wavelength cavity (a) and a quarter-wavelength cavity (b) with ratios $E_{\mathrm{ch}}/E_{\mathrm{J}}=4$ (top solid line), 7 (dashed line), 10 (dash-dotted line), and 15 (bottom dotted line), respectively.

we can set $n_g=1/2$ and $\Phi_c=0$, and derive the Hamiltonian from Eq. (3) to get (up to second order in ξ)

$$H_2=(\hbar\omega-|\xi|^2E_{\mathrm{J}}\sigma_x)a^\dagger a-E_{\mathrm{J}}\left(1+\frac{|\xi|^2}{2}\right)\sigma_x -E_{\mathrm{J}}\sigma_x\left(\frac{\xi^2}{2}a^2+\frac{\xi^{*2}}{2}a^{\dagger 2}\right). \tag{13}$$

If the system is initially in the coherent state $|\gamma\rangle$ and if the charge qubit is in the ground state $|g\rangle$, we can entangle qubit states with superpositions of two different squeezed coherent states with an evolution time t as

$$|\psi(t)\rangle=\frac{1}{2}\left[e^{-i\theta t}\left|\gamma,-i\frac{\xi^{*2}E_{\mathrm{J}}}{\hbar}t\right\rangle+e^{i\theta t}\left|\gamma,i\frac{\xi^{*2}E_{\mathrm{J}}}{\hbar}t\right\rangle\right]|g\rangle +\frac{1}{2}\left[e^{-i\theta t}\left|\gamma,-i\frac{\xi^{*2}E_{\mathrm{J}}}{\hbar}t\right\rangle-e^{i\theta t}\left|\gamma,i\frac{\xi^{*2}E_{\mathrm{J}}}{\hbar}t\right\rangle\right]|e\rangle, \tag{14}$$

where

$$\theta=E_{\mathrm{J}}\left(1+\frac{|\xi|^2}{2}\right), \tag{15a}$$

$$\left|\gamma,\mp i\frac{\xi^{*2}E_{\mathrm{J}}}{\hbar}t\right\rangle=U_{\pm}(t)|\gamma\rangle, \tag{15b}$$

and

$$U_{\pm}(t)=\exp\left\{-it\left(\omega\mp\frac{|\xi|^2E_{\mathrm{J}}}{\hbar}\right)a^\dagger a\right\} \times\exp\left\{\mp i\frac{E_{\mathrm{J}}}{\hbar}\left(\frac{\xi^2}{2}a^2+\frac{\xi^{*2}}{2}a^{\dagger 2}\right)t\right\}. \tag{15c}$$

Here, $|\gamma,\mp i\xi^{*2}E_{\mathrm{J}}t/\hbar\rangle$ denote squeezed coherent states, and the degree of squeezing [33,34] is determined by the time-dependent parameter $|\xi|^2E_{\mathrm{J}}t/\hbar$. A superposition of two squeezed coherent states can be obtained by the measurement on the charge qubit. However, we should note that if we keep to first order in $|\xi|=\pi|\eta|/\Phi_0$ the expansions of Eq. (3), the interaction between the cavity field and the charge qubit is switchable [e.g., the last term in Eq. (6) vanishes for $\Phi_c=0$]. But if we keep terms up to second order in $|\xi|$ for the expansions of Eq. (3), then the qubit-field coupling is not switchable.

IV. DISCUSSIONS

Our analytical expressions show how to prepare the Schrödinger states for the system of the microwave cavity field and the superconducting charge qubit; we further show that the superpositions of two macroscopically distinguishable states can also be created by measuring the charge states. However, similarly to optical cavity QED [27], prepared superpositions of states are limited by the following physical quantities: the Rabi frequency $|\Omega|=|\xi|E_{\mathrm{J}}$ (which determines the quantum operation time t_q of two charge qubit states through the cavity field), the lifetime t_{d} of the cavity field, and the lifetime T_1 and dephasing time T_2 of the charge qubit, as well as the measurement time τ_{m} on the charge qubit.

We now estimate the Rabi frequency $|\Omega|$ in the microwave regime for a standing-wave field in the cavity. A SQUID with an area of about 100 $(\mu\mathrm{m})^2$ is assumed to be placed in the middle of the cavity. In the microwave regime with different ratios of $E_{\mathrm{ch}}/E_{\mathrm{J}}$, we provide a numerical estimate of $|\Omega|/2\pi$ for $\omega=4E_{\mathrm{ch}}/\hbar$ in a full-wavelength cavity, shown in Fig. 1(a), and a quarter-wavelength cavity, shown in Fig. 1(b). The results reveal that a shorter wavelength of cavity field corresponds to a larger Rabi frequency $|\Omega|$. For example, in the full-wavelength cavity and the case of the ratio $E_{\mathrm{ch}}/E_{\mathrm{J}}=4$, $|\Omega|/2\pi$ with microwave length 0.1 cm is of the order of 10^6 Hz, and yet it is about 10 Hz for a microwave wavelength of 5 cm. In both cases, the transition times from $|0\rangle|e\rangle$ to $|1\rangle|g\rangle$ are about 10^{-6} s and 0.1 s, respectively, where $|0\rangle$ ($|1\rangle$) is the vacuum (single-photon) state. The experiment for this scheme should be easier for shorter wavelengths than for long wavelengths. Since the cavity field has higher energy for the shorter wavelength, so it is better to choose the material with a larger superconducting energy gap to make the Josephson junction for the experiment in the region of the shorter microwave wavelengths. For a fixed wavelength, the effect of the ratios $E_{\mathrm{ch}}/E_{\mathrm{J}}$ on the coupling between the cavity field and the charge qubit is not so large. However, decreasing the volume V of the cavity can also increase the coupling.

In order to obtain a SMS, the readout time τ_m of the charge qubit should be less than the dephasing time T_2 of the charge qubit (because the relaxation time T_1 of the charge qubit is longer than its dephasing time T_2) and the lifetime time t_d of the cavity field. For example, in Ref. [16] with a set of given parameters, the estimated time, $\tau_m = 4$ ns, is less than $T_2 = 5$ ns [10]. For a good cavity [35], the quality factor Q can reach very high values, such as $Q = 3 \times 10^8$, and then the lifetimes of the microwave field would be in the range $0.001\ \mathrm{s} \leqslant 2\pi t_d \leqslant 0.15\ \mathrm{s}$, which implies $t_m \ll t_d$. So the readout is possible within current technology. It is easier to prepare a SMS in such a system even when the coupling between the charge qubit and the cavity field is weak because, in principle, two different coherent states could be obtained with a very short time t_q such that $t_q \ll T_2$.

V. CONCLUSIONS

In conclusion, we have analyzed the generation of Schrödinger cat states via a controllable superconducting charge qubit. Based on our scheme, the SMS can be created by using one quantum operation together with the quantum measurements on the charge qubit. After the SCS's or SMS are created, the coupling between the charge qubit and the cavity field can be switched off, in principle. Because all interaction terms of higher order in $\xi = \pi\eta/\Phi_0$ are negligible for the coupling constant, $|\xi| = \pi|\eta|/\Phi_0 \ll 1$. This results in a switchable qubit-field interaction. This means *sudden switching* of the flux on time scales of the inverse Josephson energy ($>$GHz). At present this is difficult but could be realized in the future.

We have also proposed a scheme to generate superpositions of two squeezed coherent states if we can keep the expansion terms in Eq. (3) up to second order in $\xi = \pi\eta/\Phi_0$. However, in this case the interaction between the cavity field and the charge qubit cannot be switched off. By using the same method employed for trapped ions [36], we can measure the decay rate of the SMS and obtain the change of the Q value due to the presence of the SQUID.

Also, the generated SMS can be used as a source of optical qubits. Our suggestion is that the first experiment for generating nonclassical states via the interaction with the charge qubit should be the generation of superpositions of two macroscopically distinct coherent states. It needs only one quantum operation, and the condition for the coupling between the cavity field and the charge qubit can be slightly relaxed. This proposal should be experimentally accessible in the near future.

ACKNOWLEDGMENTS

We acknowledge comments on the manuscript from X. Hu, S.C. Bernstein, and C.P. Sun. This work was supported in part by the National Security Agency (NSA), Advanced Research and Development Activity (ARDA) under Air Force Office of Research (AFOSR) Contract No. F49620-02-1-0334, and by National Science Foundation Grant No. EIA-0130383.

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